Extension of Single Phase Surface Tension Model to Variable Resolution SPH

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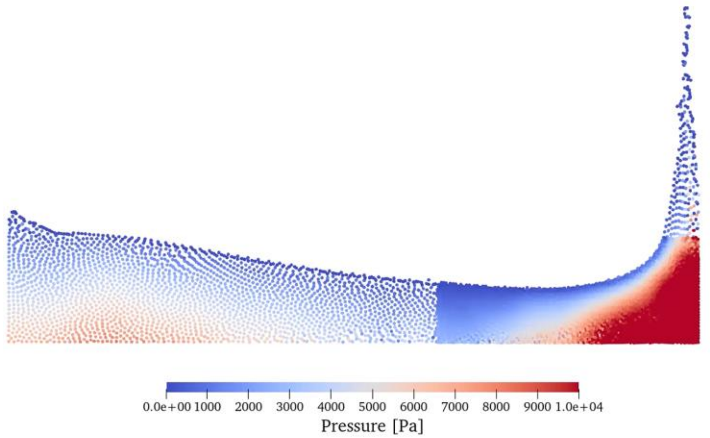
Motivation

- Bond number (Bo) = $\frac{\rho g L^2}{\sigma}$

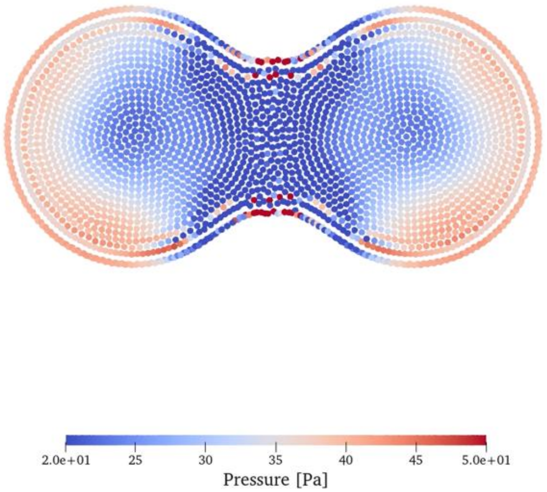
  ρ: density, σ: surface tension coefficient, L: characteristic length
  - $Bo \gg 1$: Gravity dominated flows
  - $Bo \ll 1$: Surface tension dominated flows

- Surface tension dominated flows require fine resolution and small time step size.
  → High computation cost!

- Therefore, variable resolution SPH is a reasonable and efficient strategy for surface tension dominant regimes.



Dam-break simulation with variable resolution scheme



Droplet coalescence simulation

Surface Tension Model

1. Computing a correction matrix and a minimum eigenvalue

$$L_i^{-1} = \sum_j (r_j - r_i) \otimes \nabla_i W_{ij} V_j$$

$$\lambda_i = \begin{cases} \text{Solving quadratic equation} & \text{in } 2D \\ \text{Jacobi rotation method} & \text{in } 3D \end{cases}$$

2. Computing a surface normal vector

$$\nabla\lambda_i = \begin{cases} \sum_j (\lambda_j - \lambda_i)(L_i \nabla W_{ij}) V_j & \text{if } \lambda_i \geq \lambda_{threshold} \\ \sum_j \lambda_j (L_i \nabla W_{ij}) V_j & \text{otherwise} \end{cases} \qquad (\lambda_{threshold} = 0.7)$$

$$\hat{\boldsymbol{n}}_i = \begin{cases} -\dfrac{\nabla\lambda_i}{\|\nabla\lambda_i\|} & \text{if } \|\nabla\lambda_i\| > \epsilon \dfrac{\lambda_i}{h} \\ 0 & \text{otherwise,} \end{cases} \qquad (\epsilon = 0.1)$$

3. Computing a curvature

$$\kappa_i = \sum_j S_{ij} (\hat{\boldsymbol{n}}_j - \hat{\boldsymbol{n}}_i) \cdot (L_i \nabla \mathrm{W_{ij}}) \mathrm{V_j}$$

$$S_{ij} = \begin{cases} 1 & \text{if } \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j \geq \cos\alpha_t \text{ and } \|\hat{\boldsymbol{n}}_i\| > 0 \text{ and } \|\hat{\boldsymbol{n}}_j\| > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\cos\alpha_t = \begin{cases} -\dfrac{1}{2} & \text{in } 2D \\ -\dfrac{1}{3} & \text{in } 3D \end{cases}$$

4. Computing a surface tension force

$$\phi = \frac{1}{2\sum_j W_{ij} V_j} \qquad \delta_i = 2\max(1, \phi_i) \left\| \sum_j \nabla W_{ij} V_j \right\|$$

$$\boldsymbol{F}_{st,i} = -\sigma \kappa_i \hat{\boldsymbol{n}}_i \delta_i$$

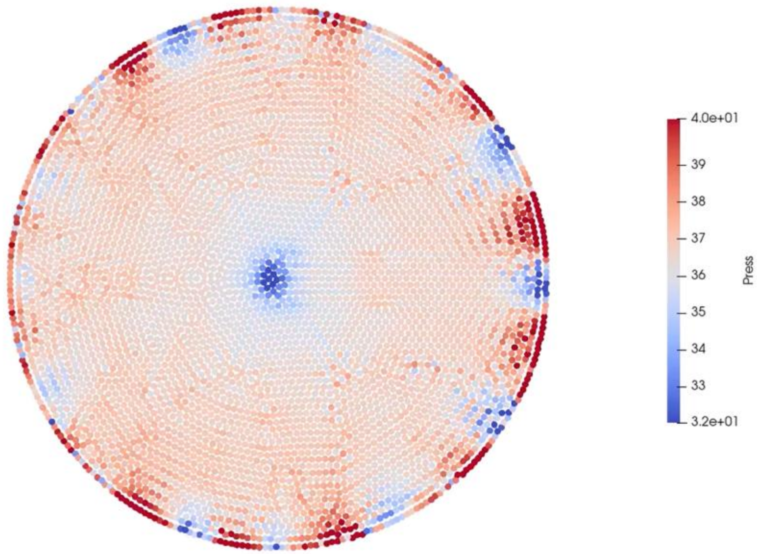
Cen, Chunze, et al. "A single-phase GPU-accelerated surface tension model using SPH." *Computer Physics Communications* 295 (2024): 109012.

