SPH for new generation fluid-structure interaction solvers and reliable design of advanced coastal/offshore structures

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  - ***Reliability***: Stability, Accuracy, Choice of Governing Equations, Satisfaction of Fluid-Structure Interface Boundary Conditions
  - ***Adaptivity***: Adaptive refinement of computational resolutions, Enhancement of Applicability
  - ***Generality***: 3D Extensions, Composite Structures, Anisotropic Structures
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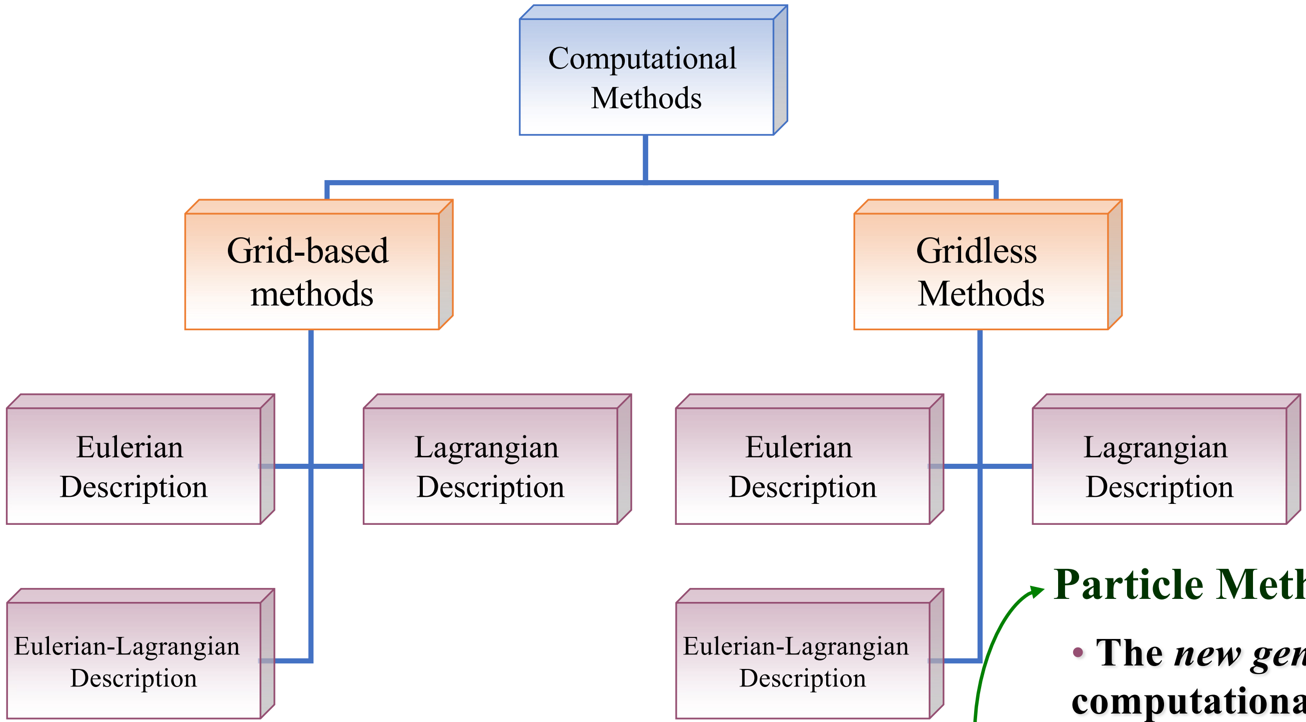
Categorization of Computational Methods



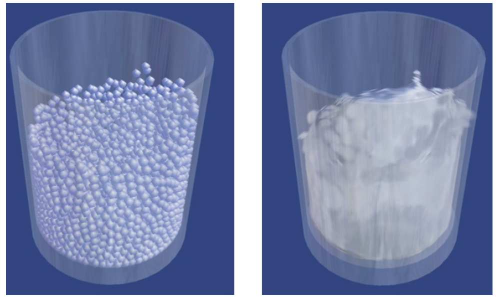
Particle Methods

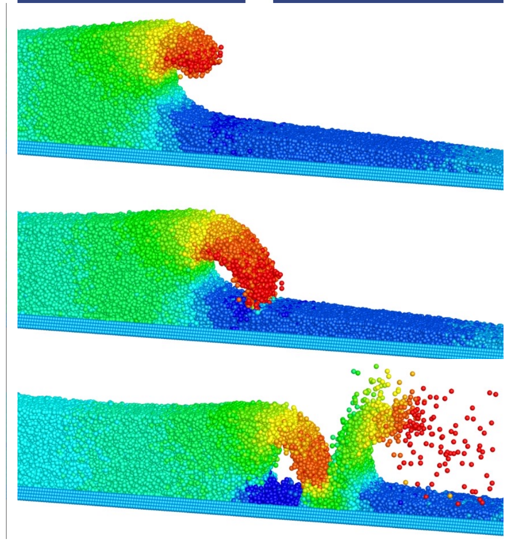
- **The *new generation* computational methods**
- **Calculation points (nodes or particles) are employed without use of any grid.**

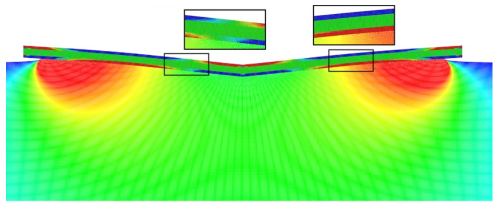
- Smoothed Particle Hydrodynamics (SPH) method (Gingold and Monaghan, 1977)
- Moving Particle Semi-implicit (MPS) method (Koshizuka and Oka, 1996)

Lagrangian Meshfree or Particle Methods

- Key Idea for meshfree (gridless) methods = providing stable and accurate numerical solutions for integral equations or PDEs with a set of arbitrarily distributed nodes or particles.
- Lagrangian Meshfree Methods or Particle Methods: Treating *computational points* (particles) in a *Lagrangian* manner
- Convection without numerical diffusion, flexibility and robustness in treating complex geometries, moving interfaces & complex physics (e.g., fracture)



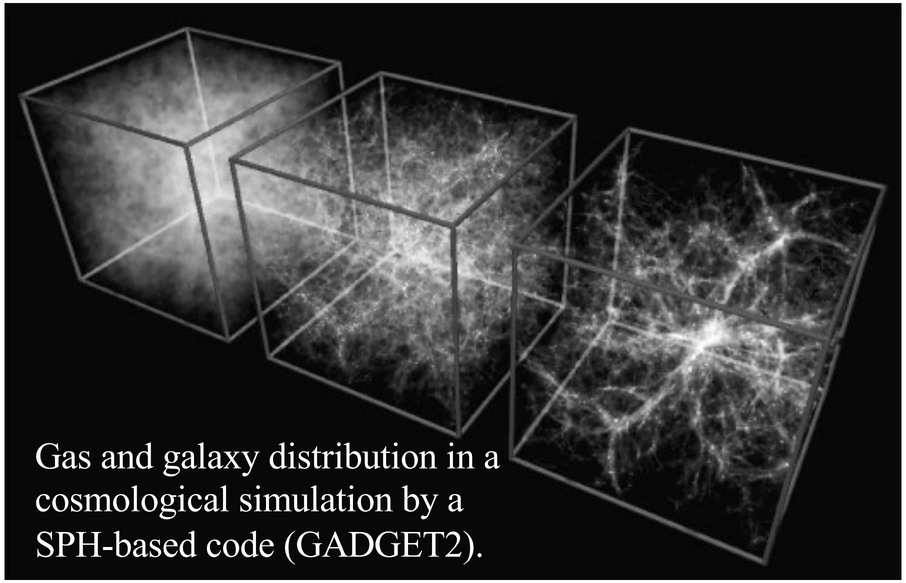




- *Particle Methods* or *Lagrangian Meshfree methods* provide substantial potential for a wide range of problems, especially those characterized by ***large deformations***, ***moving interfaces*** & ***complex topological changes***.

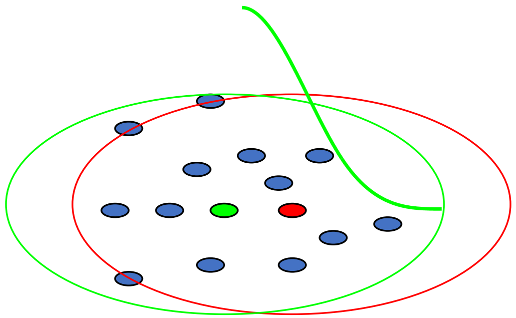
Smoothed Particle Hydrodynamics (SPH)

- Proposed in 1977, originally for astrophysical applications by Gingold & Monaghan.
- **Professor Joseph J. Monaghan**
  Emeritus Professor at School of Mathematical Sciences, Monash University, Australia
  Ph.D. in applied mathematics from Cambridge, UK



Gas and galaxy distribution in a cosmological simulation by a SPH-based code (GADGET2).

$W(r-r',h) =$ Smoothing Kernel

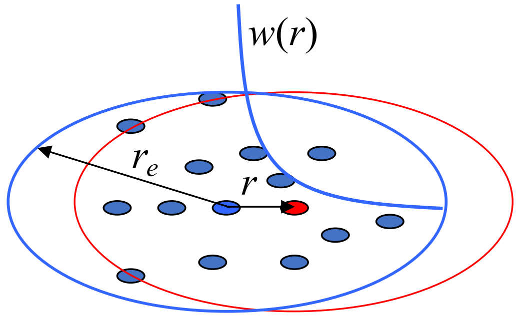


$$\langle f(r)\rangle = \int f(r')W(r-r',h)\,dr'$$

Moving Particle Semi-implicit (MPS)

- A macroscopic, deterministic gridless particle method proposed by Koshizuka and Oka (1996) initially for the simulation of **incompressible free-surface fluid flows**
- Computational elements = discrete number of particles of fluid followed in time
- Kernel-based interpolation of physical variables solely based on a local weighted averaging process
- Simplified differential operator models
- Solution process: ***semi-implicit***

Professor Seiichi Koshizuka
The University of Tokyo



$$w(r) = \begin{cases} \dfrac{r_e}{r} - 1 & 0 \le r < 1 \\ 0 & r_e < r \end{cases}$$

S. Koshizuka and Y. Oka, *Nuclear Science and Engineering*, 123, 421-434, 1996

Incompressible SPH (ISPH)

- A macroscopic, deterministic gridless particle method proposed by Shao and Lo (2003) for the simulation of ***incompressible free-surface fluid flows***
- *Inspired by MPS context, SPH-based differential operator models*
- Solution process: ***semi-implicit***
- MPS and ISPH are founded on a rigorous mathematical context, namely, ***Helmholtz-Leray Decomposition***

Dr. Songdong Shao
The University of Sheffield, UK

• S. Shao, E.Y.M. Lo, *Advances in Water Resources*, 26, 787-800, 2003

An excellent book on

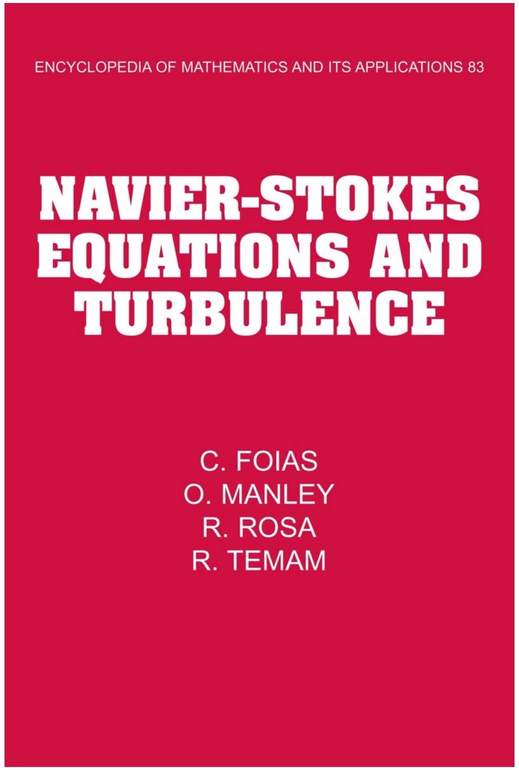
Helmholtz-Leray decomposition

Navier-Stokes Equations and Turbulence

by

C. Foias, O. Manley, R. Rosa and R. Temam

Series: Encyclopedia of Mathematics and its Applications
Paperback: 364 pages
Publisher: Cambridge University Press; 1 edition (June 12, 2008)
Language: English
ISBN-10: 0521064600
ISBN-13: 978-0521064606



Helmholtz-Leray decomposition

Helmholtz decomposition resolves a vector field ***u*** into the sum of a gradient and a curl vector.

