A fluid-structure interaction model for free-surface flows and flexible structures

JOSEPH O'CONNOR

Overview

Motivation

Method

XML Examples

More Examples

Additional Information

Overview

Motivation

Method

XML Examples

More Examples

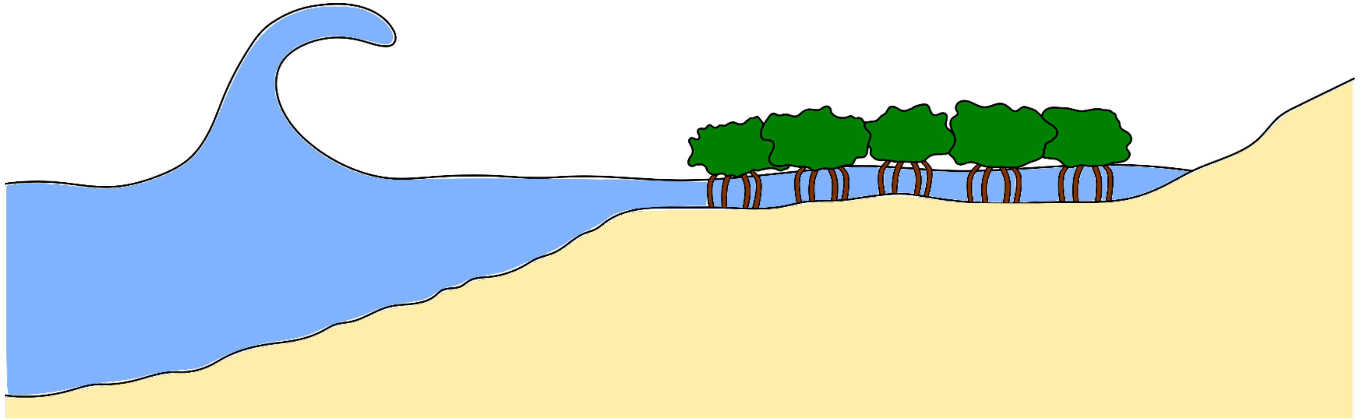
Additional Information

Motivation for Flexible FSI

- Many real-world problems are governed by flexible fluid-structure interactions
  - Vegetation
  - Biological flows
  - Coastal infrastructure
  - Many more…

- Coupling with Project Chrono provides an extensive set of features to solve a vast range of multiphysics problems

- However, we would also like an approach that is fully contained within DualSPHysics
  - Unified framework
  - Can run entirely on GPU
  - Natural boundary conditions
  - Robust fluid-structure coupling



Overview

Motivation

Method

XML Examples

More Examples

Additional Information

Structural Modelling with SPH

- Opted for an SPH-based approach to model the structure:
  - Easier integration within DualSPHysics
  - Monolithic / unified schemes provide enhanced stability over partitioned approaches
  - Better suited to modelling additional complex processes (e.g. fracture)

- Momentum equation for a continuum:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{1}{\rho}\nabla \cdot \boldsymbol{\sigma} + \mathbf{g}$$

- Can split stress tensor into an isotropic and deviatoric part and solve just like a fluid (with different state equation, constitutive model and Jaumann stress rate)

- However, there are three problems with this approach: 1) tensile instability; 2) linear inconsistency; 3) rank deficiency / hourglassing

Tensile Instability

- Solution is to adopt a Total Lagrangian approach (Belytschko et al. 2000, Rabczuk et al. 2004)

- Reformulate momentum equation with respect to a reference (initial) configuration:

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{1}{\rho_0}\nabla_0 \cdot \mathbf{P} + \mathbf{g}$$

- Cauchy stress tensor is replaced with nominal (first Piola-Kirchoff) stress tensor and standard SPH discretisation is applied

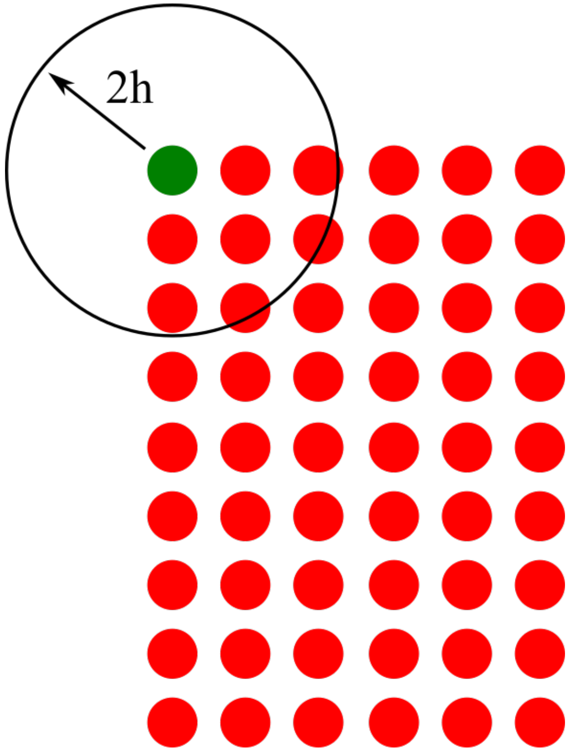
- Everything is measured with respect to initial configuration:
  - No need to recompute neighbouring particles
  - No need to recompute kernel derivatives
  - No need to compute continuity equation for the structure

Linear Inconsistency

- Boundaries are a big problem for structural dynamics with SPH due to incomplete support
- Need to reproduce gradient of a linear field (Randles & Libersky 1996)
- Introduce a kernel correction:

$$\tilde{\nabla}_a W_{ab} = \mathbf{L}_a^{-1} \nabla_a W_{ab}$$

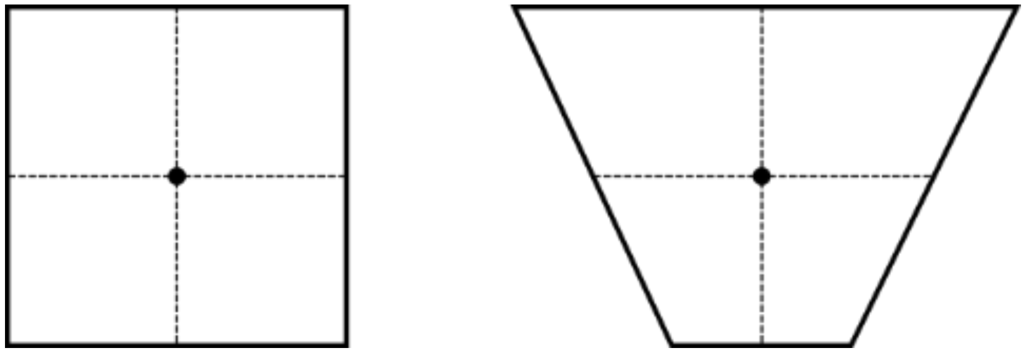
$$\mathbf{L}_a = \sum_b \frac{m_b}{\rho_b} \mathbf{x}_{ba} \otimes \nabla_a W_{ab}$$



Rank Deficiency / Hourglassing

- Rank-deficiency leads to zero-energy modes which are not suppressed and eventually become unstable (similar to reduced order elements in FEM)

- Options for suppressing these modes are:
  - Stress integration points
  - Reformulate into mixed-base set
  - Corrective force



- The corrective force approach penalises any deformation which is not described exactly by the deformation gradient (Ganzenmuller 2015)

- Corrective force approach is easy to implement and efficient however it modifies the effective stiffness of the flexible structure and introduces a tuning parameter

Discretisation and Material Model

- Finally, the discrete form of the momentum equation for the structure is:

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = \sum_b m_{0b} \left(\frac{\mathbf{P}_a \mathbf{L}_{0a}^{-1}}{\rho_{0a}^2} + \frac{\mathbf{P}_b \mathbf{L}_{0b}^{-1}}{\rho_{0b}^2} \right) \cdot \nabla_{0a} W_{0ab} + \frac{\mathbf{f}_a^{HG}}{m_{0a}} + \mathbf{g}$$

- The first Piola-Kirchhoff stress is related to the second Piola-Kirchhoff stress:

$$\mathbf{P} = \mathbf{F}\mathbf{S}$$

- The second Piola-Kirchhoff stress is related to the Green-Lagrange strain via the Saint Venant-Kirchhoff constitutive model:

$$\mathbf{S} = \lambda \mathrm{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E}$$

- Where the Green-Lagrange strain and deformation gradient are given by:

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right) \qquad \mathbf{F} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{x}_0}$$