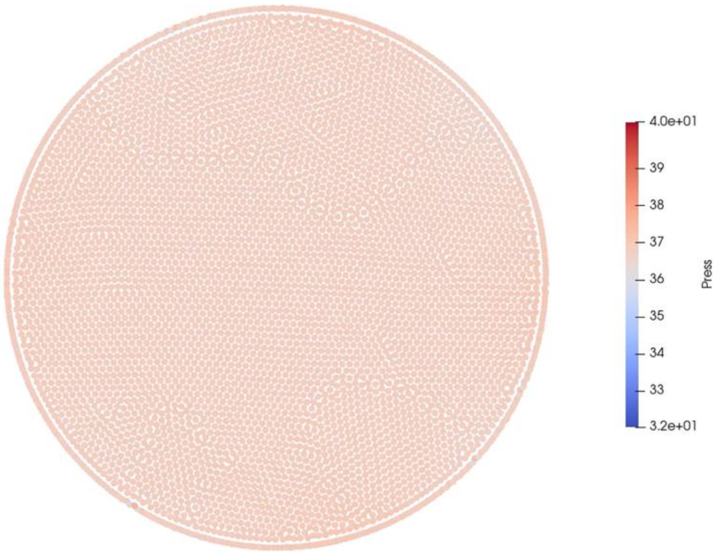
Particle Shifting

Simulation Setup

- Initial pressure = 36.4 Pa (analytical solution)
- Droplet diameter = 0.004 m
- Particle diameter = 5e-5 m
- hdp = 1.3
- Viscosity = $1.0 \times 10^{-6}\ m^2/s$
- Surface tension coeff. = 0.0728 N/m
- Shifting Threshold = 1.5 (default value for 2D case)

- Max. system speed = 0.5 m/s
- End time = 0.2 s





Pressure distribution for different particle shifting algorithms

$$\delta r = -\mathrm{Ah}\|u_i\|\Delta t \nabla C_i$$
$$(A = 2.0)$$

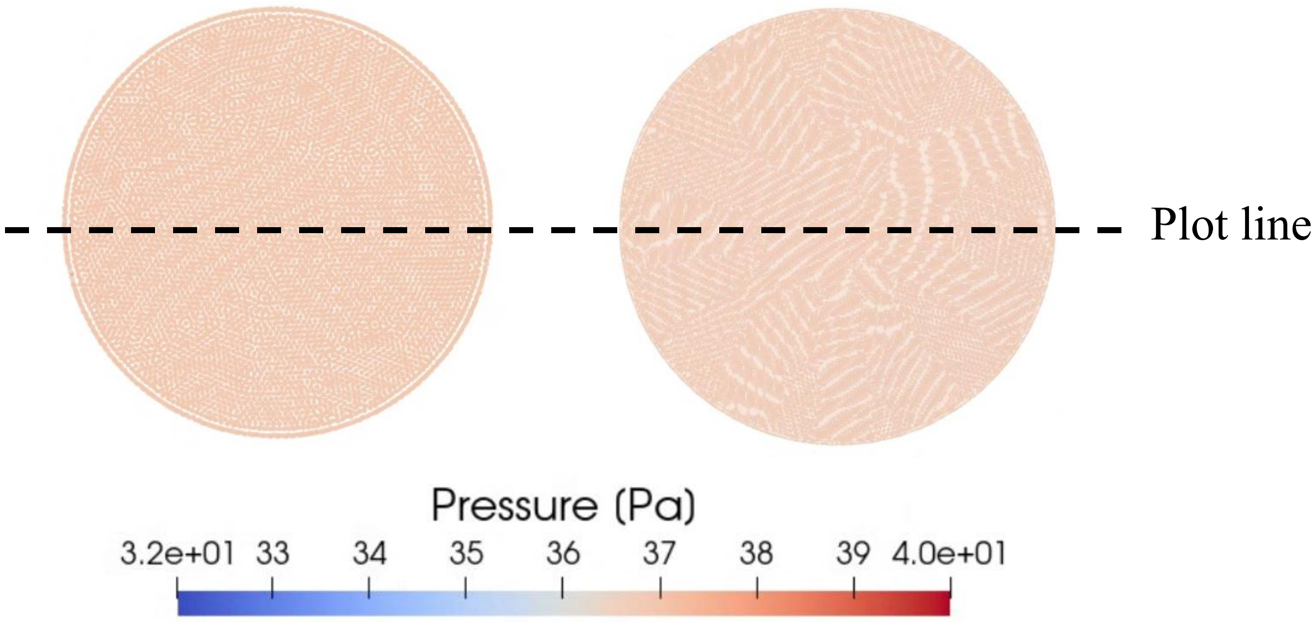
Default particle shifting in DualSPHysics
A. Skillen et. al., 2013