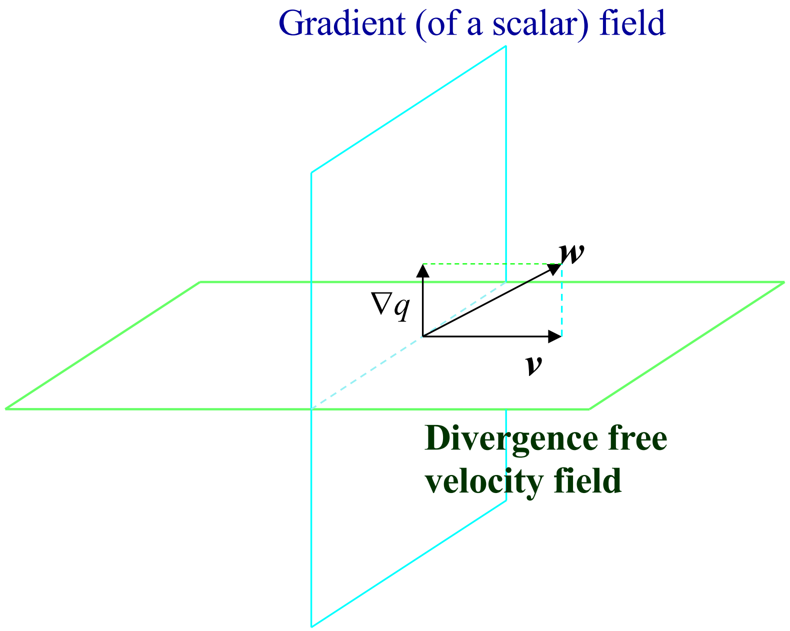
Helmholtz-Leray decomposition is an appropriate generalization, which is valid for the **vector field defined on a bounded set**, taking into account the boundary conditions.

$$\boldsymbol{w} = \nabla q + \boldsymbol{v} \qquad (1)$$

where $\boldsymbol{w}$ = vector field; q = scalar; $\boldsymbol{v}$ = curl vector.

The assumption, div $\boldsymbol{v} = 0$, leads to the following equation:

$$\Delta q = \operatorname{div} \ \boldsymbol{w} \qquad (2)$$



Helmholtz-Leray decomposition

Considering non-slip case as a bounded set,

$$\boldsymbol{v}\cdot\boldsymbol{n}=0 \quad \text{on} \quad \partial\Omega \qquad\qquad \text{or,} \qquad\qquad \frac{\partial q}{\partial \boldsymbol{n}}=\boldsymbol{w}\cdot\boldsymbol{n} \quad \text{on} \quad \partial\Omega \qquad (3)$$

We conclude q is solution of the Neumann problem Eqs. (2) and (3).

The necessary consistency condition

$$\int_{\Omega} \operatorname{div} \boldsymbol{w}\; dx=\int_{\partial\Omega} \boldsymbol{w}\cdot\boldsymbol{n}\; dS(x)$$

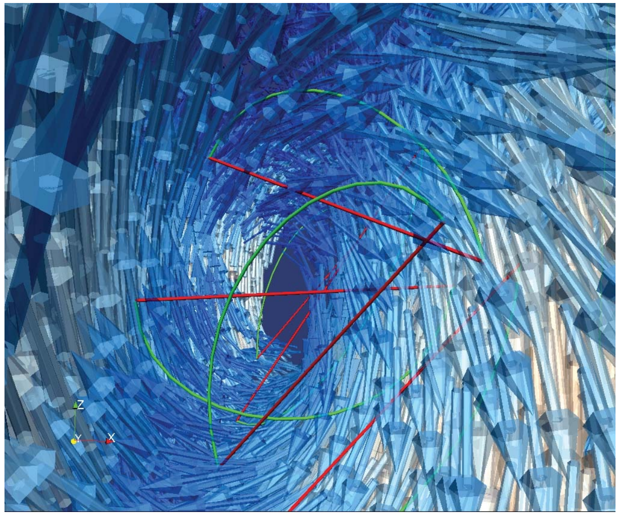
follows from the divergence theorem. Thus, q is **uniquely/exactly defined** up to an additive constant and $\boldsymbol{v}$ is equally well-defined.

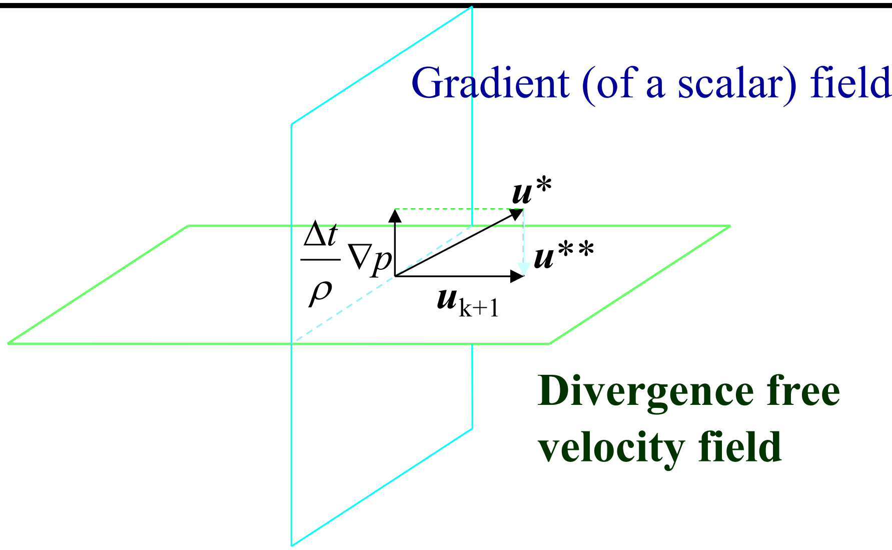
Also, if **connectivity condition**, i.e. boundary $\partial\Omega$ of Ω is connected (no holes in Ω), is true, the conditions

$$\operatorname{div} \boldsymbol{v}=0 \quad \text{in} \quad \Omega \qquad \text{and} \qquad \boldsymbol{v}\cdot\boldsymbol{n}=0 \quad \text{on} \quad \partial\Omega$$

imply the uniqueness of $\boldsymbol{v}$.

Projection-Based methods for incompressible fluids (ISPH or MPS)





Helmholtz-Leray decomposition $\quad \boldsymbol{u}^* = \boldsymbol{u}_{sol} + \boldsymbol{u}_{irrot} = \boldsymbol{u}_{sol} + \nabla\phi \qquad \boldsymbol{u}^* = \boldsymbol{u}_{k+1} + \frac{\Delta t}{\rho}\nabla p$

Unique/exact solution under the conditions of:

$$\begin{cases} \nabla \cdot \boldsymbol{u} = 0 \quad \text{in} \ \Omega \\ \boldsymbol{u} \cdot \boldsymbol{n} = 0 \quad \text{on} \ \partial\Omega \\ \text{Connectivity (boundary } \partial\Omega \text{ of } \Omega \text{ is connected)} \end{cases}$$

$\left\{ \begin{array}{l} \boxed{\text{Enhanced pressure field}} \\ \boxed{\text{Enhanced volume conservation}} \end{array} \right.$

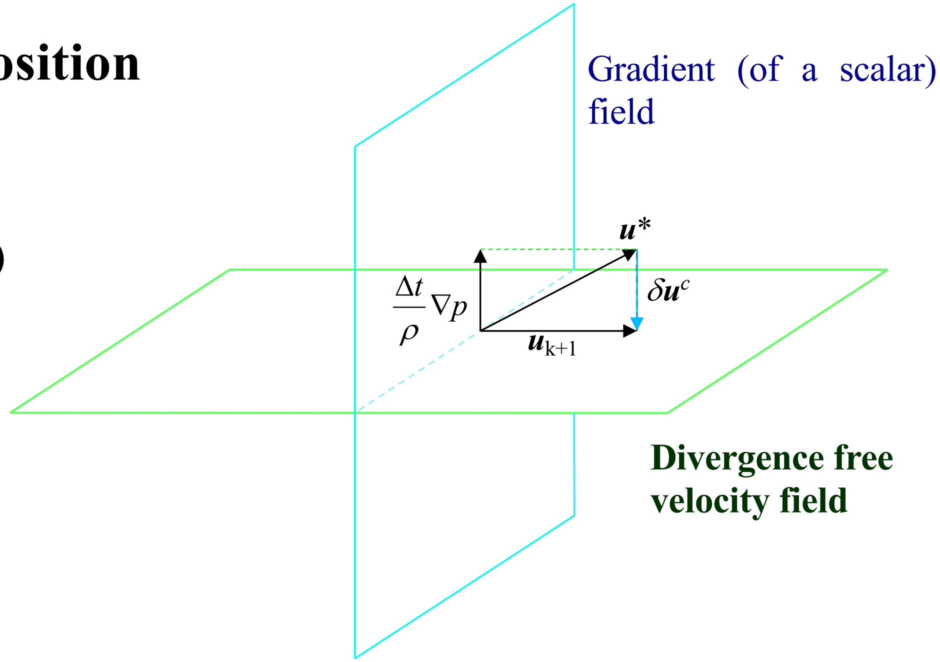
With respect to explicit SPH

Derivation of Poisson Pressure Equation in Projection Particle Methods

Helmholtz-Leray decomposition

$$\boldsymbol{u}^* = \boldsymbol{u}_{sol} + \boldsymbol{u}_{irrot} = \boldsymbol{u}_{sol} + \nabla \phi$$

sol = solenoidal (divergence-free)
irrot = irrotational



$$\boldsymbol{u}^* = \boldsymbol{u}_{k+1} + \frac{\Delta t}{\rho} \nabla p_{k+1}$$

$$\delta \boldsymbol{u}^c = \boldsymbol{u}_{k+1} - \boldsymbol{u}^* = -\frac{\Delta t}{\rho} \nabla p_{k+1}$$

Continuity equation

$$\left(\frac{D\rho}{Dt}\right)^c + \rho \nabla \cdot \delta \boldsymbol{u}^c = 0$$

$$\nabla \cdot \boldsymbol{u}_{k+1} = 0$$

Poisson Pressure Equation (PPE)

$$\nabla^2 p_{k+1} = \frac{1}{\Delta t} \left(\frac{D\rho}{Dt}\right)^c$$

c = correction step

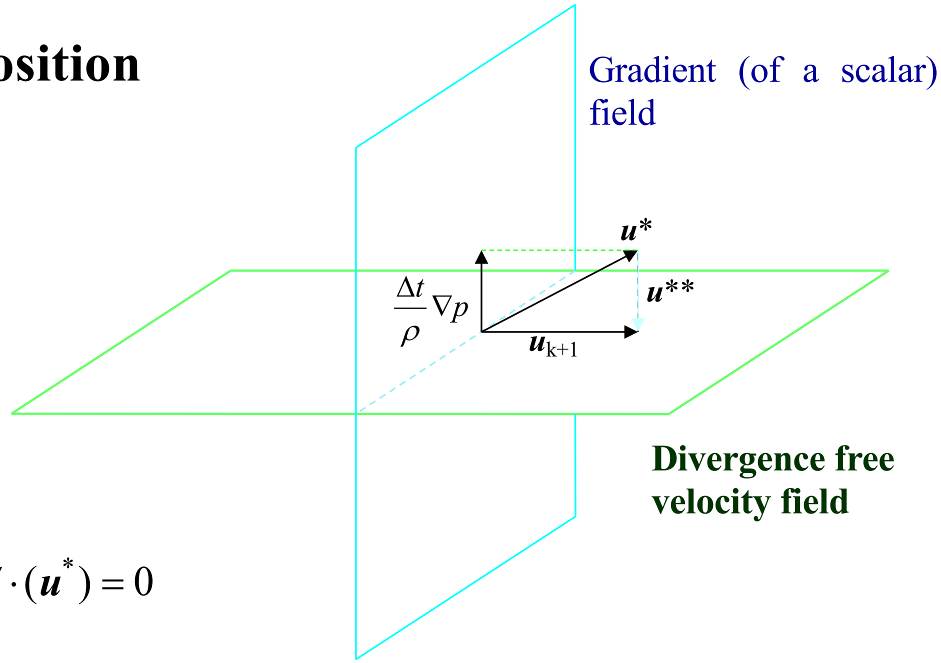
Derivation of Poisson Pressure Equation in Projection Particle Methods

Helmholtz-Leray decomposition

$$\boldsymbol{u}^* = \boldsymbol{u}_{k+1} + \frac{\Delta t}{\rho}\nabla p$$

$$\boldsymbol{u}_{k+1} - \boldsymbol{u}^* = \boldsymbol{u}_k^{**} = -\frac{\Delta t}{\rho}\nabla p$$

$$\Rightarrow \frac{\Delta t}{\rho}\left(\nabla^2 p_{k+1}\right)_i = \nabla \cdot \boldsymbol{u}^*$$



$$\frac{1}{\rho_0}\left(\frac{\mathrm{D}\rho}{\mathrm{D}t}\right)^* + \nabla\cdot(\boldsymbol{u}^*) = \frac{1}{n_0}\left(\frac{\mathrm{D}n}{\mathrm{D}t}\right)^* + \nabla\cdot(\boldsymbol{u}^*) = 0$$