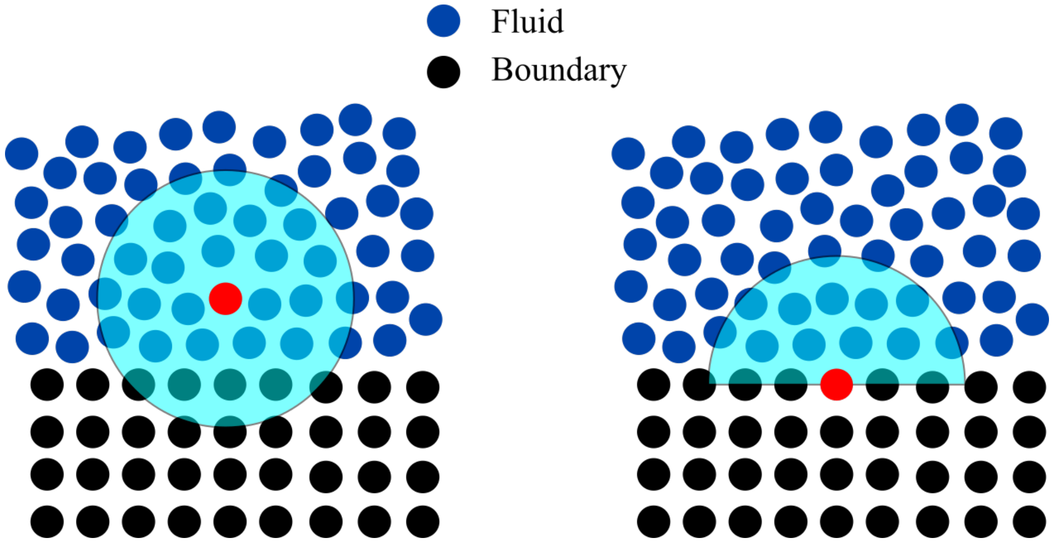
Dynamic Boundary Condition

- The dynamic boundary condition is the basic pre-existing boundary condition within DualSPHysics
- Density of boundary particles is evolved via the continuity equation as normal
- Momentum equation is not computed for boundary particles

Fluid

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{u}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}$$

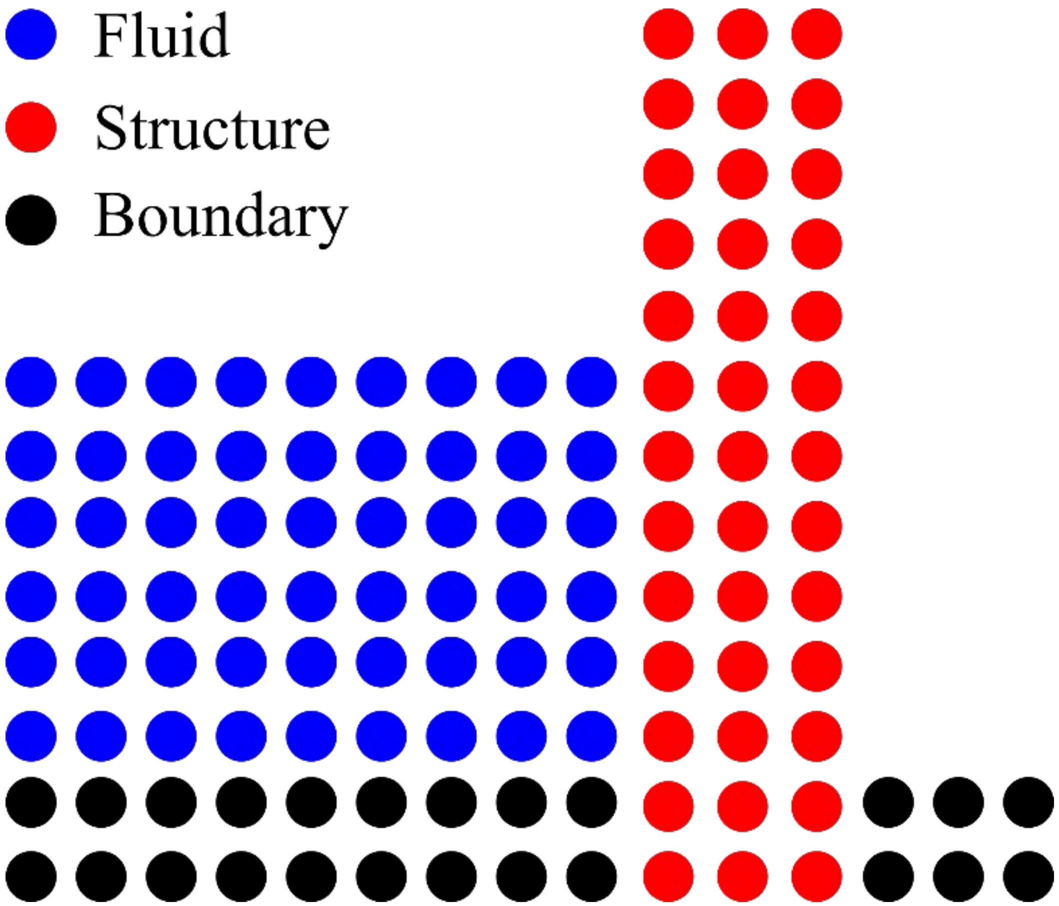


Boundary

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \nabla \cdot \mathbf{u}$$

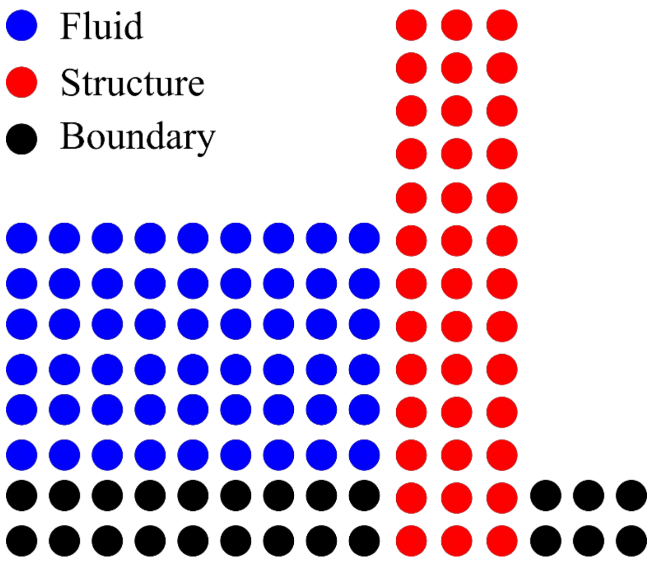
Fluid-Structure Coupling

- The fluid-structure coupling is handled via the same approach (dynamic boundary condition)
- Fluid see structural particles as normal boundary particles (with a velocity)
- Structure sees fluid particles in the same way that a boundary particle sees the fluid
- Momentum equation is integrated for structure particles but not for boundary
- No need to know geometric information about interface (e.g. surface normals)



Fluid-Structure Coupling

- Total force on a particle is sum of contributions from neighbouring fluid, structure and boundary particles
- Note that the last two terms in the structure momentum equation use the Total Lagrangian form



Fluid Particle

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{p_a + p_b}{\rho_a \rho_b}\right) \nabla_a W_{ab} - \sum_b m_b \left(\frac{p_a + p_b}{\rho_a \rho_b}\right) \nabla_a W_{ab} - \sum_b m_b \left(\frac{p_a + p_b}{\rho_a \rho_b}\right) \nabla_a W_{ab}$$

Structure Particle

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{p_a + p_b}{\rho_a \rho_b}\right) \nabla_a W_{ab} + \sum_b m_{0b} \left(\frac{\mathbf{P}_a \mathbf{L}_{0a}^{-1}}{\rho_{0a}^2} + \frac{\mathbf{P}_b \mathbf{L}_{0b}^{-1}}{\rho_{0b}^2}\right) \cdot \nabla_{0a} W_{0ab} + \sum_b m_{0b} \left(\frac{\mathbf{P}_a \mathbf{L}_{0a}^{-1}}{\rho_{0a}^2} + \frac{\mathbf{P}_b \mathbf{L}_{0b}^{-1}}{\rho_{0b}^2}\right) \cdot \nabla_{0a} W_{0ab}$$

Overview

Motivation

Method

XML Examples

More Examples

Additional Information

Case Setup in XML

- There are three main steps to setting up a case involving flexible fluid-structure interaction

```xml
        <commands>
            <mainlist>
                <setdrawmode mode="full" />
                <!--Clamp-->
                <setmkbound mk="0" />
                <drawsphere radius="0.05">
                    <point x="0.2" y="0.0" z="0.2" />
                </drawsphere>
                <!--Flexible Structure-->
                <setmkbound mk="1" />
                <drawbox>
                    <boxfill>solid</boxfill>
                    <point x="0.2" y="-0.01" z="0.19" />
                    <size x="0.4" y="0.02" z="0.02" />
                </drawbox>
            </mainlist>
        </commands>
    </geometry>
    <motion>
        <objreal ref="1">
            <begin mov="1" start="0" />
            <mvnull id="1" />
        </objreal>
    </motion>
</casedef>
<execution>
    <special>
        <flexstrucs>
            <flexstrucbody mkbound="1" mkclamp="0">
                <density value="1000.0" comment="Mass density" units_comment="kg/m^3" />
                <youngmod value="1.4e6" comment="Young's Modulus" units_comment="Pa" />
                <poissratio value="0.4" comment="Poisson ratio" />
                <constitmodel value="1" comment="Constitutive model 1:Plane Strain (2D),
                <hgfactor value="0.0" comment="Hourglass correction factor: keep as low
            </flexstrucbody>
        </flexstrucs>
    </special>
```