$$\delta r = -D\nabla \mathrm{C} = -\mathrm{Ah}^2\nabla C$$
$$(A = 0.05)$$

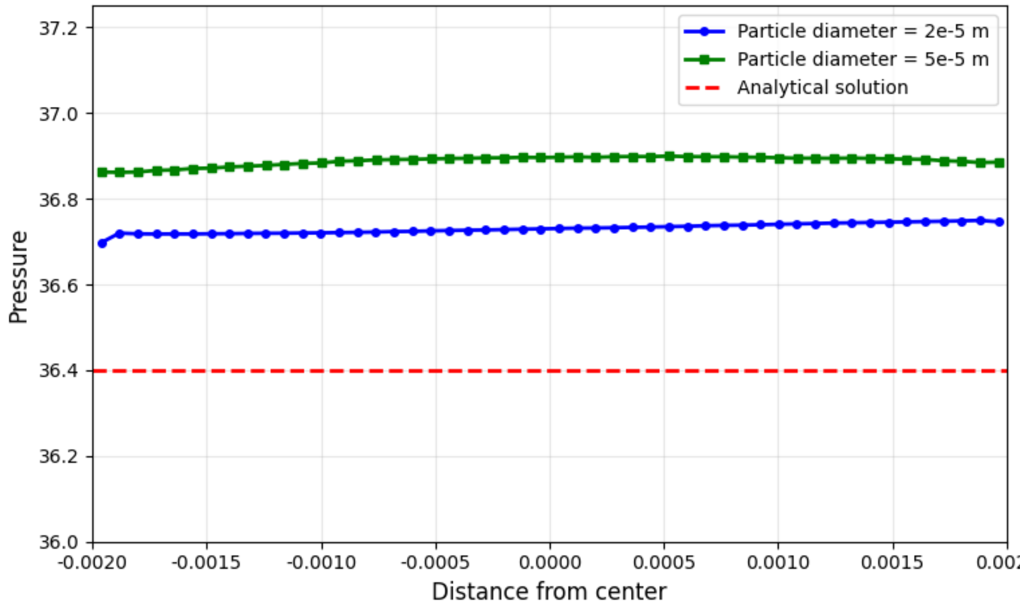
Particle shifting without the velocity term
Lind et al., 2012

Validation of Surface Tension model at Single Resolution

$$P_{analy} = \frac{2\sigma}{R} = \frac{2 \times 0.0728}{0.004} = 36.4\ Pa$$



Pressure contour at t = 1s
Particle diameters are 5×10^{-5} m (left) and 2×10^{-5} m (right)



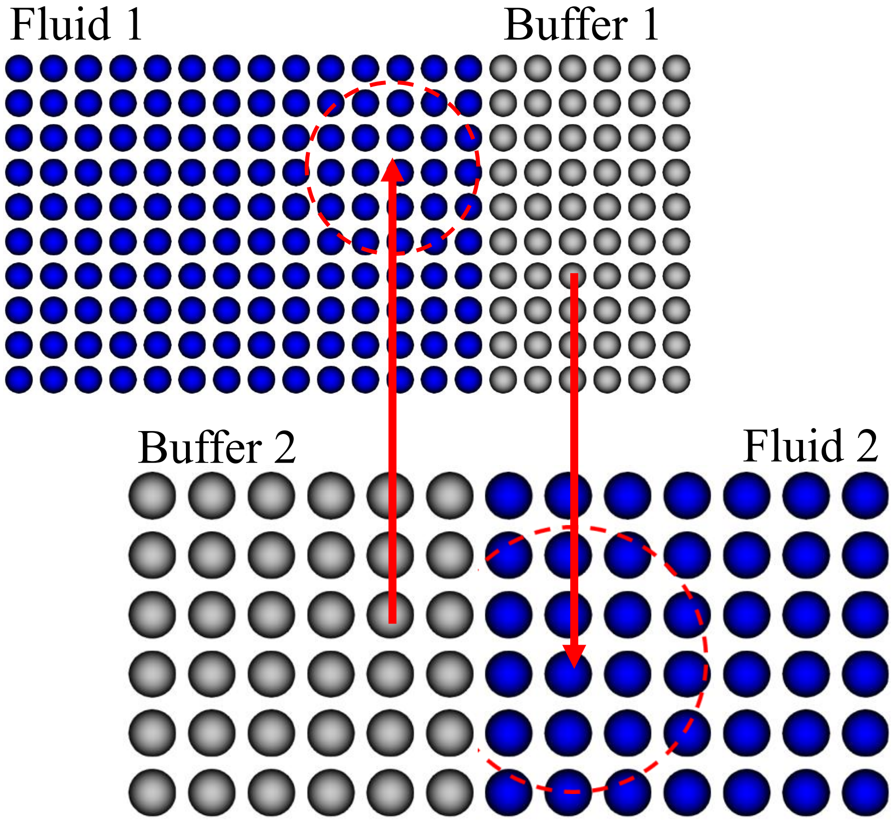
Comparison of pressure distributions along the diameter

Particle diameter	5×10^{-5} m	2×10^{-5} m
RMSE	0.4960	0.3344
NRMSE	0.01362	0.009186