* Corresponds to the intermediate calculated quantities

$$\frac{\Delta t}{\rho}\left(\nabla^2 p_{k+1}\right)_i = \nabla\cdot\boldsymbol{u}^* = -\frac{1}{n_0}\left(\frac{Dn}{Dt}\right)^* \; ; \; n = \sum_{i\neq j} w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) \longrightarrow \frac{\mathrm{D}n}{\mathrm{D}t} = \sum_{i\neq j}\frac{\mathrm{D}w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right)}{\mathrm{D}t}$$

MPS-HS

↓

Refined Scheme

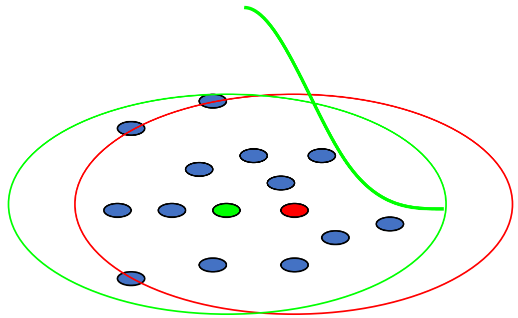
$$\frac{\Delta t}{\rho}\left(\nabla^2 p_{k+1}\right)_i = \nabla\cdot\boldsymbol{u}^* = \frac{1}{n_0}\left(\frac{n_0 - n^*}{\Delta t}\right) \longrightarrow \text{Original MPS}$$

ISPH and MPS – Projection-based fluid models

■ ISPH (Incompressible SPH) and MPS (Moving Particle Semi-implicit) methods

- Possible instabilities and inaccuracies (e.g., noise in physical fields including pressure field)
- Issues with convergence and conservation
- New schemes, methodologies, algorithms

$W(r-r',h) =$ Smoothing Kernel



$$\langle f(r) \rangle = \int f(r') W(r-r',h)\, dr'$$

Enhanced schemes & algorithms

☐HS ☐HL ☐ECS ☐GC ☐DS

☐FDS ☐OPS ☐CIECS

Enhanced ISPH or
Enhanced MPS

Khayyer et al., *European Journal of Mechanics - B/Fluids*, 2017
Khayyer et al., *Coastal Engineering* 2018.

Particle Methods for Fluid-Structure Interactions

■ Fluid Structure Interaction (FSI)

- ➢ Coastal/Ocean engineering problems
  - ✓ Storm surge impact on breakwaters
  - ✓ Sloshing in liquid containers
  - ✓ Slamming on ship hulls
  - ✓ Wave-current interaction with offshore wind turbines
- ➢ Strongly entangled mutual interactions and complex hydrodynamic-structure systems
- ➢ Environmental & Experimental Limitations
- ➢ Computational Engineering for FSI
- ➢ Comprehensive Structural Response
- ➢ *Hydroelastic FSI Solvers*

Entirely Lagrangian Meshfree FSI Solvers

- Entirely Lagrangian Meshfree FSI Solvers

1. Why an entirely Lagrangian meshfree FSI solver?
   - **Violent flows** with large/abrupt hydrodynamics loads and consequently **large structural deformations**
   - Precise satisfaction of fluid-structure **interface boundary conditions**
   - An integrated solver, enhanced **applicability/adaptivity**

2. Why a projection-based method for the fluid phase?
   - Not only because of relatively **accurate pressure field/volume conservation**, but also because of the advantage they bring about for a **consistent coupling** in between fluid and structure.

Entirely Lagrangian Meshfree FSI Solvers - Key Aspects

- Key aspects for development of reliable/efficient entirely Lagrangian meshfree hydroelastic FSI solvers

✓ **Stability and accuracy** (enhanced/consistent schemes)

1. **Choice of governing equations** for accurate reproduction of nonlinear structure responses with strict preservation of conserved physical quantities such as energy, linear and angular momenta.

2. Imposition of accurate and consistent fluid-structure **interface boundary conditions** (normal stress/volume continuities)

3. Enhancement of **adaptivity/applicability** corresponding to adaptive refinement of fluid/structure domains

4. **Generality** of FSI solvers corresponding to extension to **3D** simulations, arbitrary **choice of constitutive equations**, and reproduction of FSI comprising of **composite structures**

Entirely Lagrangian Meshfree FSI Solvers - Key Aspects

- Key aspects for development of reliable/efficient entirely Lagrangian meshfree hydroelastic FSI solvers

  ✓ **Stability and accuracy** (enhanced/consistent schemes)

  1. **Choice of governing equations**
  2. Accurate imposition of fluid-structure **interface boundary conditions** (normal stress/volume continuities)

  I. Reliability

  3. Enhancement of **adaptivity/applicability** corresponding to adaptive refinement of fluid/structure domains

  II. Adaptivity

  4. **Generality** of FSI solvers corresponding to extension to **3D** simulations, arbitrary **choice of constitutive equations**, and reproduction of FSI comprising of **composite structures**

  III. Generality

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

- Newtonian/Hamiltonian formulations for ***structure***
  - ***Newtonian (classical) mechanics***
    - Linear/angular momenta conservations
    - Notion of force
  - ***Hamiltonian mechanics***
    - Energy, linear and angular momenta conservations
    - More flexibility for extension of model to complex physical systems, such as nonlinear deformation
    - Minimum number of equations for description of the dynamics
    - Numerically more challenging

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

Newtonian Mechanics

Fluid phase

$$\nabla \cdot \boldsymbol{u} = 0$$

Continuity

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{g} \boxed{+\boldsymbol{a}^{SF}}$$

Navier-Stokes

Fluid-Structure Interaction

Structure phase

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)^S = -\frac{1}{\rho^S}\nabla \cdot \boldsymbol{\sigma}^S + \boldsymbol{g} \boxed{+\boldsymbol{a}^{FS}}$$

Conservation of linear momentum

$$\frac{D}{Dt}\left(\boldsymbol{I}^S \cdot \boldsymbol{\omega}^S\right) = \frac{D}{Dt}\left(\boldsymbol{r} \times m\boldsymbol{u}\right)^S$$

Conservation of angular momentum

$$\boldsymbol{\sigma}^S = \lambda^S tr(\boldsymbol{\varepsilon}^S)\boldsymbol{I} + 2\mu^S \boldsymbol{\varepsilon}^S$$

ρ : density	$\boldsymbol{g}$: gravitational acceleration vector	$\boldsymbol{\sigma}$: stress tensor	ν : kinematic viscosity
t : time	$\boldsymbol{u}$: velocity vector	$\boldsymbol{\varepsilon}$: strain tensor	I : moment of inertia
p : pressure	$\boldsymbol{r}$: position vector	m : mass	λ, μ : Lame's constants

Khayyer et al., *Journal of Fluids & Structures*, 2021.

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

Structure Phase

Newtonian

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_S = -\frac{1}{\rho_S}\nabla\cdot\boldsymbol{\sigma}_S + \boldsymbol{g} + \boldsymbol{a}_{F\,to\,S}$$

Conservation of linear momentum

$$\boldsymbol{\sigma}_S = \lambda_S tr(\boldsymbol{\varepsilon}_S)\boldsymbol{I} + 2\mu_S\boldsymbol{\varepsilon}_S$$

$$\left(\frac{D}{Dt}I\boldsymbol{\omega}\right)_S = \frac{D}{Dt}(\boldsymbol{r}\times m\boldsymbol{u})$$

Conservation of angular momentum

Hamiltonian

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_S = -\frac{1}{\rho_S}\left(\frac{\partial\psi}{\partial\boldsymbol{r}}\right)_S + \boldsymbol{g} + \boldsymbol{a}_{F\,to\,S}$$

Hamiltonian-based conservation of linear momentum

ψ : Strain energy density function

Khayyer et al., *Journal of Hydrodynamics*, 2018.

Khayyer et al., *Journal of Fluids & Structures*, 2021.

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

Hamiltonian Structure Model

■ Conservation of linear momentum

$$\frac{D\boldsymbol{u}^S}{Dt} = -\frac{1}{\rho^S}\underline{\frac{\partial \psi^S}{\partial \boldsymbol{r}}} + \boldsymbol{g} + \boldsymbol{a}^{FS}$$

$$\frac{\partial \psi^S}{\partial \boldsymbol{r}} = \frac{\partial \psi^S}{\partial \boldsymbol{F}^S} : \frac{\partial \boldsymbol{F}^S}{\partial \boldsymbol{r}} = \boldsymbol{P}^S : \frac{\partial \boldsymbol{F}^S}{\partial \boldsymbol{r}}$$

$\boldsymbol{P}^S$ = first Piola-Kirchhoff stress tensor

$\boldsymbol{S}^S$ = second Piola-Kirchhoff stress tensor

$\boldsymbol{E}^S$ = Green-Lagrange strain tensor

$$\boldsymbol{P}^S = \boldsymbol{F}^S \cdot \boldsymbol{S}^S$$

$$\boldsymbol{S}^S = \underline{\frac{\partial \psi^S}{\partial \boldsymbol{E}^S}} = \lambda^S tr\left(\boldsymbol{E}^S\right) + 2\mu^S \boldsymbol{E}^S$$

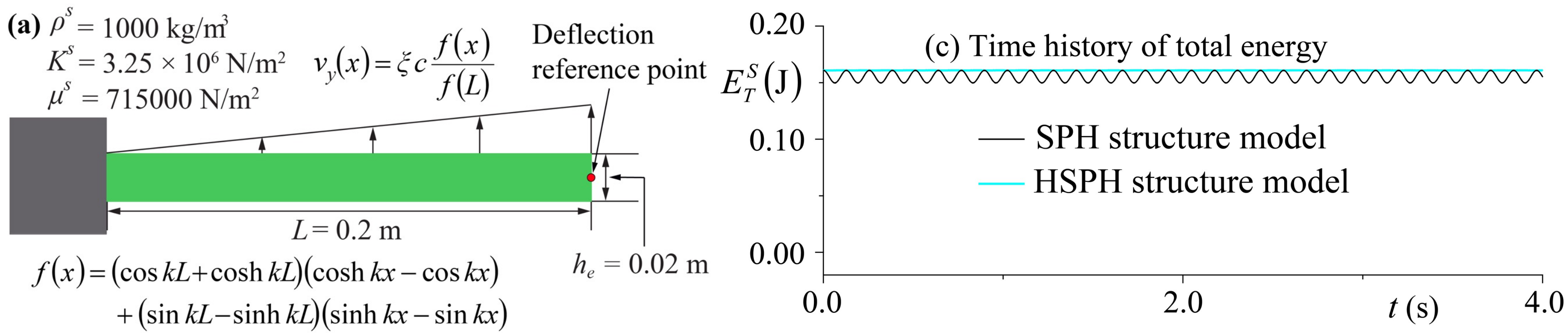
$$\boldsymbol{E}_i^S = \frac{1}{2}\left\{\left(\boldsymbol{F}_i^S\right)^T \cdot \boldsymbol{F}_i^S - \boldsymbol{I}\right\}$$

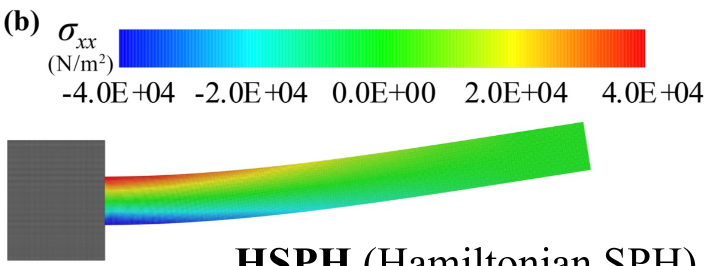
- Saint Venant-Kirchhoff hyperelastic model
- Finite strain, geometrical non-linearity