Geometry definition

Tag as moveable object

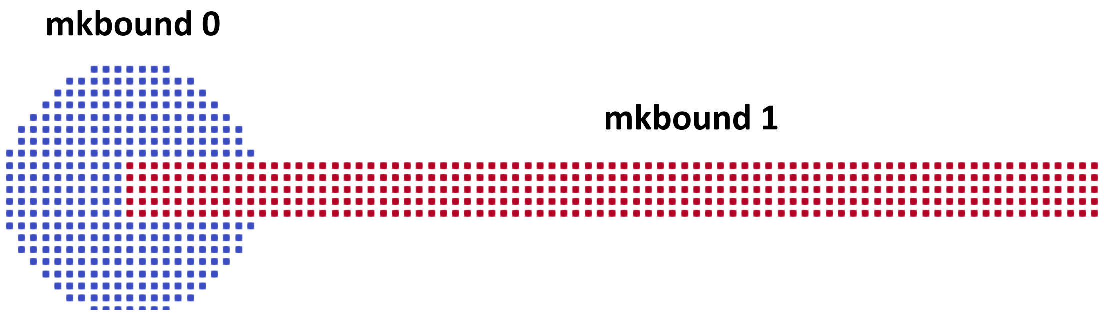
Flexible structure definition

Example 1 (Turek & Hron CSM3)

Geometry Definition

```
<mainlist>
    <setdrawmode mode="full" />
    <!--Clamp-->
    <setmkbound mk="0" />
    <drawsphere radius="0.05">
        <point x="0.2" y="0.0" z="0.2" />
    </drawsphere>
    <!--Flexible Structure-->
    <setmkbound mk="1" />
    <drawbox>
        <boxfill>solid</boxfill>
        <point x="0.2" y="-0.01" z="0.19" />
        <size x="0.4" y="0.02" z="0.02" />
    </drawbox>
</mainlist>
```

- Define the geometry of the clamp first
- Embed the flexible structure within the clamp



Tag as Moveable Object

```
<motion>
    <objreal ref="1">
        <begin mov="1" start="0" />
        <mvnull id="1" />
    </objreal>
</motion>
```

- Tag flexible structure as moveable object
- The **mvnull** label informs DualSPHysics the motion will be calculated during runtime

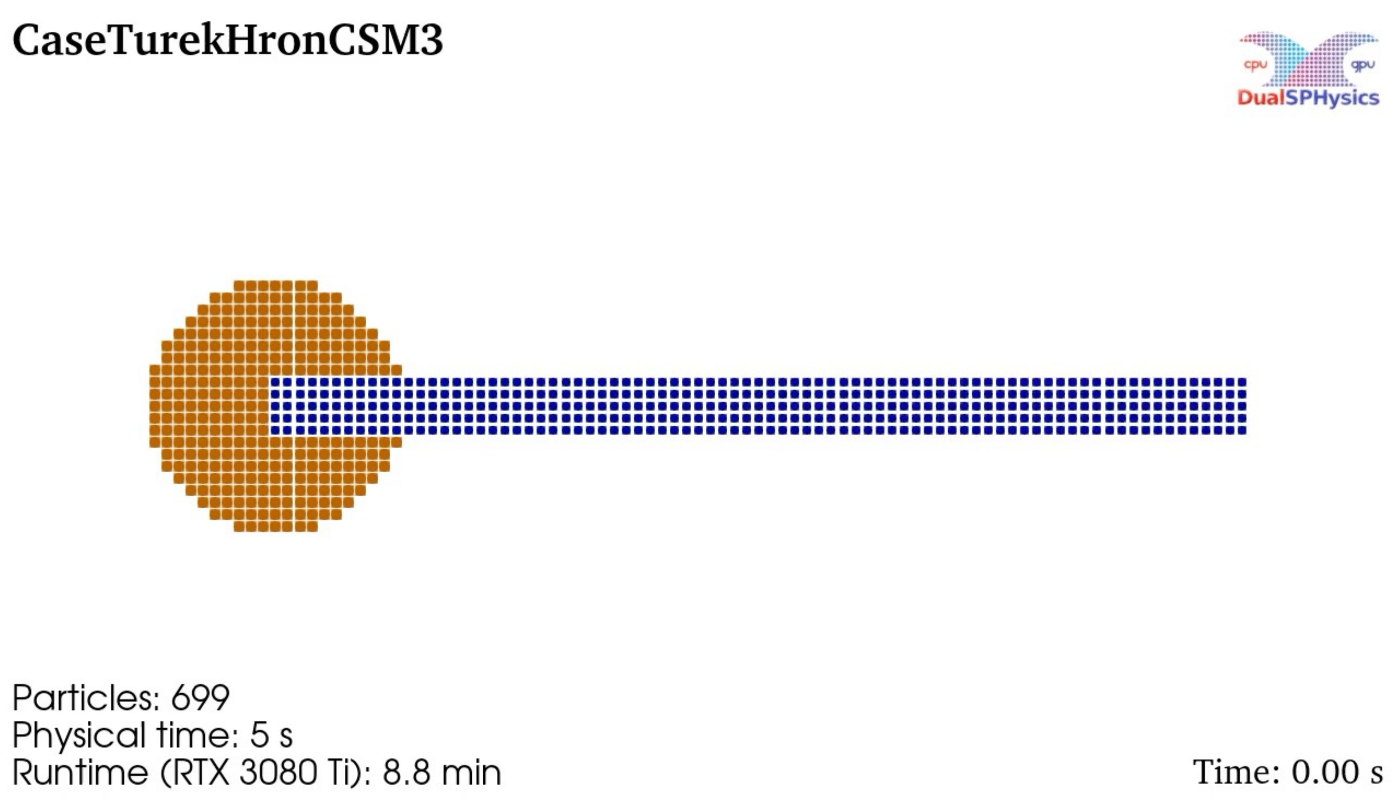
Example 1 (Turek & Hron CSM3)

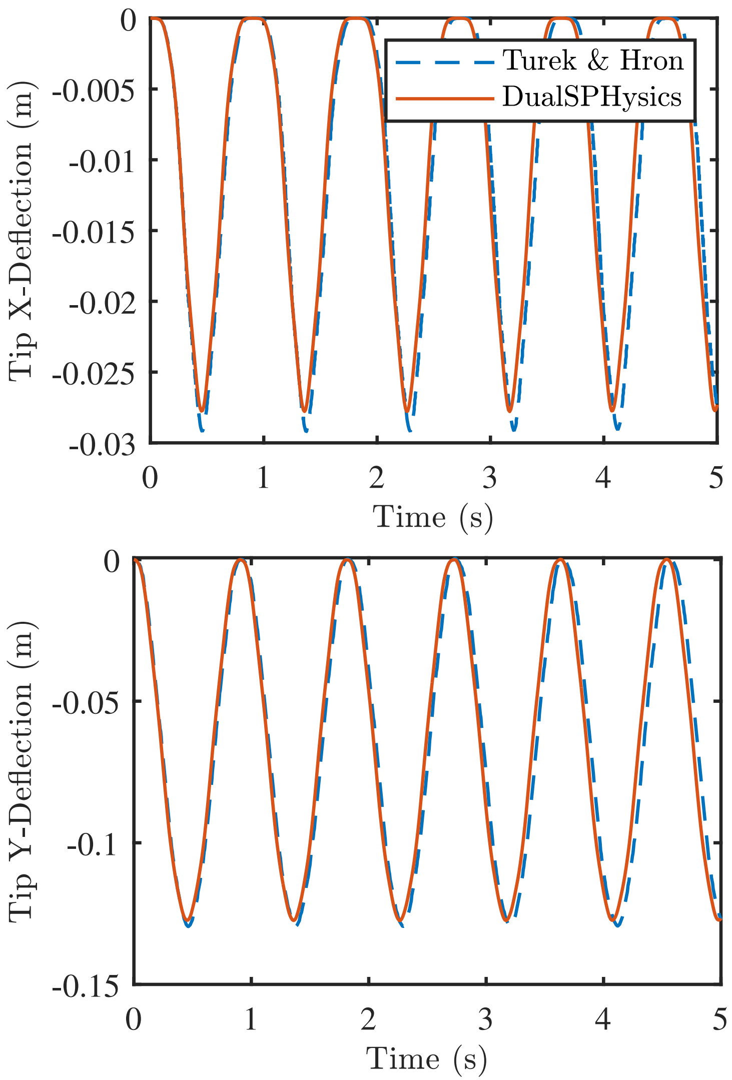
Flexible Structure Definition

```
<special>
    <flexstrucs>
        <flexstrucbody mkbound="1" mkclamp="0">
            <density value="1000.0" comment="Mass density" units_comment="kg/m^3" />
            <youngmod value="1.4e6" comment="Young's Modulus" units_comment="Pa" />
            <poissratio value="0.4" comment="Poisson ratio" />
            <constitmodel value="1" comment="Constitutive model 1:Plane Strain (2D),
            <hgfactor value="0.0" comment="Hourglass correction factor: keep as low
        </flexstrucbody>
    </flexstrucs>
</special>
```