RMSE and NRMSE of pressure results

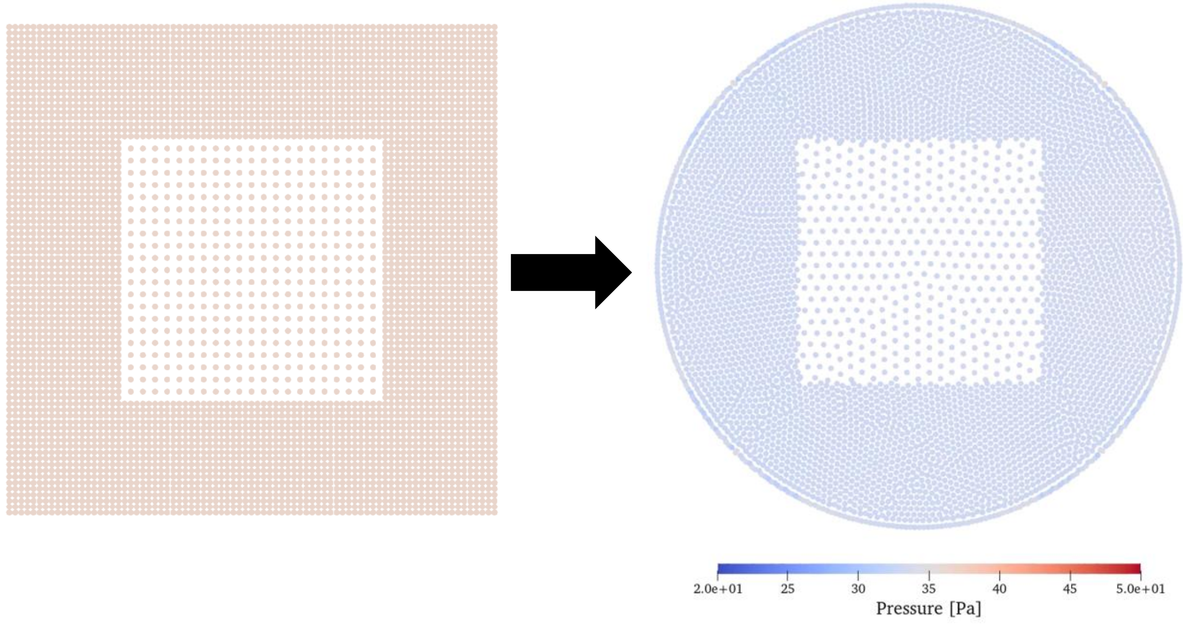
Variable resolution

- Variable resolution in DualSPHysics



Variable resolution in DualSPHysics using buffer particles

- Setup for variable resolution

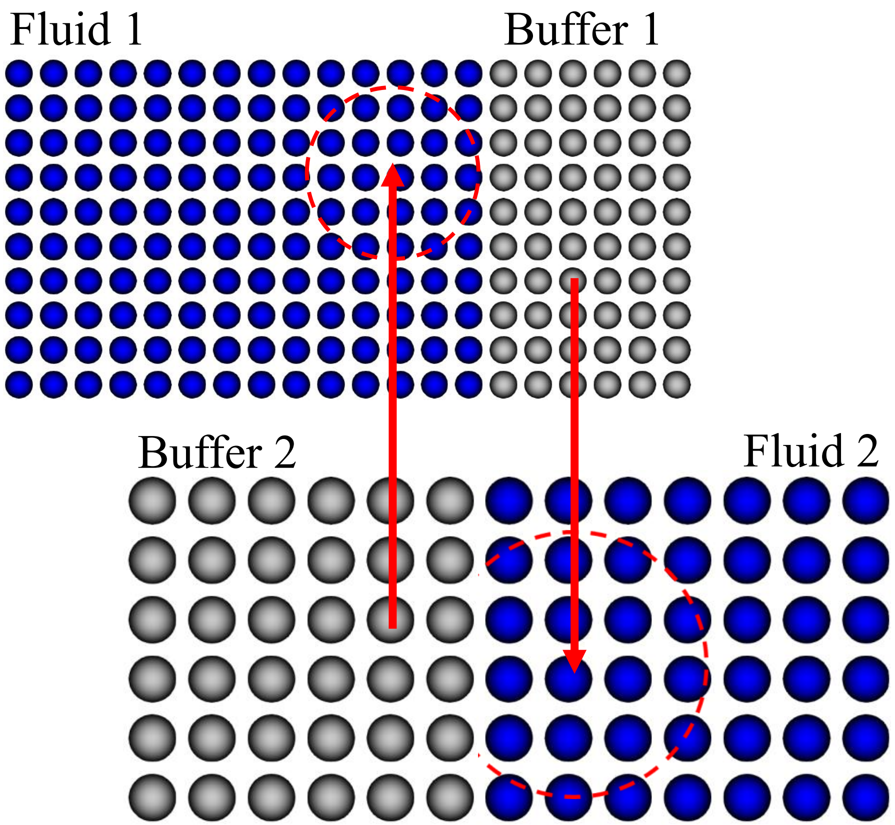


Droplet oscillation simulation
with an embedded variable resolution region

Ricci, Francesco, Renato Vacondio, and Angelantonio Tafuni. "Multiscale smoothed particle hydrodynamics based on a domain-decomposition strategy." Computer Methods in Applied Mechanics and Engineering 418 (2024): 116500.