Variationally Consistent Framework

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

■ Dynamic response of a free oscillating cantilever plate





HSPH (Hamiltonian SPH) structure model

✓Stable and smooth stress field

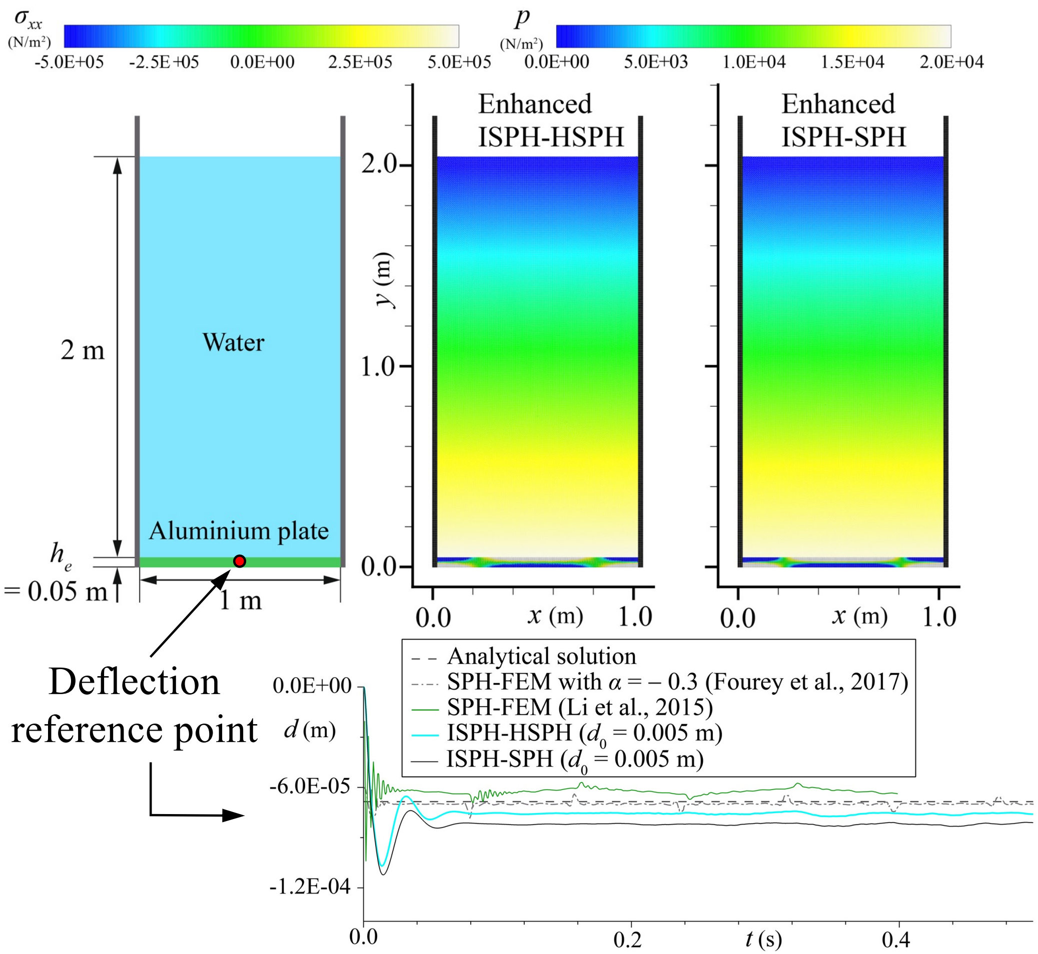
✓Accurate energy conservation

Khayyer et al., *Journal of Hydrodynamics*, 2018.

Gray et al., *Comput. Methods Appl. Mech. Eng.*, 2001

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

- Hydrostatic water column on an elastic plate (Fourey et al., 2017)



✓Stable stress/ pressure fields

✓ISPH-HSPH provides more accurate deflection time history than ISPH-SPH.

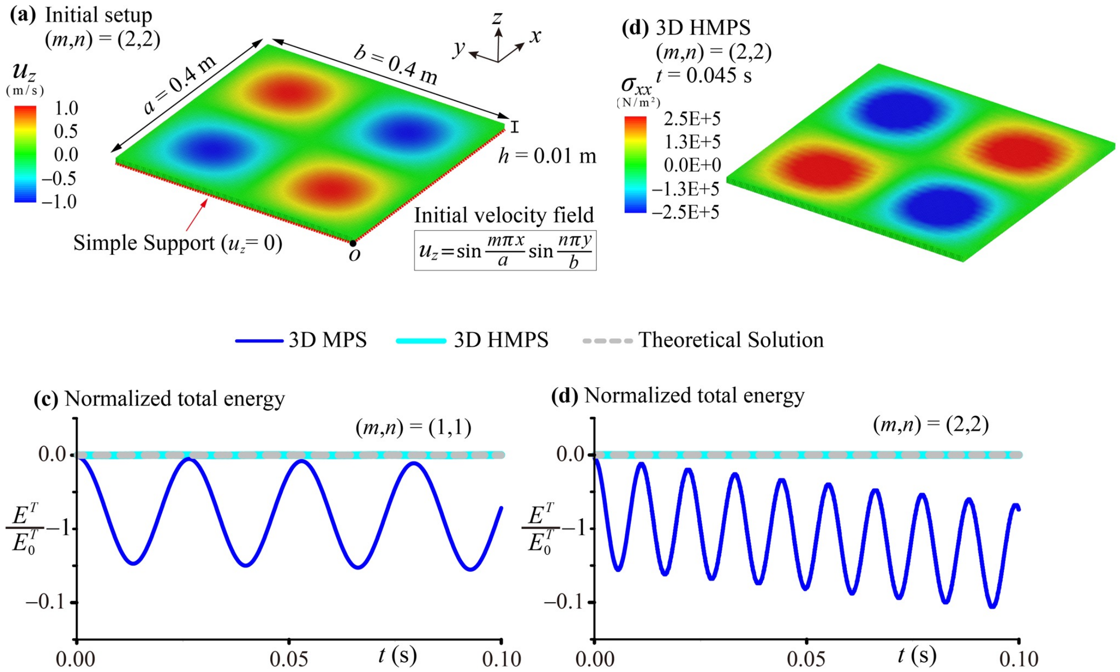
Khayyer et al., Journal of Hydrodynamics, 2018.

Fouray et al., Comput Phys Comm, 2017.

Key aspects – I. ***Reliability*** – Choice of governing equations (e.g. Newtonian or Hamiltonian frameworks)

- Free vibration of a thin plate

HMPS vs MPS



✓ Smooth stress field

✓ HMPS is clearly shown to outperform MPS in terms of conservation of energy.

Khayyer et al., *Journal of Fluids and Structures*, 2021.

Key aspects – I. ***Reliability*** – Satisfaction of interface boundary conditions

- Fluid-Structure interface boundary conditions
  - Velocity and normal stress continuities

$$\begin{cases} \boldsymbol{u}^S = \boldsymbol{u}^F \\ \boldsymbol{\sigma}^S \cdot \boldsymbol{n}^S = -\boldsymbol{\sigma}^F \cdot \boldsymbol{n}^F \end{cases}$$

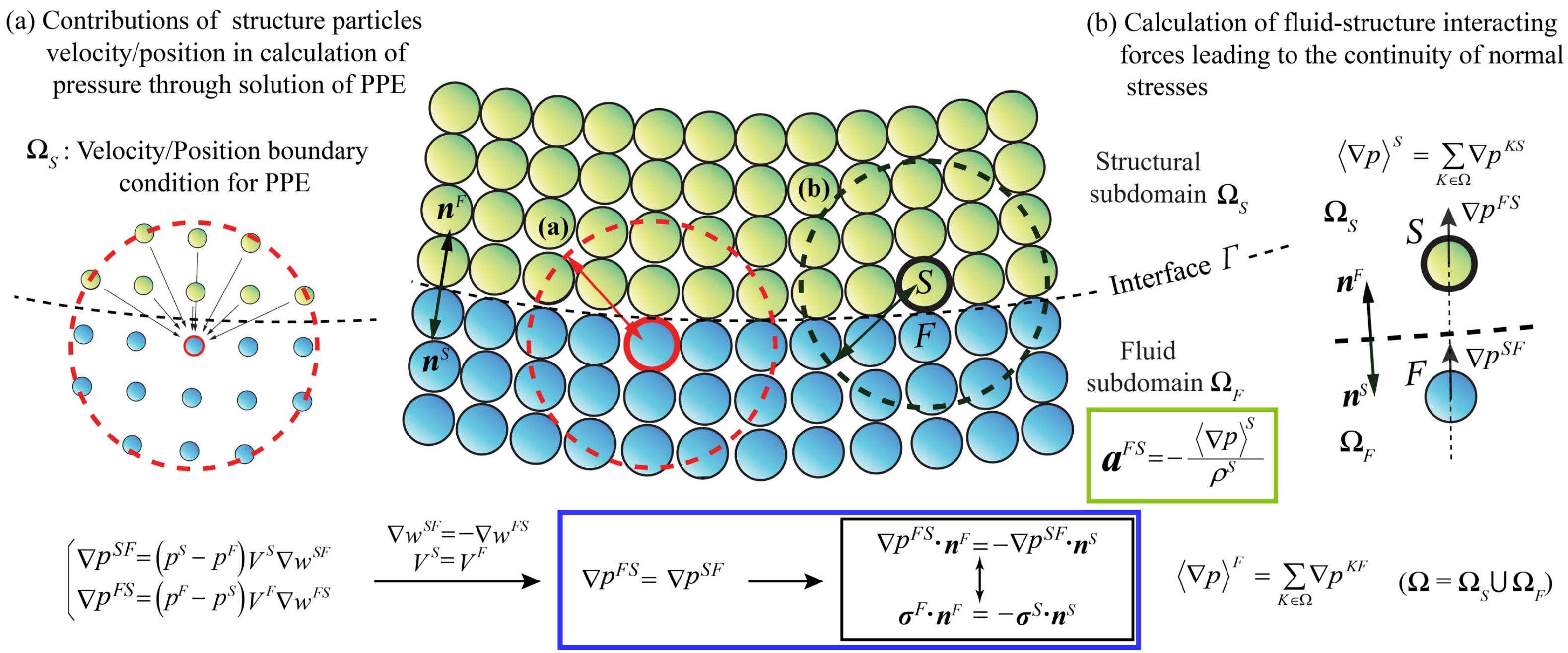
  - Conservation of volume at the interface
  - Spatially continuous transfer of momentum

- **F**luid-**S**tructure **A**cceleration-based (**FSA**) coupling
- **P**ressure-**I**ntegration (**PI**) coupling

Key aspects – I. *Reliability* – Satisfaction of interface boundary conditions

- **Fluid-Structure Acceleration-based (FSA) coupling**

Velocity continuity ($\boldsymbol{u}^F = \boldsymbol{u}^S$)

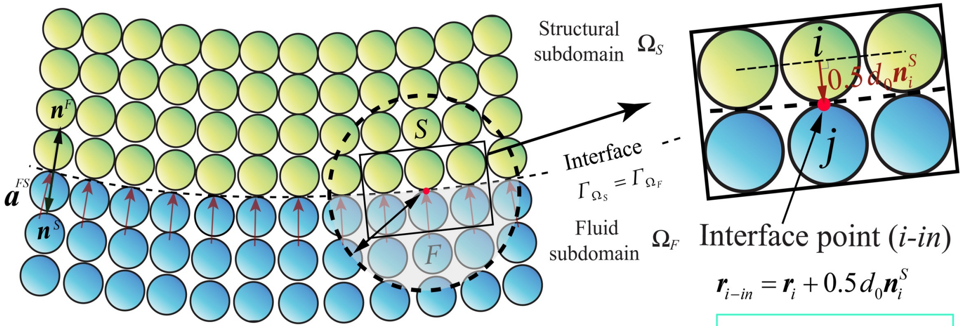


Normal stress continuity
at interface

Khayyer et al., *Computer Physics Communications*, 2018.

Key aspects – I. ***Reliability*** – Satisfaction of interface boundary conditions

- **P**ressure-**I**ntegration (**PI**) coupling



$$\boldsymbol{r}_{i-in} = \boldsymbol{r}_i + 0.5\, d_0 \boldsymbol{n}_i^S$$

$$\langle \nabla p \rangle_i = \int_{\Omega_S} p_j \nabla w_{ij} dV + \boxed{\int_{\Gamma_{\Omega_S}} p_j w_{ij} \boldsymbol{n}_j dA}$$

$$\boxed{p_{i-in} = \frac{\sum_{j \in \Omega_F} p_j w_{i-in\, j}}{\sum_{j \in \Omega_F} w_{i-in\, j}}}$$

Averaging

$$\boxed{\int_{\Gamma_{\Omega_S}} p_j w_{ij} \boldsymbol{n}_j dA} \approx \frac{\sum_{j \in \Gamma_{\Omega_S}} p_j w_{ij} \boldsymbol{n}_j^S A_j^S}{\sum_{j \in \Omega_S} w_{ij} V_j^S} \approx \frac{\boxed{p_{i-in}} \sum_{j \in \Gamma_{\Omega_S}} w_{ij} \boldsymbol{n}_j^S A_j^S}{\sum_{j \in \Omega_S} w_{ij} V_j^S} \approx \frac{p_{i-in} \boldsymbol{n}_i^S A_i^S}{V_i^S}$$

$$(\boldsymbol{\sigma} \cdot \boldsymbol{n})_i^S = -(\boldsymbol{\sigma} \cdot \boldsymbol{n})_i^F = -p_{i-in} \boldsymbol{n}_i^F = -\left(\frac{\boldsymbol{F}^{FS}}{A^S}\right)_i \rightarrow \boldsymbol{F}_i^{FS} = p_{i-in} \boldsymbol{n}_i^F A_i^S \rightarrow \boxed{\boldsymbol{a}_i^{FS} = \left(\frac{\boldsymbol{F}^{FS}}{m^S}\right)_i}$$

$$\boxed{\boldsymbol{\sigma}^F = \begin{bmatrix} p_{in} & 0 \\ 0 & p_{in} \end{bmatrix} \rightarrow \boldsymbol{\sigma}^S \cdot \boldsymbol{n}^S = -\boldsymbol{\sigma}^F \cdot \boldsymbol{n}^F = -p_{in} \boldsymbol{n}^F}$$

Normal stress continuity at interface (conditional)

✓Accelerations from fluid to structure are transferred only to structure boundary particles through approximation of a surface integral.