- **mkbound** and **mkclamp** are the mkbound numbers for the flexible structure and clamp
- **density** is mass density, **youngmod** is Young's modulus, and **poissratio** is the Poisson ratio
- **constitmodel** is the constitutive model (plane strain, plane stress, or St. Venant-Kirchhoff)
- **hgfactor** is the hourglass correction factor to use in the zero-energy mode suppression scheme

Example 1 (Turek & Hron CSM3)

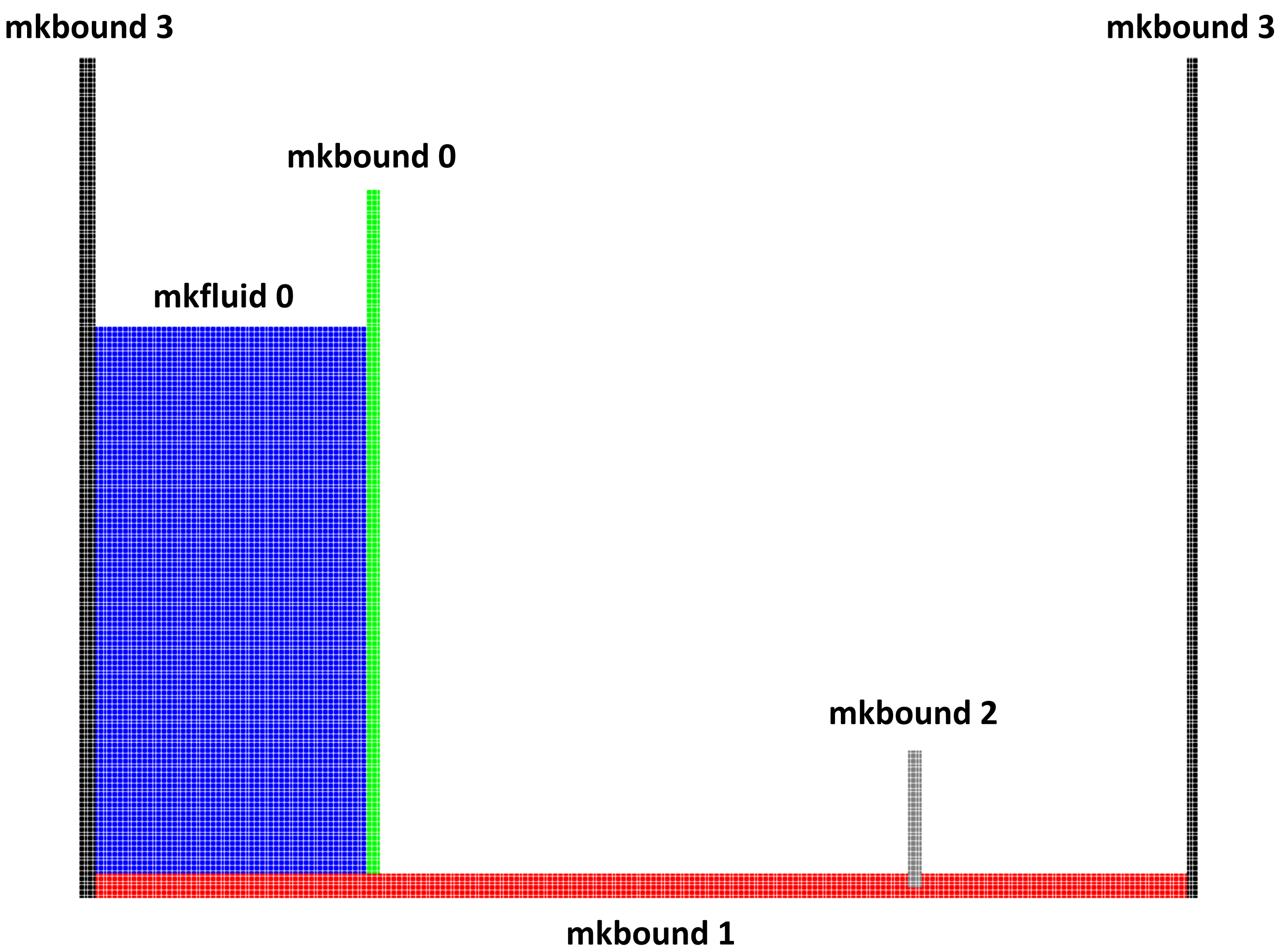




Example 2 (2D Dambreak)

Geometry Definition

```
<commands>
    <mainlist>
        <setdrawmode mode="full" />
        <setmkfluid mk="0" />
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="0.0" y="-0.01" z="0.0" />
            <size x="0.2" y="0.02" z="0.4" />
        </drawbox>
        <setmkbound mk="0" />
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="0.2" y="-0.01" z="0.0" />
            <size x="0.008" y="0.02" z="0.5" />
        </drawbox>
        <setmkbound mk="1" />
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="-0.008" y="-0.01" z="-0.016" />
            <size x="0.816" y="0.02" z="0.016" />
        </drawbox>
        <setmkbound mk="2" />
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="0.596" y="-0.01" z="-0.008" />
            <size x="0.008" y="0.02" z="0.098" />
        </drawbox>
        <setmkbound mk="3" />
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="-0.008" y="-0.01" z="-0.016" />
            <size x="0.008" y="0.02" z="0.616" />
        </drawbox>
        <drawbox>
            <boxfill>solid</boxfill>
            <point x="0.8" y="-0.01" z="-0.016" />
            <size x="0.008" y="0.02" z="0.616" />
        </drawbox>
    </mainlist>
</commands>
```



Example 2 (2D Dambreak)

Tag as Moveable Object

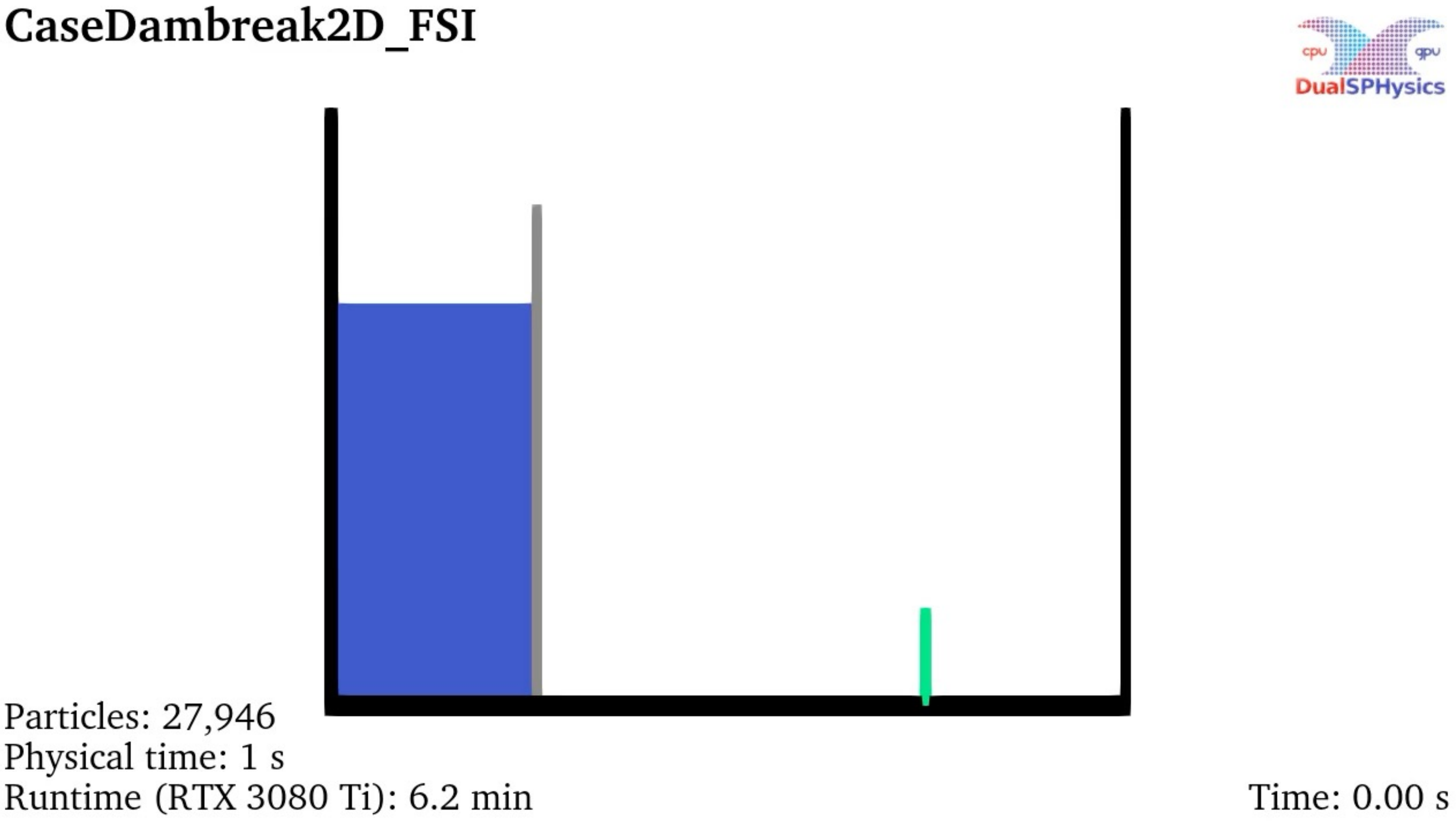
Uses realistic gate motion

```
<motion>
    <objreal ref="0">
        <begin mov="1" start="0.0" finish="0.12" />
        <mvfile id="1" duration="0.12">
            <file name="DambreakGateTimeSeries.csv" fields="2" fieldtime="0" fieldz="1" />
        </mvfile>
    </objreal>
    <objreal ref="2">
        <begin mov="1" start="0" />
        <mvnull id="1" />
    </objreal>
</motion>
```