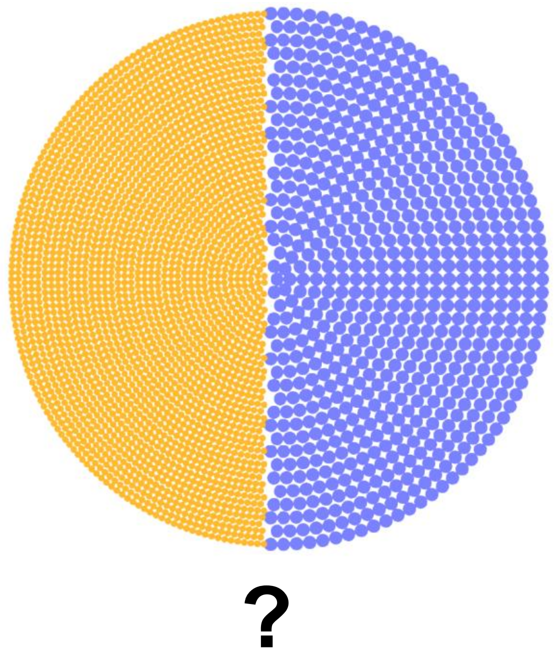
Variable resolution

- Variable resolution in DualSPHysics



Variable resolution in DualSPHysics using buffer particles

- Setup for variable resolution

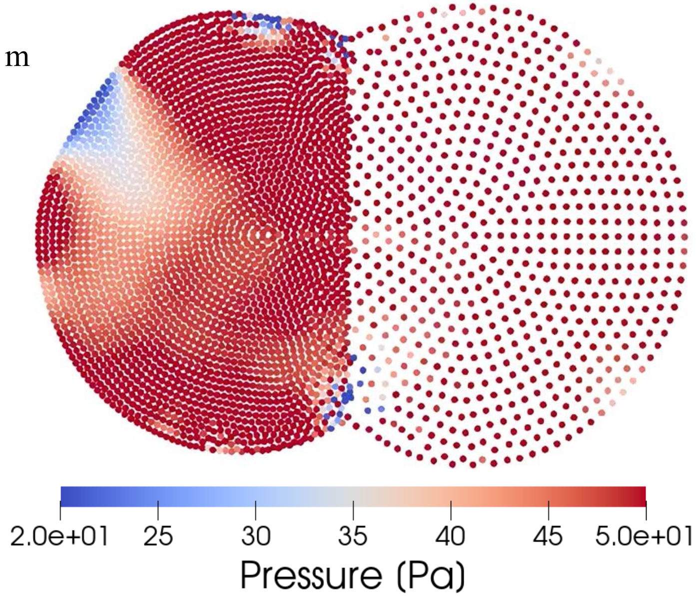


Ricci, Francesco, Renato Vacondio, and Angelantonio Tafuni. "Multiscale smoothed particle hydrodynamics based on a domain-decomposition strategy." Computer Methods in Applied Mechanics and Engineering 418 (2024): 116500.

Surface tension model with variable resolution

Case 1.
Initial Circular Shape

Simulation Setup
- Droplet diameter = 0.004 m
- Particle diameter = 5e-5 m, 1e-4 m
- hdp = 1.3
- Viscosity = $1.0 \times 10^{-5}\ m^2/s$
- Surface tension coeff. = 0.0728 N/m
- Shifting Threshold = 1.5 (default value for 2D case)
- Shifting coeff. = 0.05
- Buffer size = **2**h
- Max. system speed = 0.5 m/s
- End time = 0.05 s