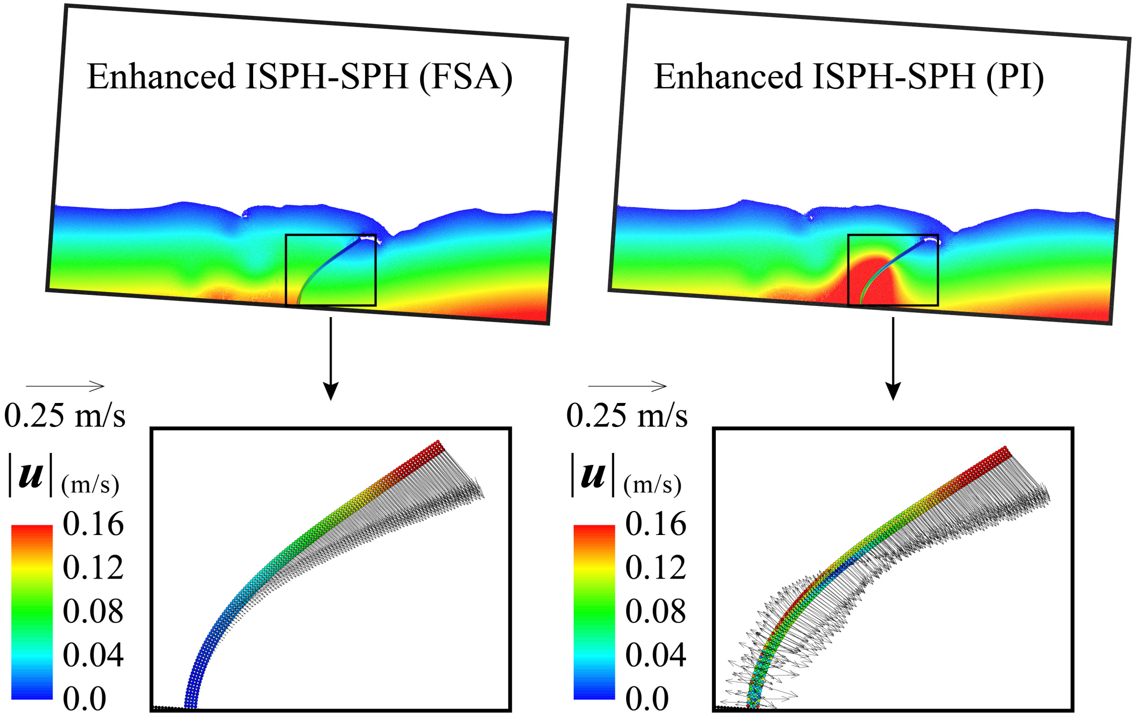
Khayyer et al., *Computer Physics Communications*, 2018.

Antoci et al, Computers and Structures, 2007.

Key aspects – I. ***Reliability*** – Satisfaction of interface boundary conditions

- **ISPH-SPH** (Khayyer et al., 2018)
  - ✓Investigation of **FSA/PI** fluid-structure coupling schemes

t = 2.32 s

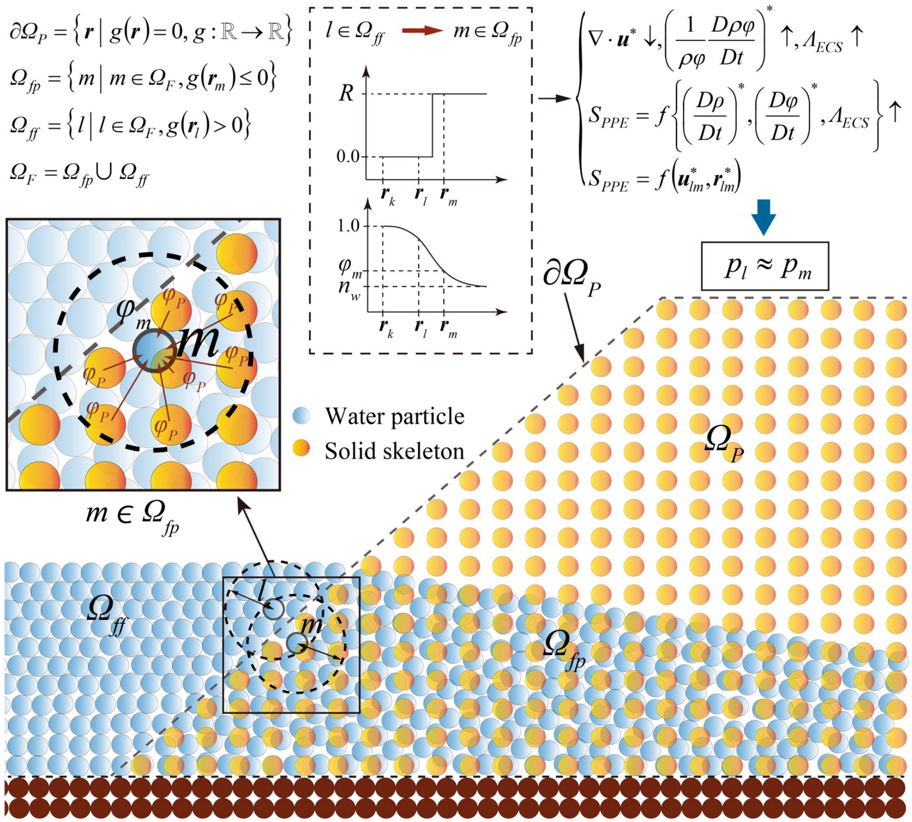


- ✓Careful investigations are carried out for **coupling schemes** in between fluid and structure phases.
- ✓FSA is shown to outperform PI for homogeneous structures.

Khayyer et al., *Computer Physics Communications*, 2018.

Key aspects – I. ***Reliability*** – Satisfaction of interface boundary conditions

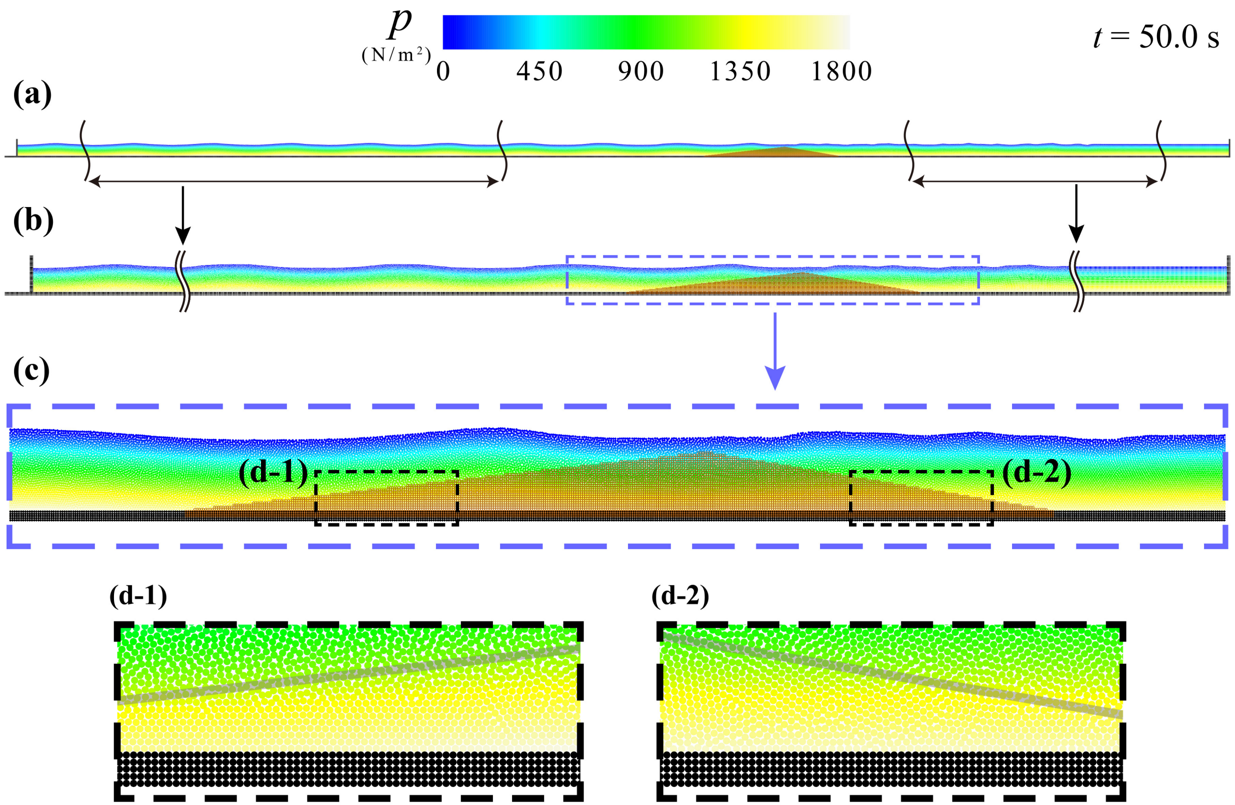
ISPH-based numerical wave flume for wave interactions with porous media of variable porosity



- ✓ Modification of source term of Poisson Pressure Equation (PPE) with consideration of fluid volume fraction (*mixture theory*).
- ✓ Fluid-porous media interface boundary conditions (velocity/stress continuities) are well satisfied.
- ✓ Fluid-porous media with variable porosities.

Khayyer et al., Development of a projection-based SPH method for numerical wave flume with porous media of variable porosity, Coastal Engineering, 2018.

Key aspects – I. ***Reliability*** – Satisfaction of interface boundary conditions



✓Stable/smooth pressure field at the fluid-porous media interface

Khayyer et al., *Coastal Engineering*, 2018.

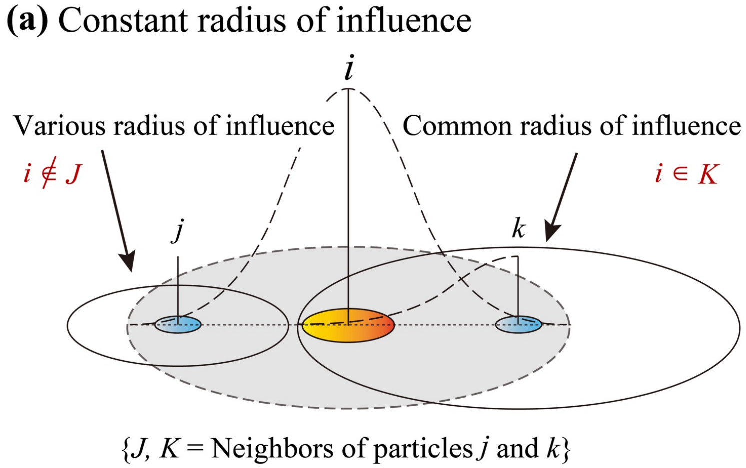
Key aspects – II. ***Adaptivity/applicability***

- Multi-resolution FSI solver
  - Computational challenges corresponding to memory/CPU time
  - ***Adaptable refinement of fluid/structure domains***
  - Wide range of applicability
    (e.g. 3D large scale FSI simulations containing thin elastic structure)
- Multi-resolution scheme
  - Common radius of influence / revised weight function
  - SPP scheme
- Fluid-Structure Coupling scheme for Multi-resolution
  - **F**luid-**S**tructure **A**cceleration-based (**FSA**) coupling scheme
  - Normal stress continuity at fluid-structure interface

Khayyer et al., *Applied Ocean Research*, 2019
Khayyer et al., *Ocean Engineering,* 2021