Flexible Structure Definition

```
<special>
    <flexstrucs>
        <flexstrucbody mkbound="2" mkclamp="1">
            <density value="1161.54" comment="Mass density" units_comment="kg/m^3" />
            <youngmod value="8.75e5" comment="Young's Modulus" units_comment="Pa" />
            <poissratio value="0.49" comment="Poisson ratio" />
            <constitmodel value="1" comment="Constitutive model 1:Plane Strain (2D),
            <hgfactor value="0.1" comment="Hourglass correction factor: keep as low a
        </flexstrucbody>
    </flexstrucs>
</special>
```

Example 2 (2D Dambreak)



Overview

Motivation

Method

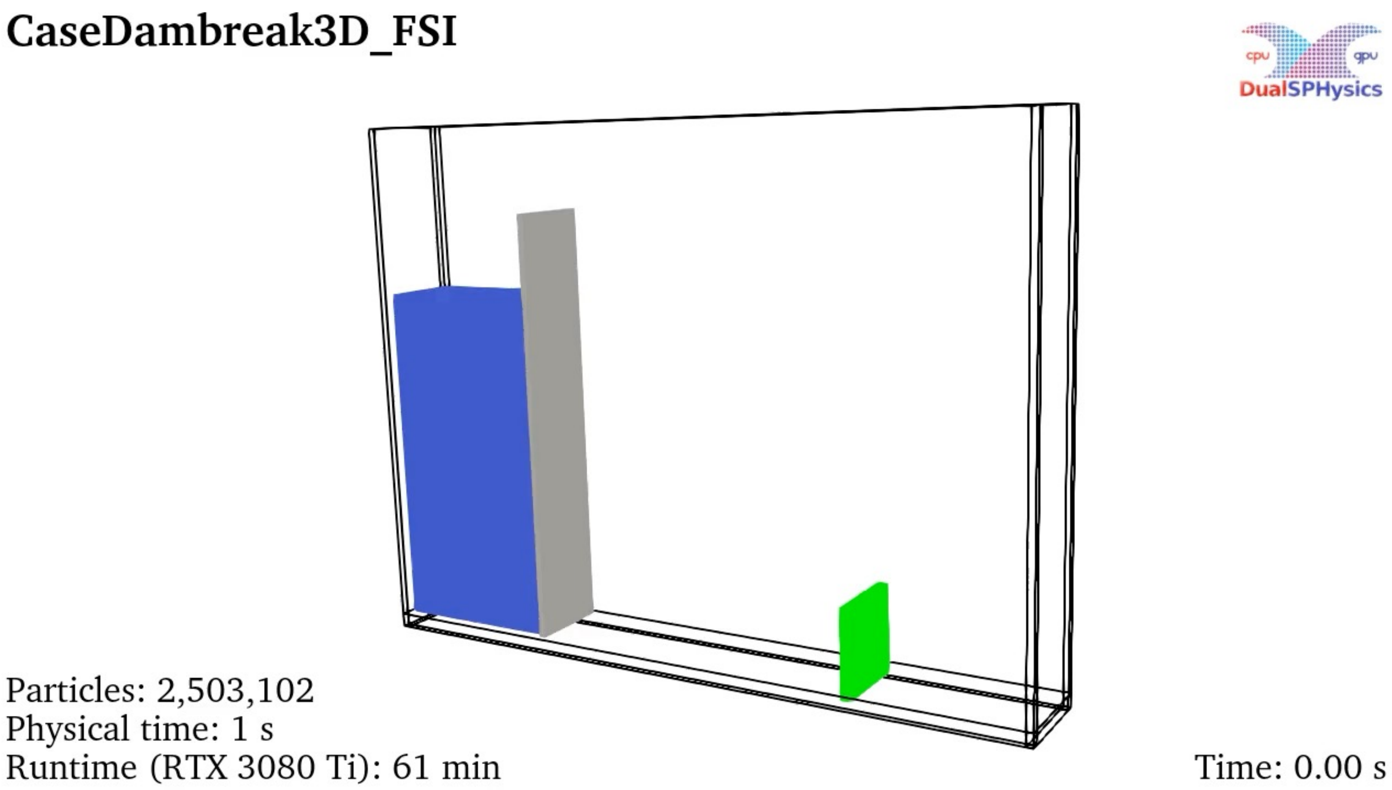
XML Examples

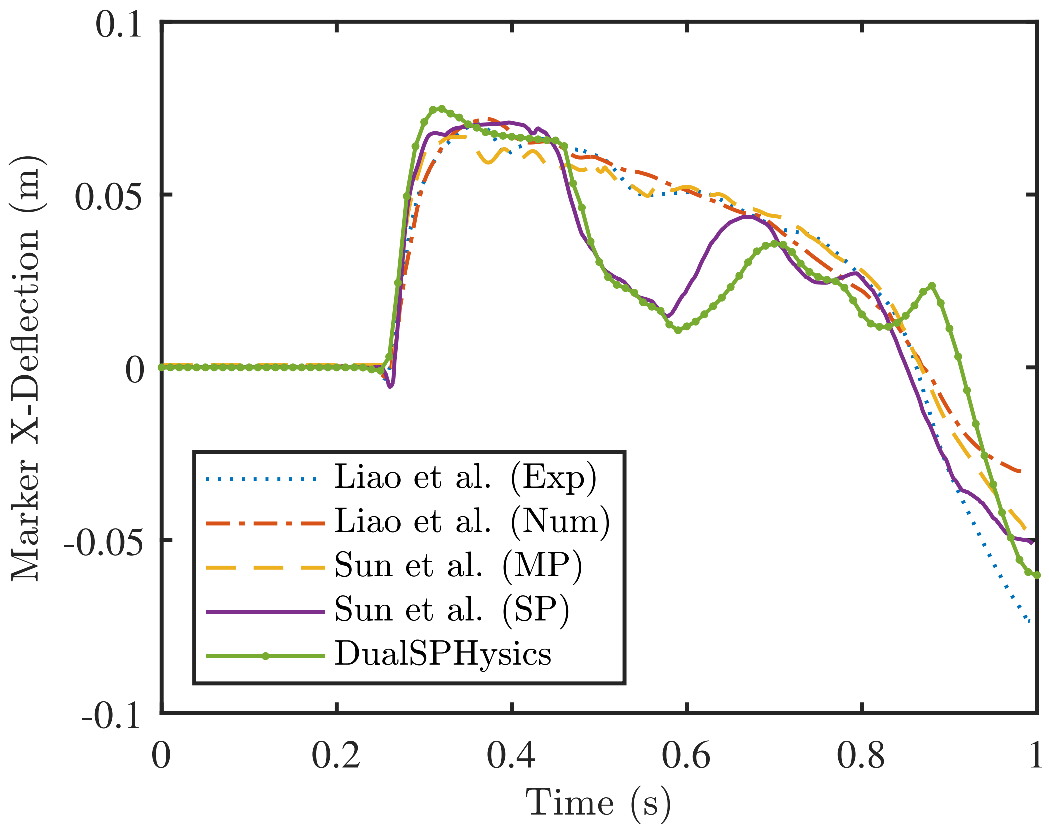
More Examples

Additional Information

3D Dambreak

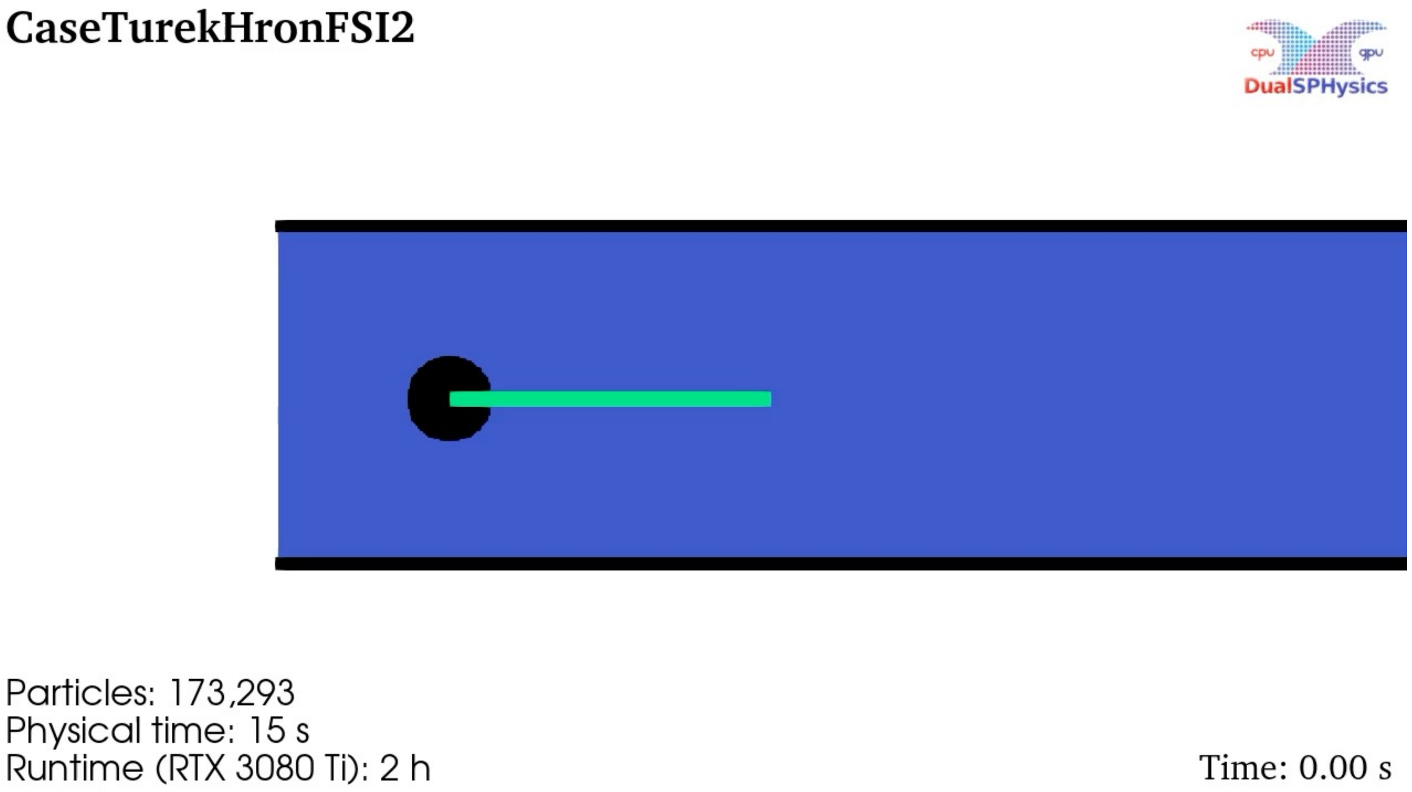
Note that the example XML is slightly modified with respect to the validation case to enable a faster run time for the example