Case 2.
Initial Square Shape

Simulation Setup
- Square length = 0.004 m
- Particle diameter = 5e-5 m, 1e-4 m
- hdp = 1.3
- Viscosity = $1.0 \times 10^{-4}\ m^2/s$
- Surface tension coeff. = 0.0728 N/m
- Shifting Threshold = 1.5 (default value for 2D case)
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- Buffer size = **2**h
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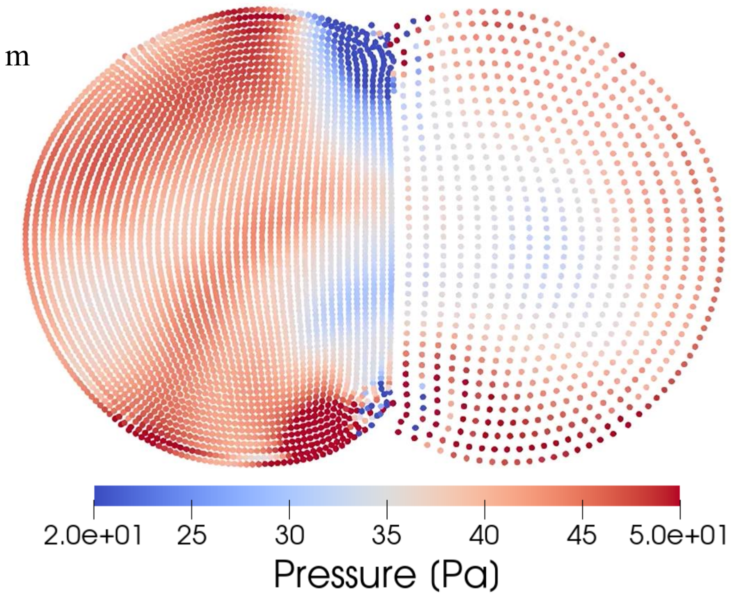
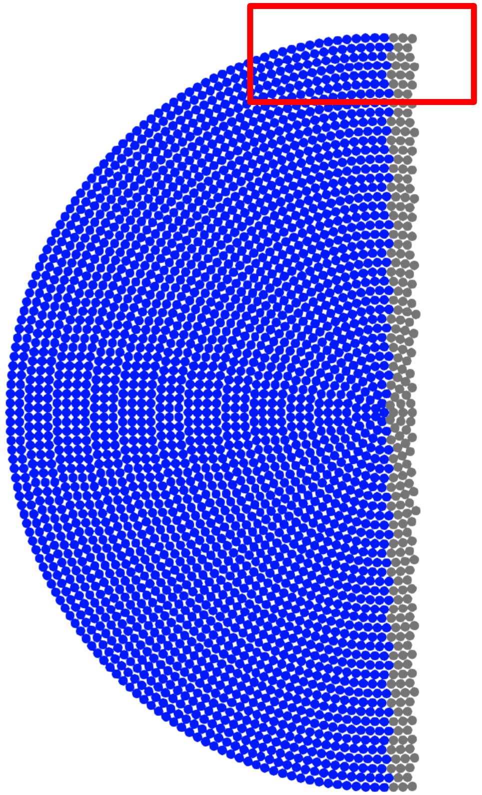
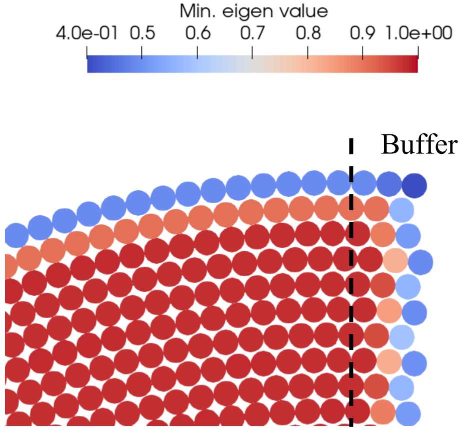
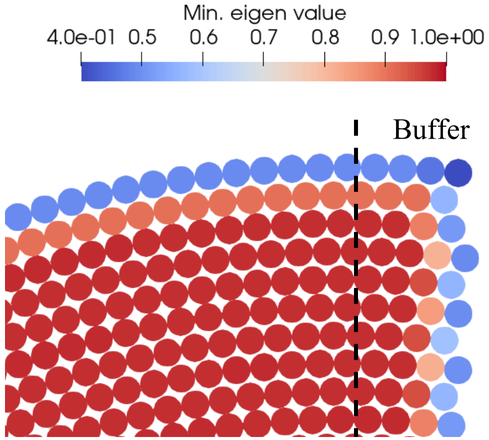
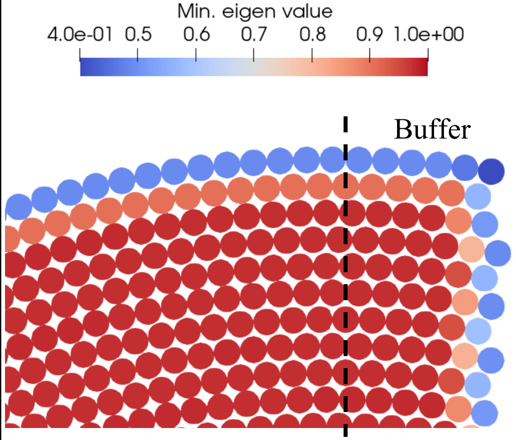
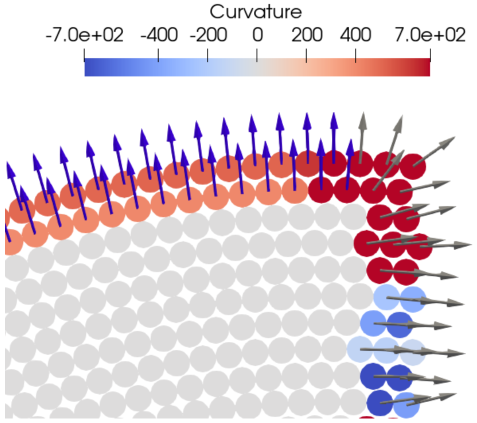
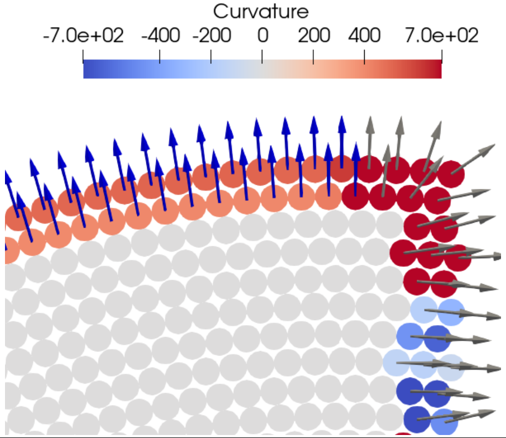
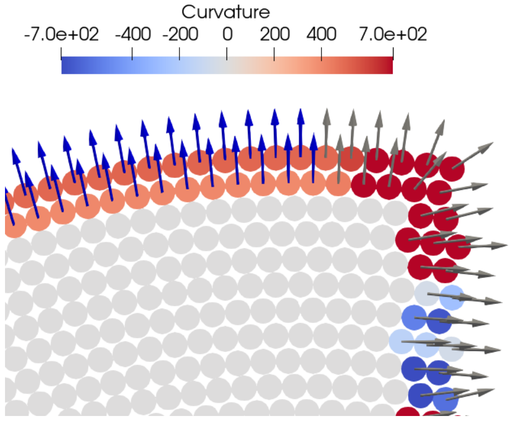