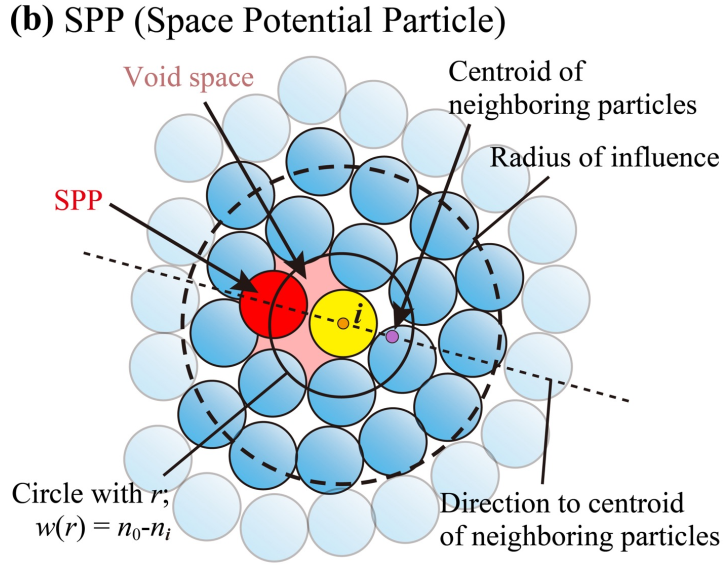
Key aspects – II. *Adaptivity/applicability*

- Multi-resolution FSI solver



Revised number density

$$n_i^{new} = \sum_{j \neq i} w_{ij}^{new} \; ; \; w_{ij}^{new} = \frac{V_j w(|\boldsymbol{r}_{ij}|, r_e)}{V_0}$$



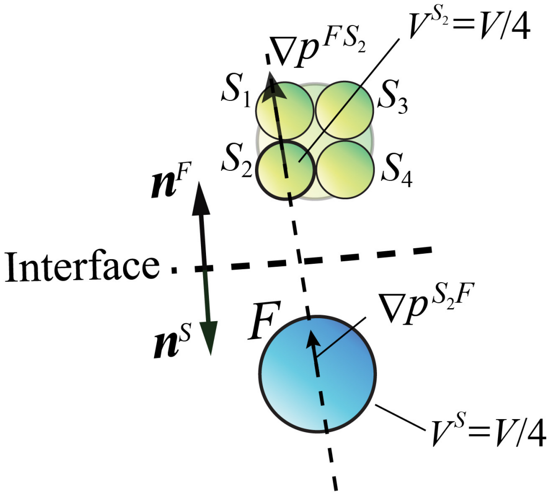
SPP: Tsuruta et al., *Int. Jour. Comp. Fluid Dyn.*, 2015

- A multi-resolution scheme comprising of revised weight function, revised number density, potential number density concepts and SPP scheme to enhance i) consistency of particle-based discretizations, ii) imposition of boundary conditions and iii) volume conservation at fluid-structure interface.

Khayyer et al., *Applied Ocean Research*, 2019

Key aspects – II. ***Adaptivity/applicability***

■ Fluid-Structure Acceleration-based (FSA) coupling



$$w_{ij}^{new}=\frac{V_j\, w\left(\left|\boldsymbol{r}_{ij}\right|, r_e\right)}{V_0}\ ;\ V^S=\frac{V^F}{4}$$

- $\rho^F V^F\left(-\frac{\nabla p^{SF}}{\rho^F}\right)\approx -V_0\sum_m^4 \nabla p^{S_mF}$
- $\rho^S V^S\left(-\frac{\nabla p^{FS}}{\rho^S}\right)\approx -\frac{1}{4}V_0\sum_m^4 \nabla p^{FS_m}$
- $\rho^F V^F\left(-\frac{\nabla p^{S_mF}}{\rho^F}\right)=\rho^{S_m} V^{S_m}\left(-\frac{\nabla p^{FS_m}}{\rho^{S_m}}\right)$

$$\Leftrightarrow \nabla p^{S_mF}=\frac{V^{S_m}}{V^F}\nabla p^{FS_m}=\frac{1}{4}\nabla p^{FS_m}$$

$\longrightarrow$ Equivalent pressure gradient

$$\nabla p^{FS}\approx \nabla p^{SF}$$

$$\downarrow$$

$$\boxed{\begin{array}{c}\nabla p^{FS}\cdot \boldsymbol{n}^F=-\nabla p^{SF}\cdot \boldsymbol{n}^S\\ \Updownarrow\\ (\boldsymbol{\sigma}^F\cdot \boldsymbol{n}^F)\cdot \boldsymbol{e}^{FS}=(-\boldsymbol{\sigma}^S\cdot \boldsymbol{n}^S)\cdot \boldsymbol{e}^{FS}\end{array}}$$

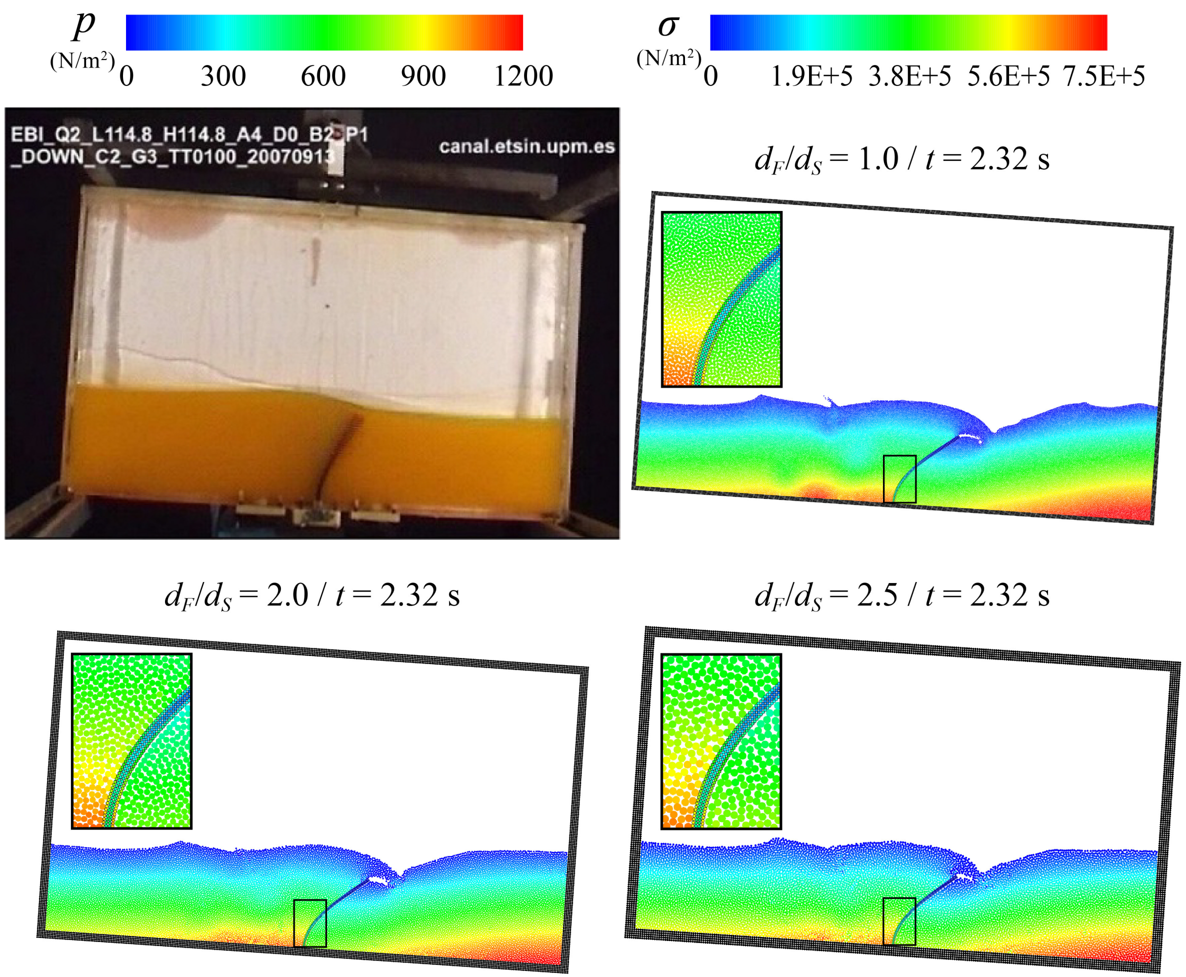
Normal stress continuity
at fluid-structure interface

- Multi-resolution implementation is conducted with careful consideration of **interface boundary conditions**.

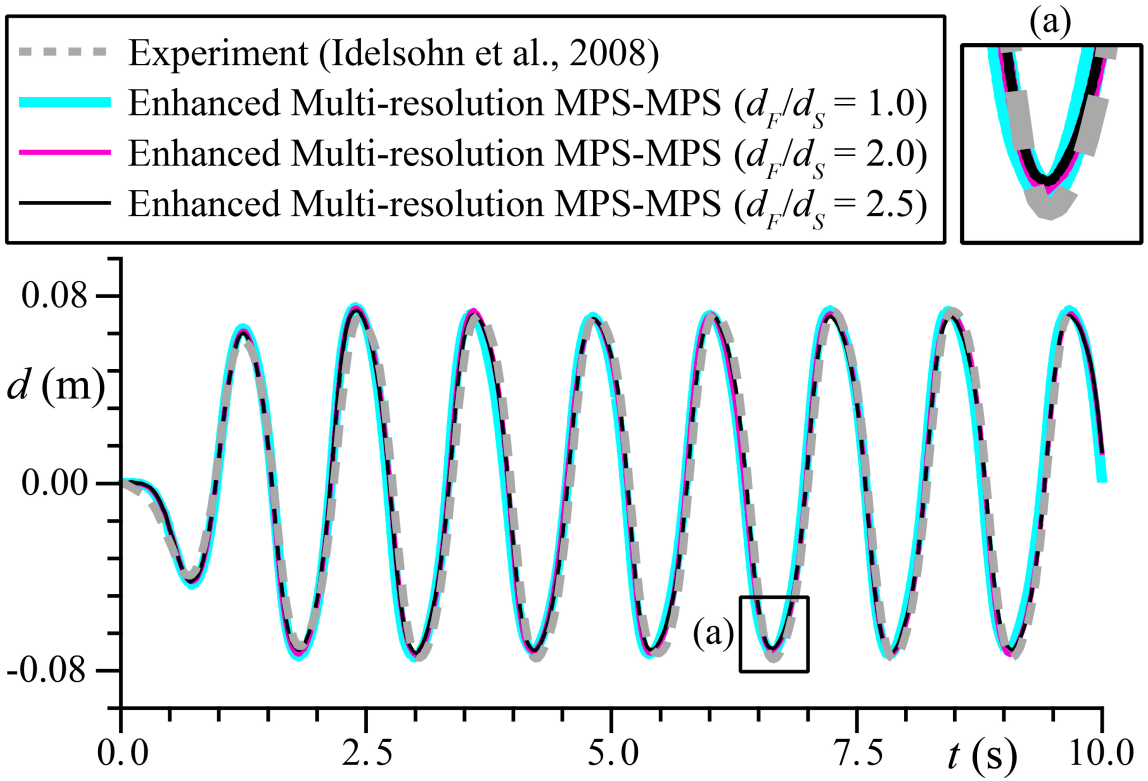
Khayyer et al., *Appl. Ocean Res.*, 2019.

Key aspects – II. *Adaptivity/applicability*

- Sloshing in a rolling tank with a bottom clamped elastic baffle
- Experiment by Idelsohn et al., Computational Mechanics, 2008
- Qualitative comparison of results by MPS-MPS FSI solver for different fluid/structure diameter ratios d_F/d_S.



Key aspects – II. ***Adaptivity/applicability***



✓ For all diameter ratios, time histories of displacement of elastic baffle's free end are in good agreement with experiment.