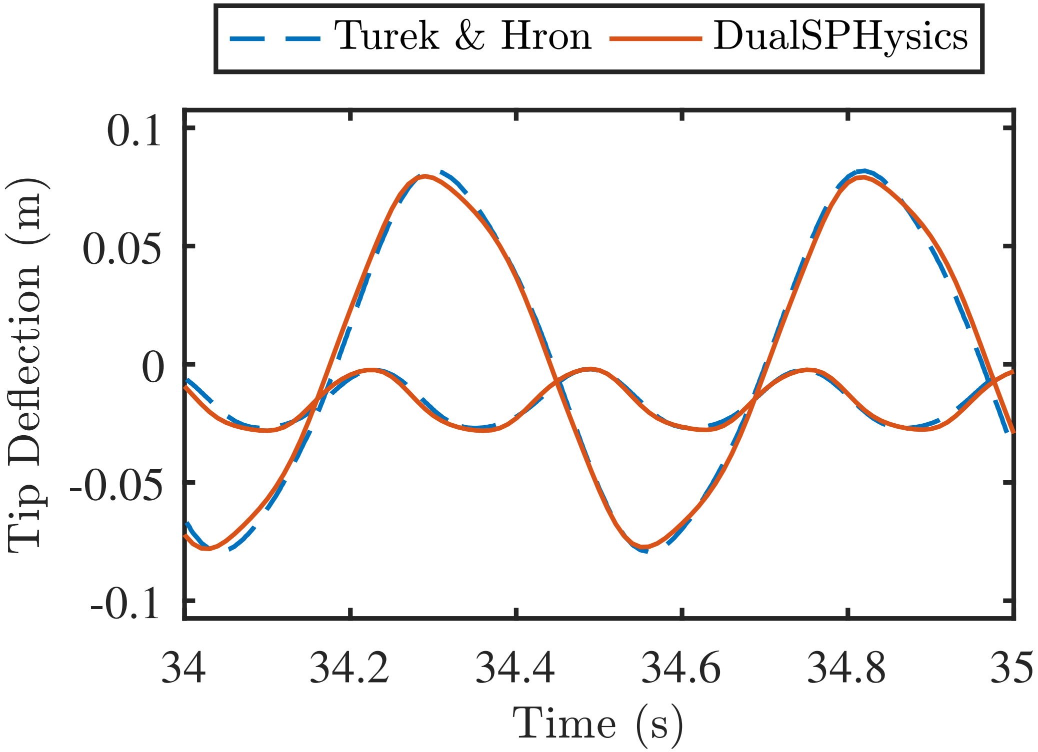




Turek & Hron FSI2

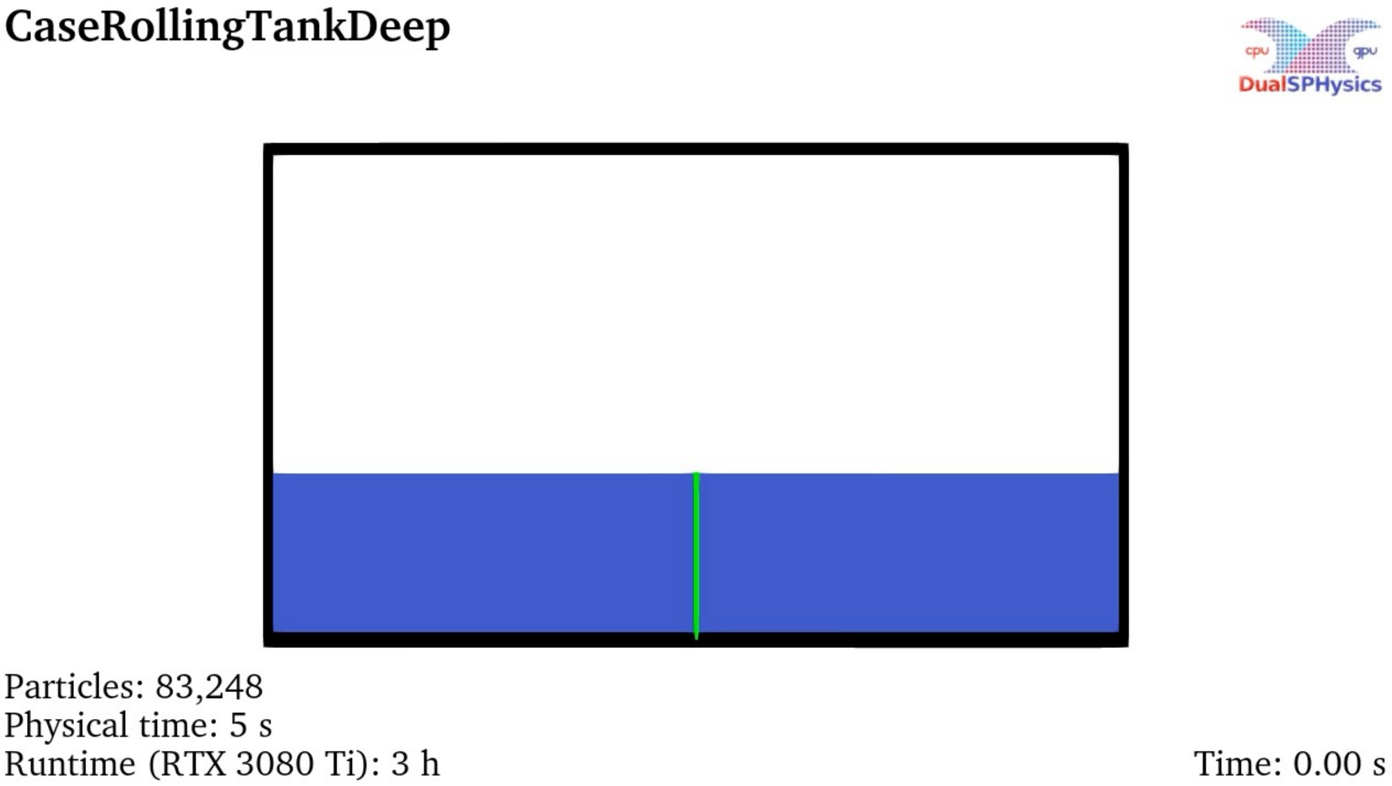
- With inlet/outlet and shifting

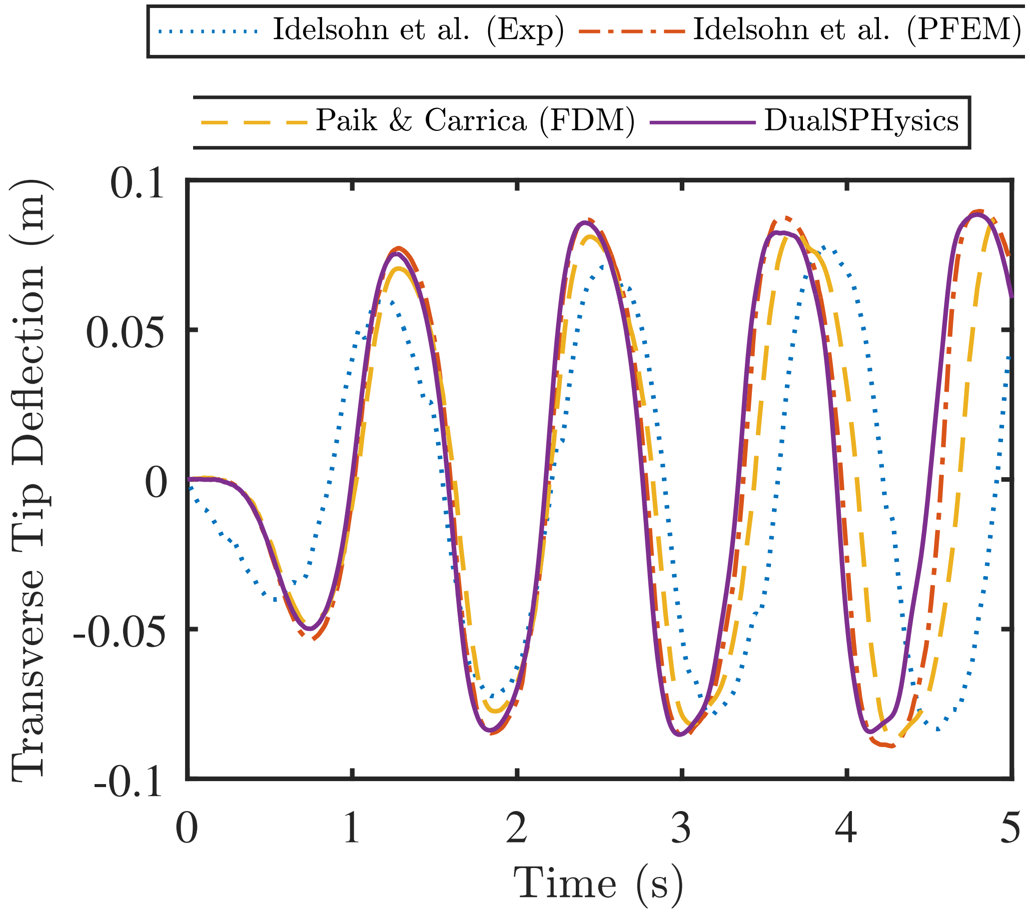