Surface tension model with variable resolution



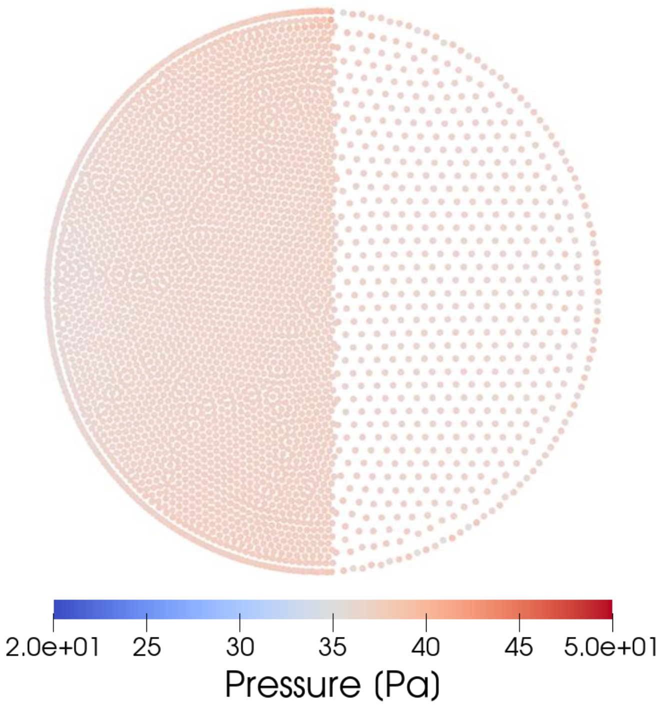
Initial particle configuration
in the fine-resolution region
with fluid (**blue**) and buffer (**gray**) particles.

Buffer size	2h	3h	4h
Minimum Eigen value			
Normal Vector & Curvature			

Surface tension model with variable resolution

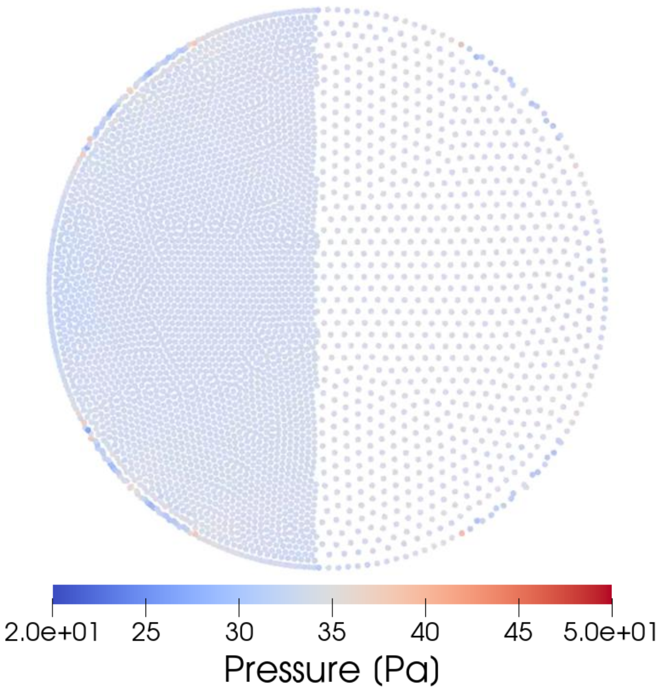
Case 1.
Initial Circular Shape

Simulation Setup
- Droplet diameter = 0.004 m
- Particle diameter = 5e-5 m, 1e-4 m
- hdp = 1.3
- Viscosity = $1.0 \times 10^{-5}\ m^2/s$
- Surface tension coeff. = 0.0728 N/m
- Shifting Threshold = 1.5 (default value for 2D case)
- Shifting coeff. = 0.05
- Buffer size = **4**h
- Max. system speed = 0.5 m/s
- End time = 0.5 s



Case 2.
Initial Square Shape

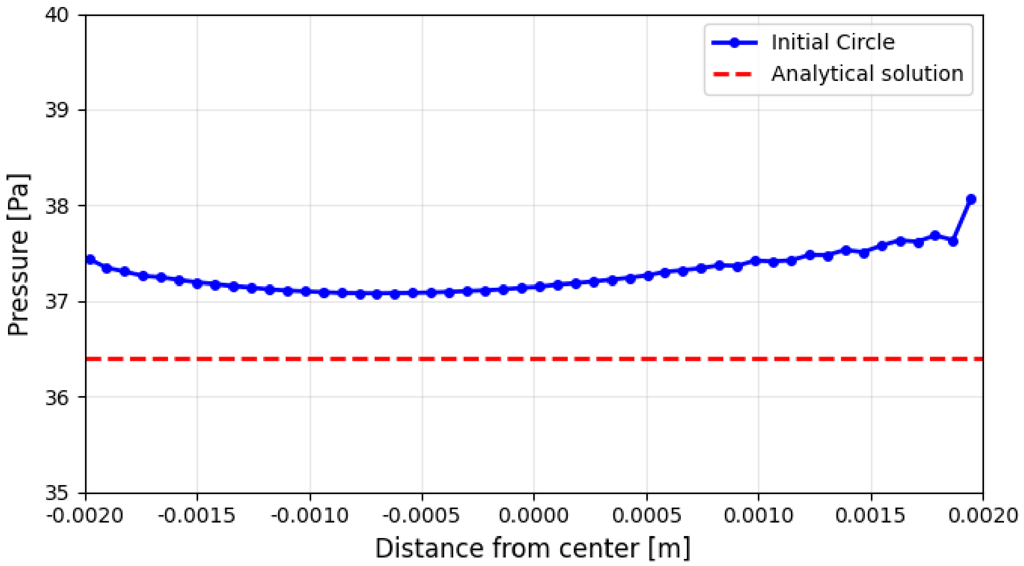
Simulation Setup
- Square length = 0.004 m
- Particle diameter = 5e-5 m, 1e-4 m
- hdp = 1.3
- Viscosity = $1.0 \times 10^{-4}\ m^2/s$
- Surface tension coeff. = 0.0728 N/m
- Shifting Threshold = 1.5 (default value for 2D case)
- Shifting coeff. = 0.05
- Buffer size = **4**h
- Max. system speed = 0.5 m/s
- End time = 0.5 s