Key aspects – II. *Adaptivity/applicability*

- Computational time

	d_F/d_S	Number of particles	CPU time for 1 second of calculation (s)				
			Prediction	Correction	Structure	Others*	**Overall**
PARDISO	1	79685	2103.27	9463.45	4374.07	2305.07	18245.93
	2	22621	587.18	2259.85	3556.50	646.82	7050.34
	2.5	15532	449.01	1563.62	3376.42	517.45	5906.59
CG	1	79685	2098.54	11888.74	4379.20	2296.74	20663.25
	2	22621	574.68	1955.40	3472.85	657.13	6660.08
	2.5	15532	440.89	1381.09	3360.10	502.91	5685.06

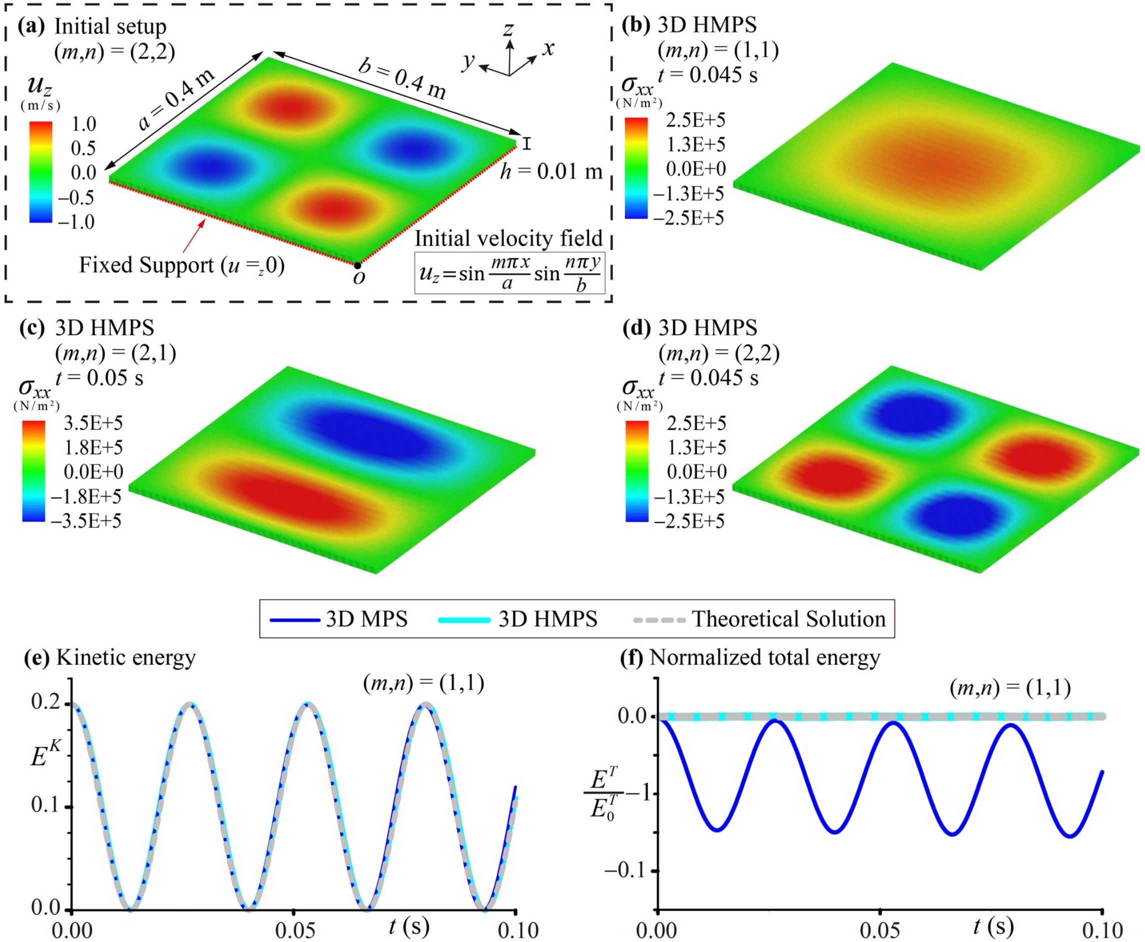
*Others include data storage, data output and computations of variables for output

✓ Clear enhancement of computational efficiency

Key aspects – III. *Generality*

- **3D** Entirely Lagrangian Meshfree projection-based hydroelastic FSI solvers
  - For application to 3D engineering problems
  - The 3D structure models are configured based on extension of the previously developed 2D MPS/HMPS structure models.
- Choices of different **constitutive equations**
  - Appropriate constitutive equations reproducing nonlinearities
- Particle-based structure models for **composite structures**
  - Composite materials are widely used in different types of engineering structures.
  - A novel structure model for simulation of laminated composite structure in the framework of HSPH (Khayyer et al., *Applied Math. Modelling*, 2021).

Key aspects – III. ***Generality*** – 3D structure models

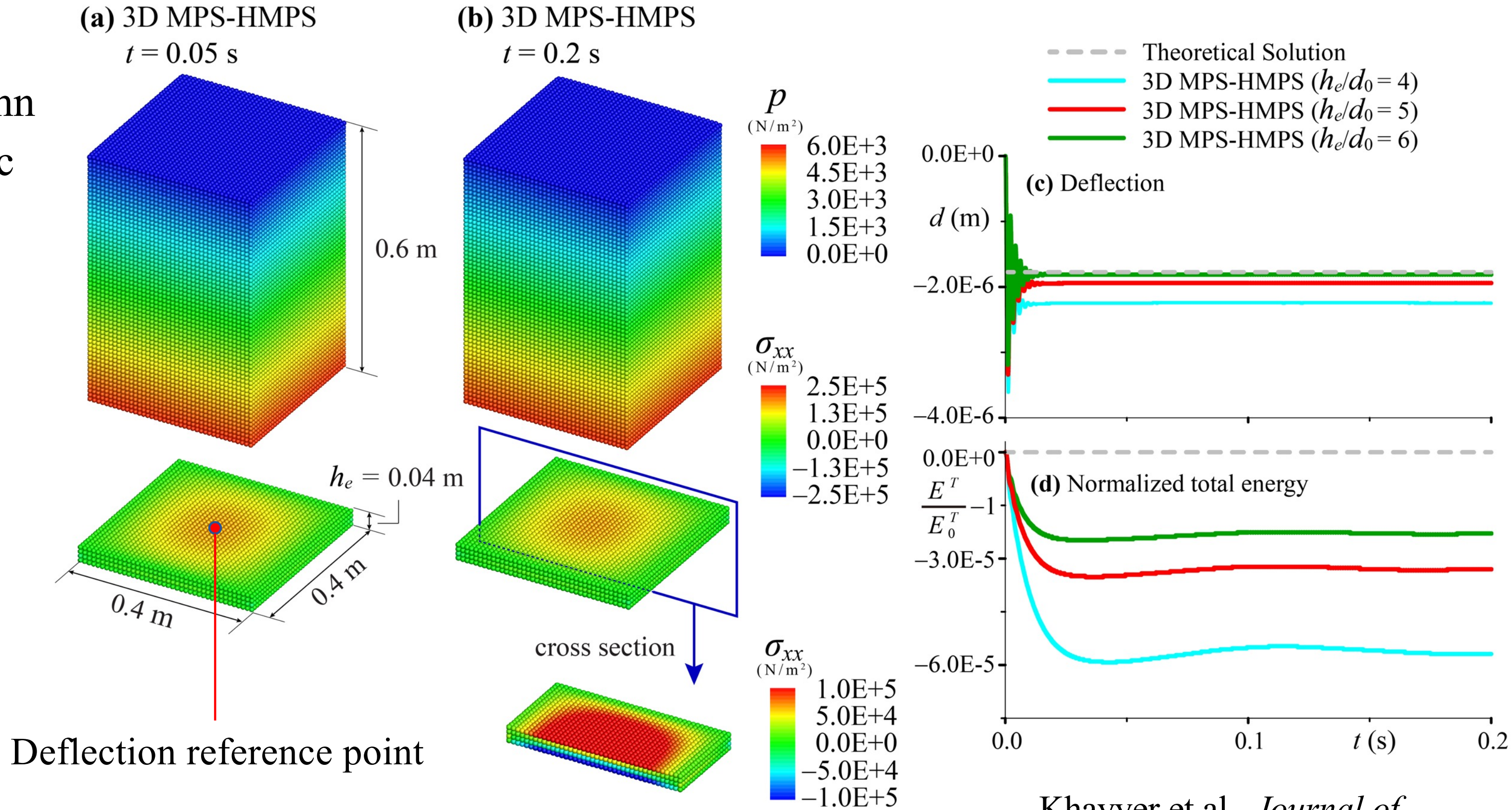


- ✓ Free vibration of a thin elastic plate
- ✓ Smooth stress field
- ✓ Accurate energy conservation by the HMPS structure model
- ✓ Detailed quantitative analyses for different vibration modes

Khayyer et al., *Journal of Fluids and Structures*, 2021.

Key aspects – III. ***Generality*** – 3D structure models

- Hydrostatic water column on an elastic plate
- 3D MPS-HMPS



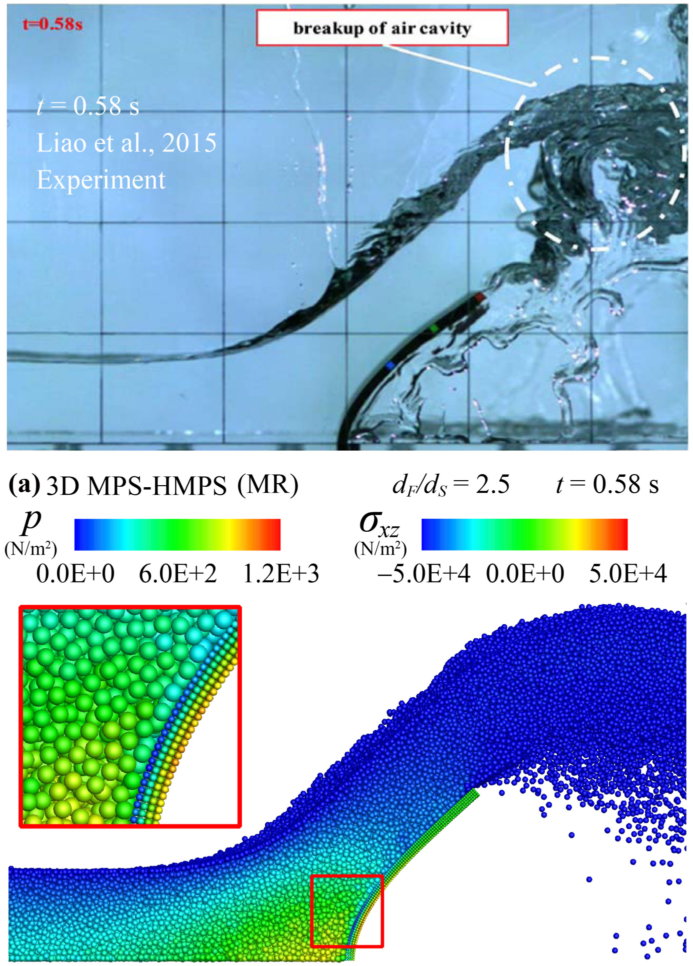
✓ Smooth stress/pressure fields

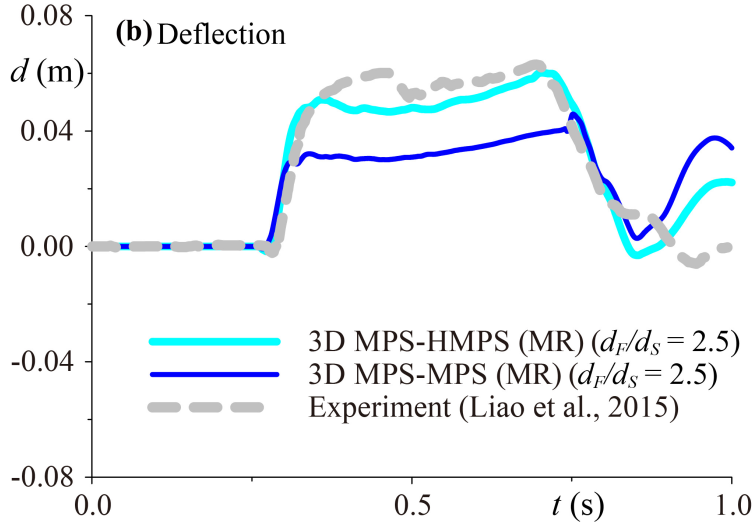
✓ Good convergence / energy conservation property

Khayyer et al., *Journal of Fluids and Structures*, 2021.

Fourey et al., *Comp. Phys. Comm.*, 217, 2017.

Key aspects – III. ***Generality & Adaptivity*** – 3D adaptive hydroelastic FSI solvers



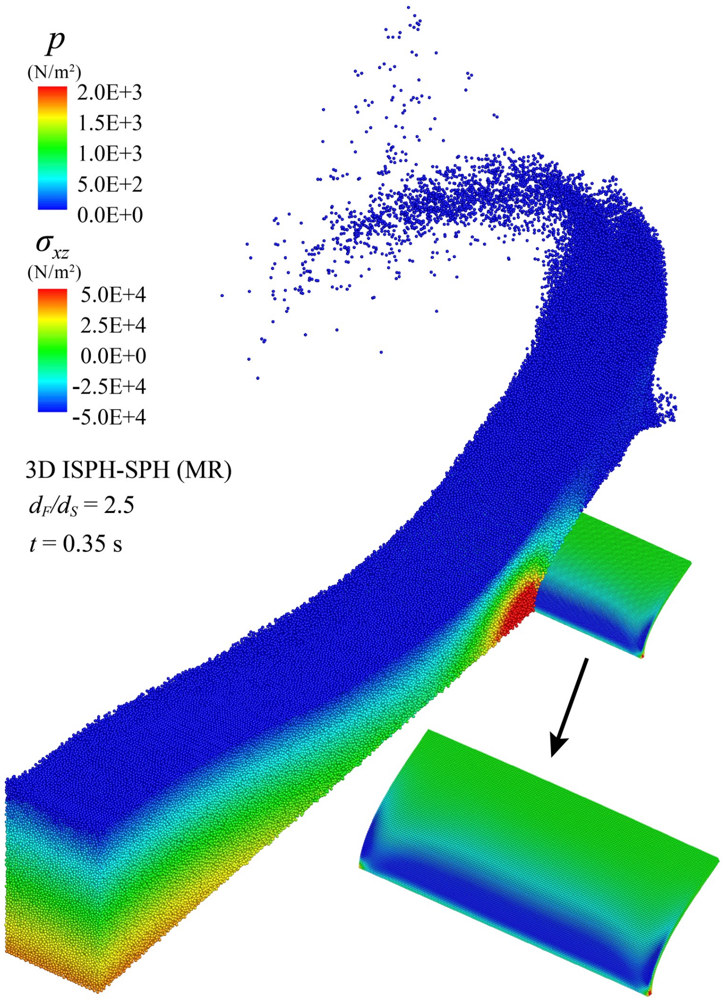


- ✓ Smooth stress/pressure fields are obtained by 3D MPS-HMPS (MR).
- ✓ MPS-HMPS provides almost acceptable deflection time history with respect to that of experiment.