Rolling Tank (Deep)

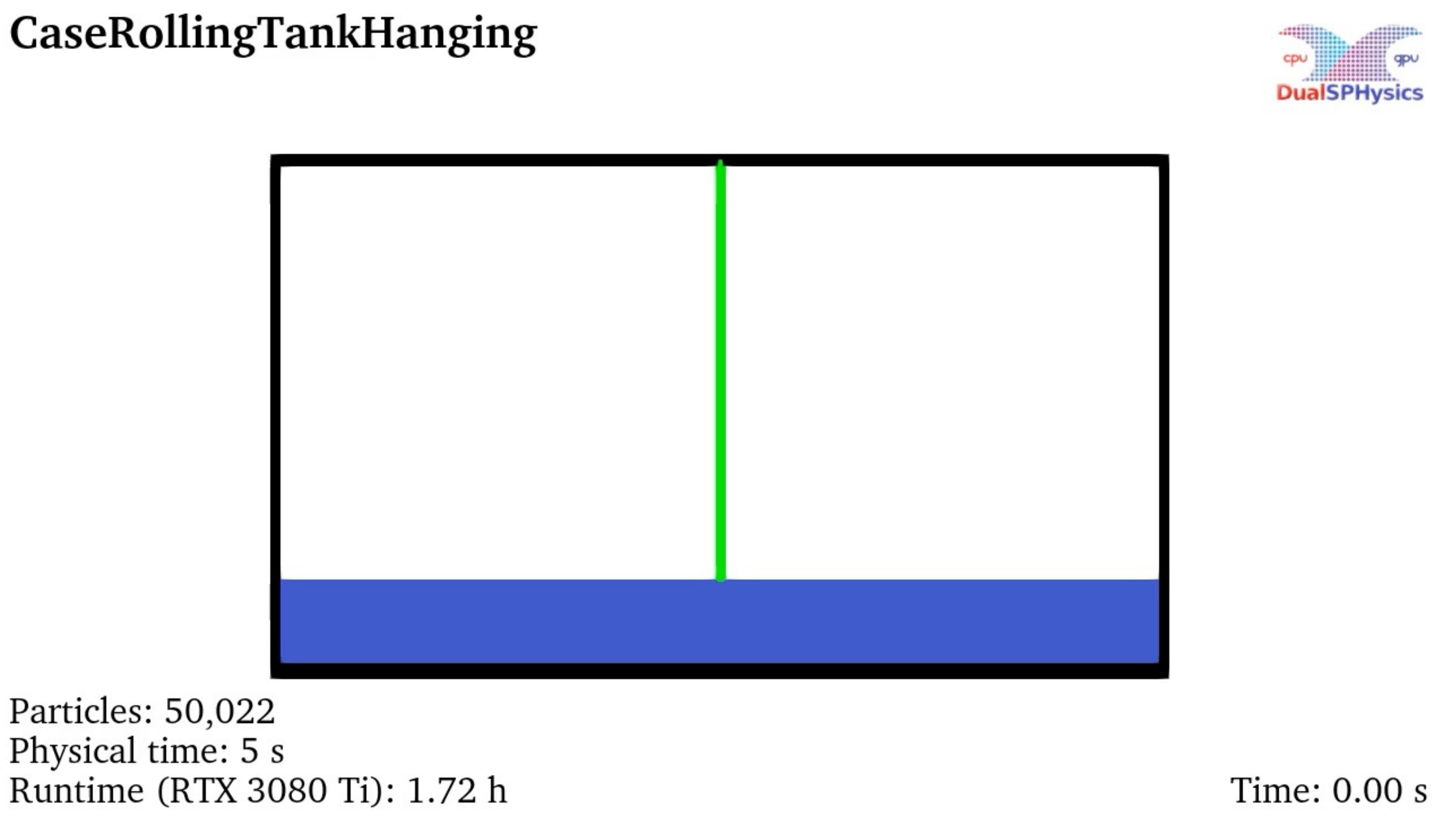
- With moving clamp

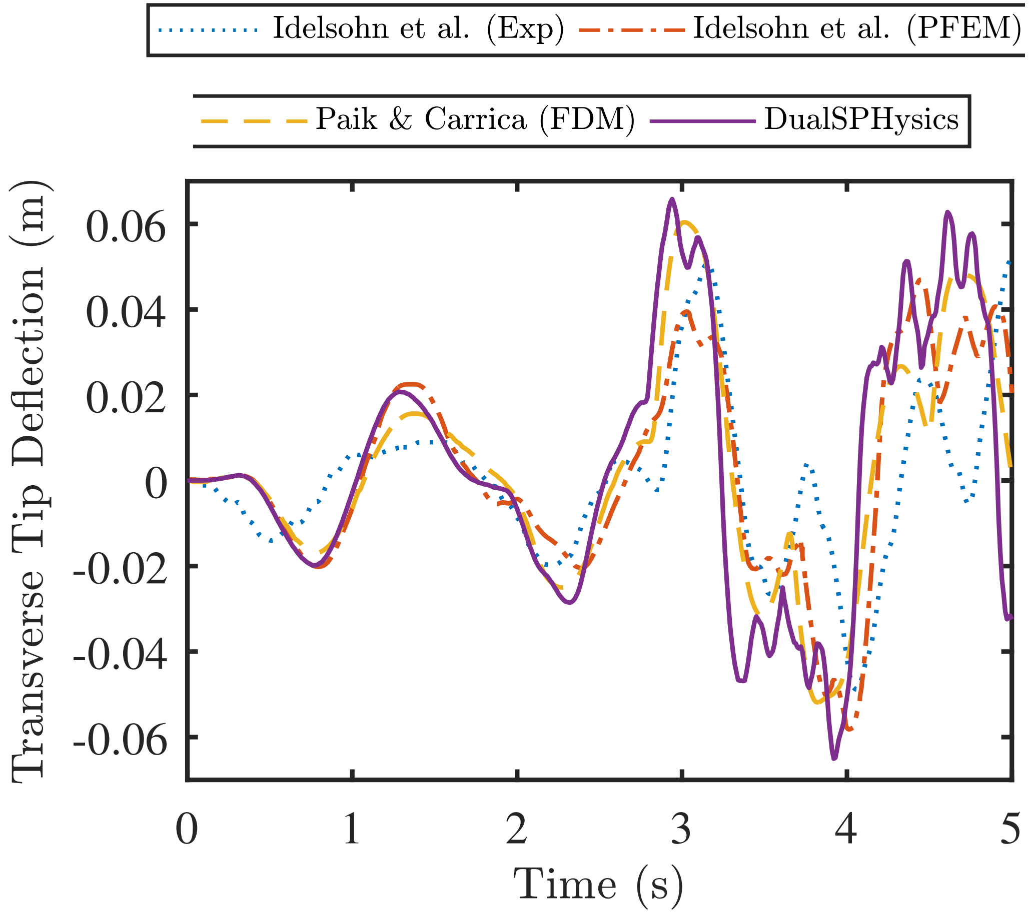




Rolling Tank (Hanging)