Surface tension model with variable resolution

Case 1.
Initial Circular Shape

$P_{analy} = 36.4\ Pa$

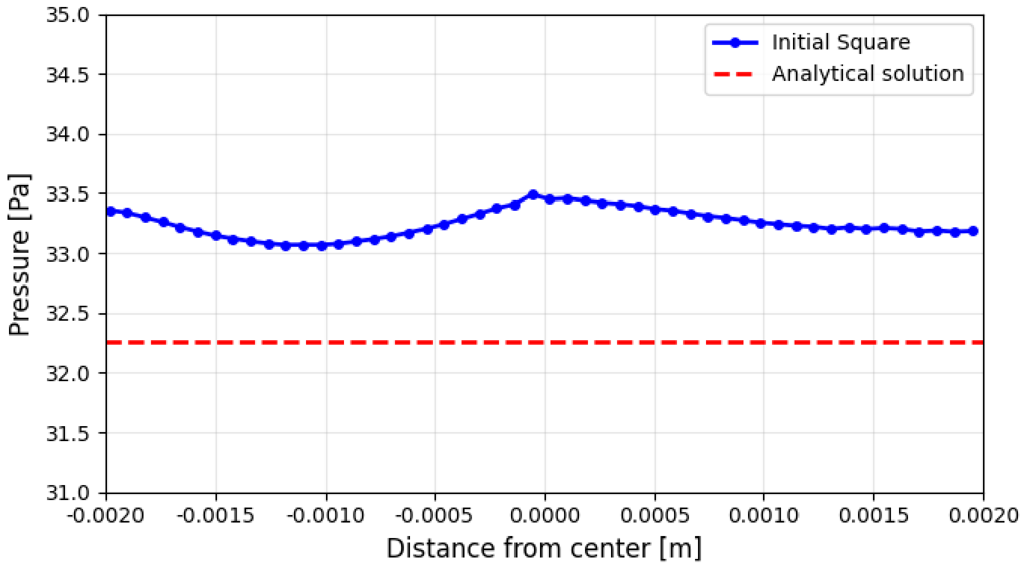


Pressure distributions along the diameter at t=0.5 s

RMSE	0.7708
NRMSE	0.02117

Case 2.
Initial Square Shape

$P_{analy} = 32.26\ Pa$

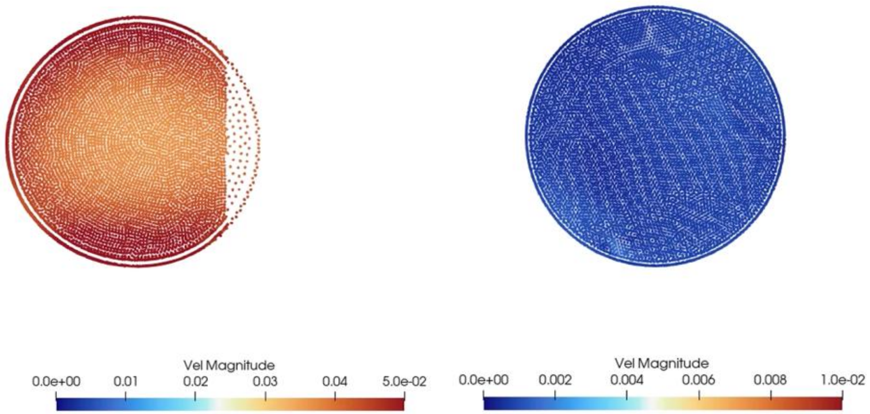


Pressure distributions along the diameter at t=0.5 s

RMSE	1.1036
NRMSE	0.03421

Limitations and Future Work

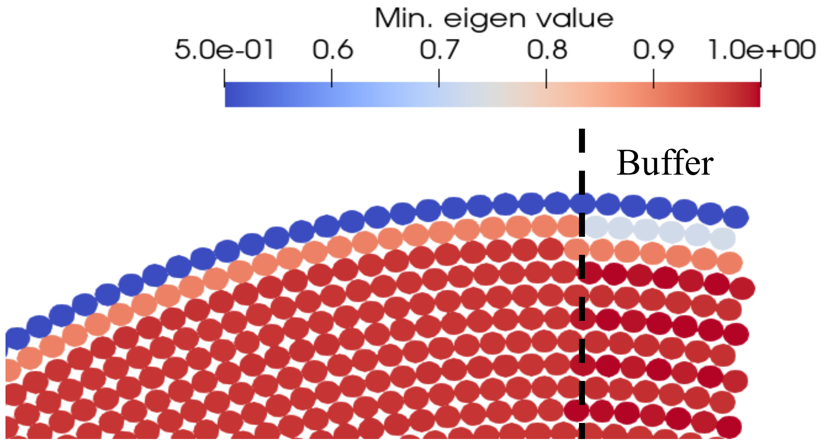
- Parasitic Velocity



Magnitude of velocity contour t = 0 to 1 s
Variable resolution (left) and single resolution (right)

Parasitic velocity is accelerated when particles are created or removed while crossing the resolution boundary.

- Advanced Algorithm for Buffer Particle
  - Enhancing the buffer size can't be an ultimate solution.
  - Modified variable resolution scheme to compute surface tension model by using data from adjacent domain.



Minimum eigenvalue contour with a modified variable-resolution scheme

  - Discrepancy across the variable resolution boundary.

Acknowledgement

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Thank you!