- With moving clamp





Overview

Motivation

Method

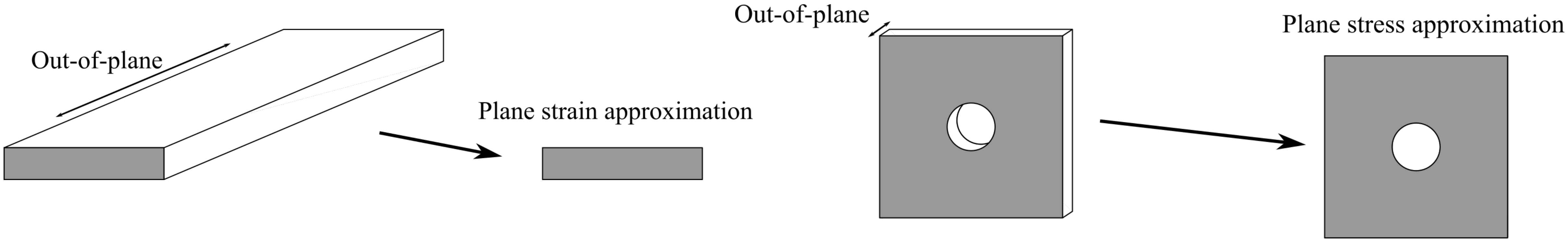
XML Examples

More Examples

Additional Information

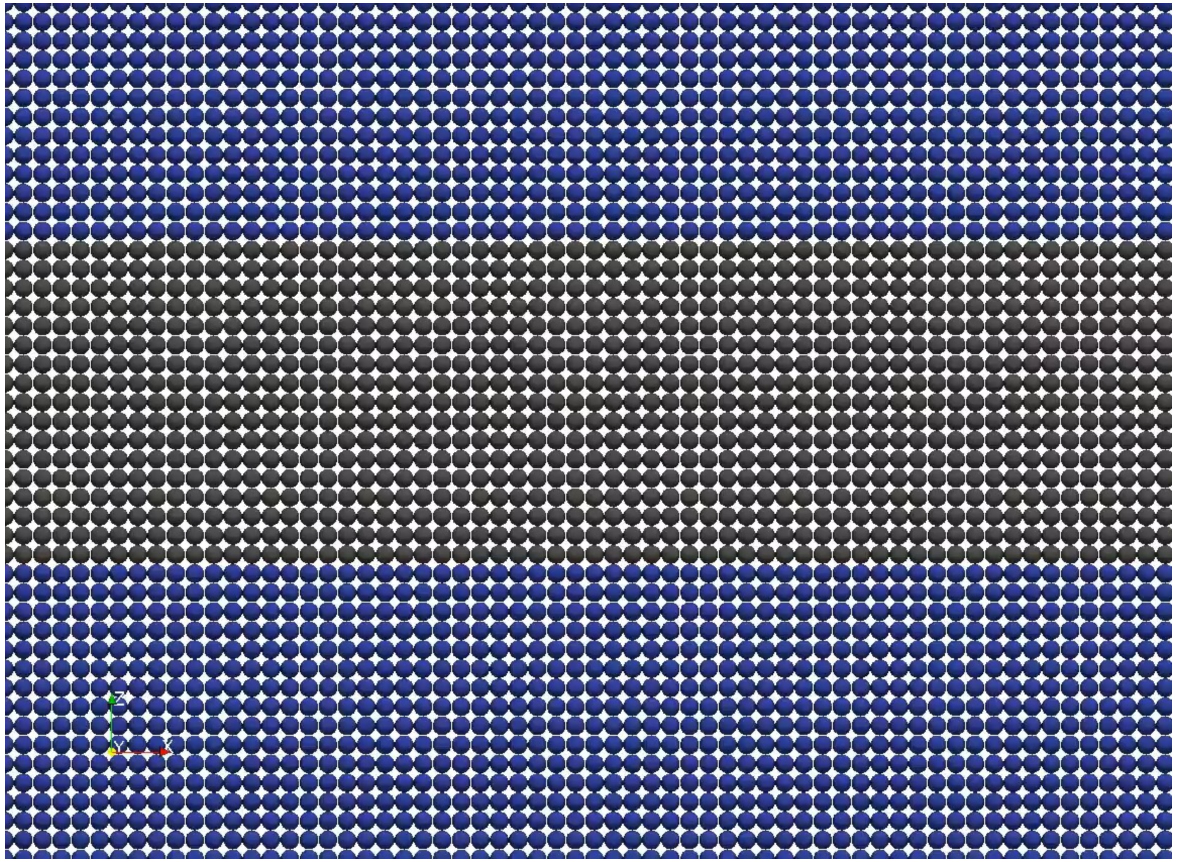
Constitutive Model

- Plane strain (2D) models the third (out-of-plane) dimension by assuming zero strain in the out-of-plane direction (suitable for problems that are very thick in the out-of-plane direction)
- Plane stress (2D) models the third (out-of-plane) dimension by assuming zero stress in the out-of-plane direction (suitable for problems that are very thin in the out-of-plane direction)
- The hyperelastic St. Venant-Kirchhoff constitutive model is the only available 3D model and is an extension of the linear elastic model to the geometrically nonlinear regime



Hourglass Suppression Scheme

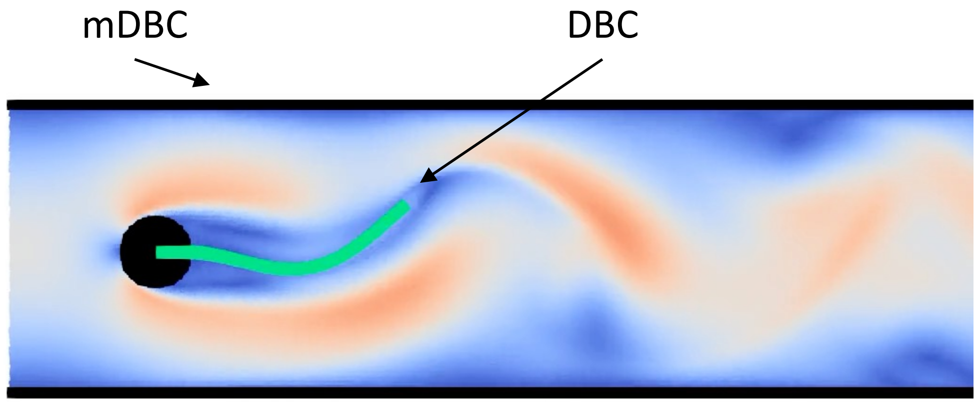
- Hourglass / zero-energy mode instability manifests as unphysical particle displacements
- The hourglass suppression scheme penalises any deformation which is not described exactly by the deformation gradient
- However, this modifies the effective stiffness of the flexible structure
- Therefore, it is recommended to first try without the correction (hgfactor = 0)
- If the instability appears, a value of 0.1 typically mitigates this instability with negligible impact on the effective stiffness



More Information

Combined Options

- The flexible FSI does not currently work with restart or symmetric/periodic BCs
- It does work with mDBC – but not on the flexible structure itself



Particle Resolution

- Generally, a minimum of four particles across the structure thickness is required
- Therefore, very thin structures will require a lot of particles and will be very expensive

Timestep Size

- There is an additional timestep constraint for the flexible structure (based on sound speed)
- High Young's modulus combined with low mass density will lead to smaller timestep

Reference & Acknowledgements

For more details, please see:

O'Connor, J. and Rogers, B.D. A fluid-structure interaction model for free-surface flows and flexible structures using smoothed particle hydrodynamics on a GPU. Journal of Fluids and Structures, 104 (103312). 2021.

Thanks To:

Prof. Benedict Rogers
University of Manchester

Dr Alejandro Crespo, **Dr José Domínguez**, **Iván Martínez Estévez**
Universidade de Vigo