Khayyer et al., *Journal of Fluids & Structures*, 2021.

Experiment: Liao et al., *Appl. Ocean Res.*, 50, 2015.

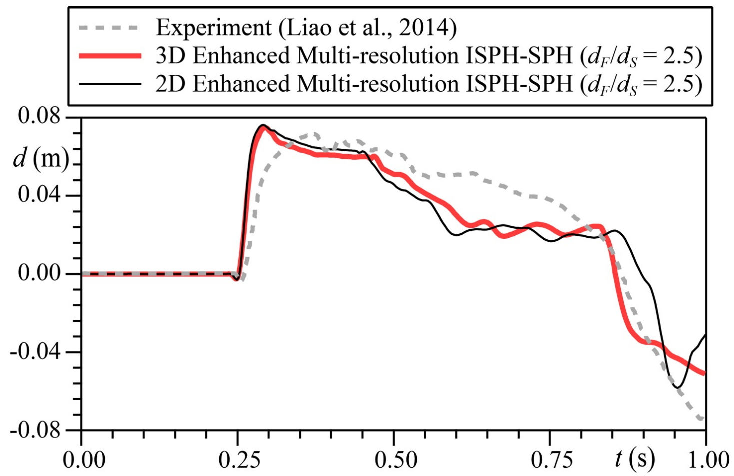
Key aspects – III. ***Generality & Adaptivity*** – 3D adaptive hydroelastic FSI solvers



- Dam break impact on an elastic plate

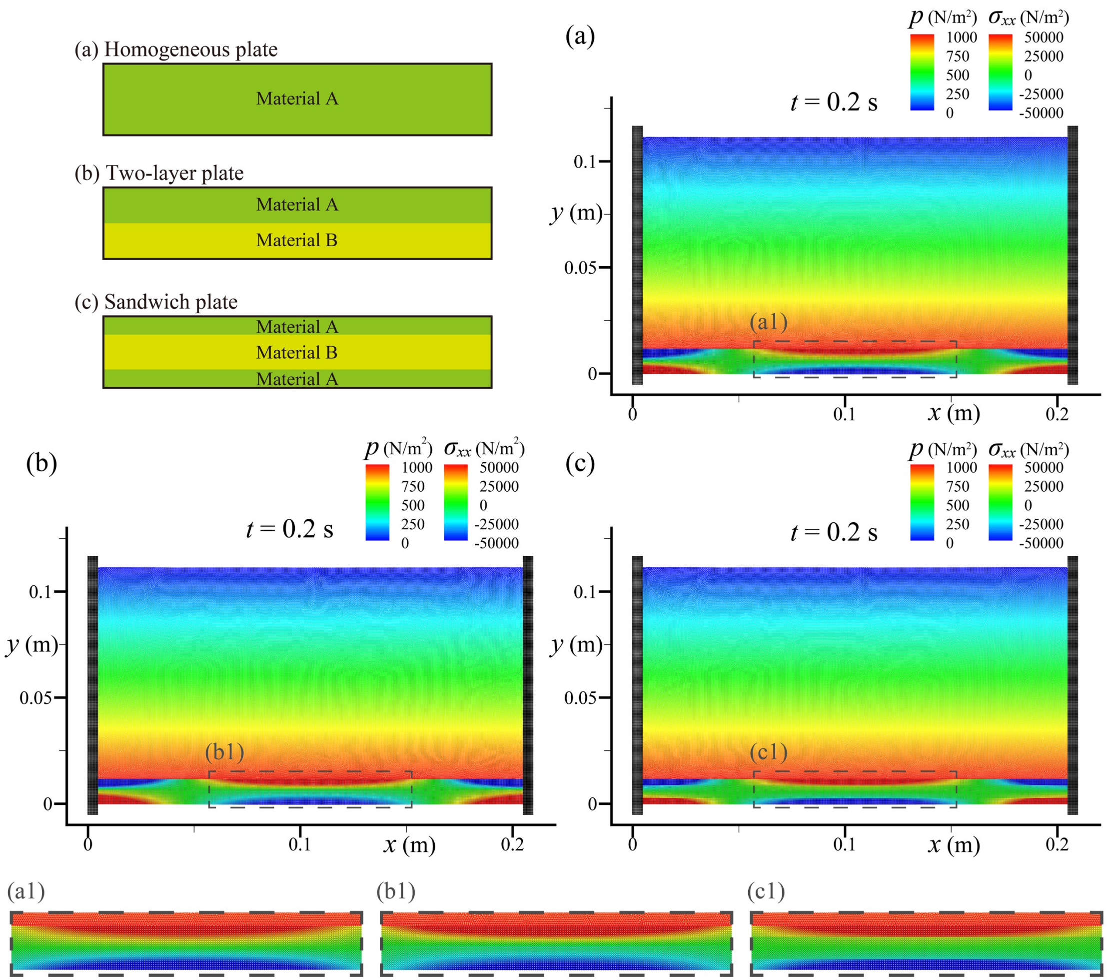
✓ Smooth stress/pressure fields are obtained by 3D ISPH-SPH (Multi-Resolution).

✓ ISPH-SPH provides acceptable *deflection time history* with respect to that of experiment.



Khayyer et al., *Ocean Engineering*, 2021.

Key aspects – III. ***Generality*** – Hydroelastic FSI solvers for ***Composite materials***

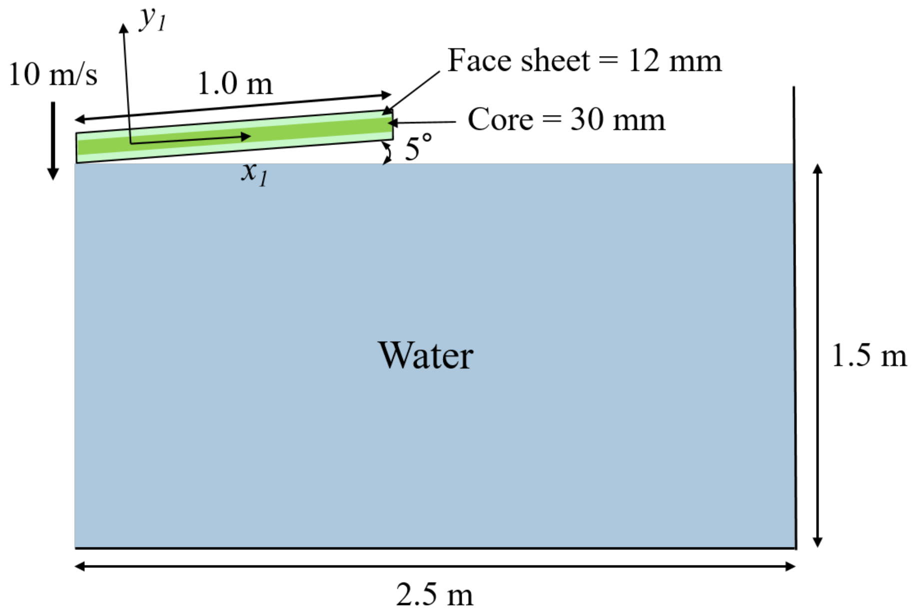


- ✓ Hydrostatic water column on *composite elastic plate*
- ✓ Smooth/noiseless stress/pressure fields
- ✓ The first entirely Lagrangian meshfree FSI solver for composite structures
- ✓ Density and Elastic Modulus ratios of up to 1:1000 tested and validated both qualitatively and quantitatively.

Key aspects – III. *Generality* – Hydroelastic FSI solvers for ***Composite materials***

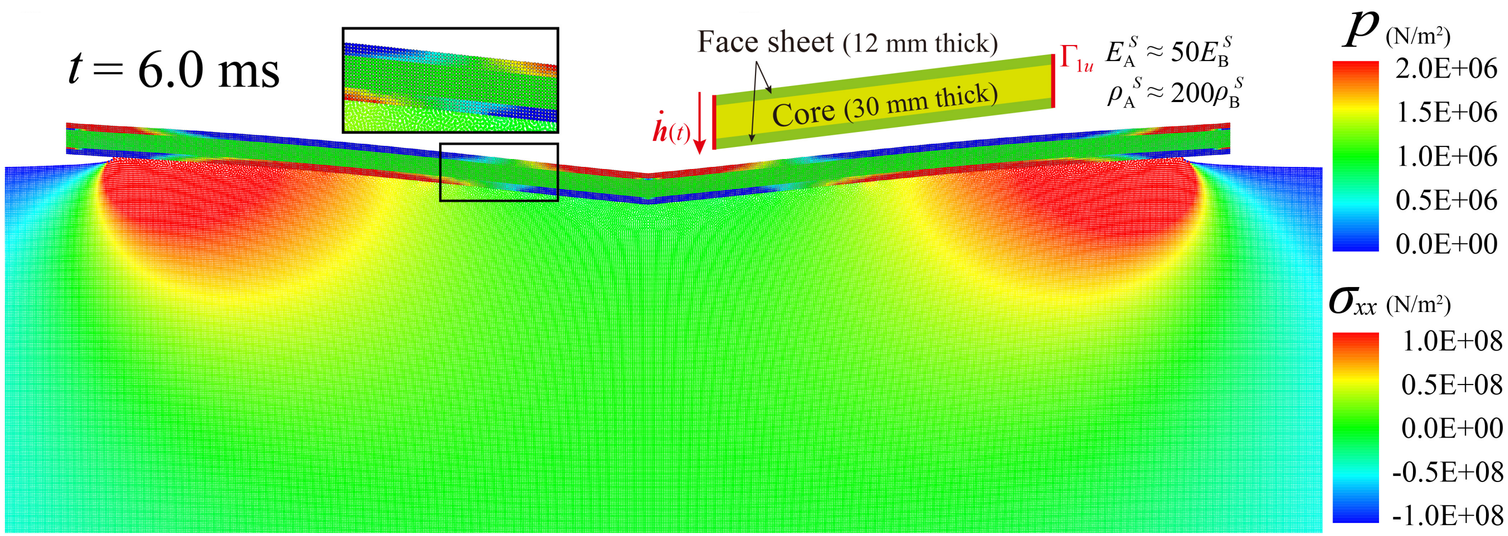
- Water slamming of a sandwich hull*

Water density	1000 (kg/m^3)
Face sheet density	31400 (kg/m^3)
Core density	150 (kg/m^3)
Face sheet Young's modulus along length	1.38×10^{11} (Pa)
Face sheet Young's modulus along thickness	8.66×10^{9} (Pa)
Core Young's modulus	2.8×10^{9} (Pa)
Poisson's ratio	0.30



*Z. Qin and R. C. Batra, "Local slamming impact of sandwich composite hulls", *International Journal of Solids and Structures*, Vol. 46, pp. 2011-2035, 2009.

Key aspects – III. *Generality* – Hydroelastic FSI solvers for ***Composite materials***



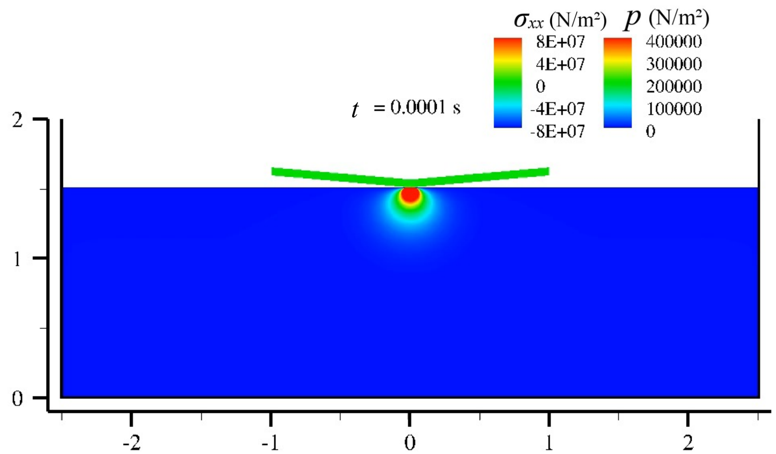
✓ The **first** 2D entirely Lagrangian meshfree **hydroelastic FSI solver** corresponding to isotropic composite materials.

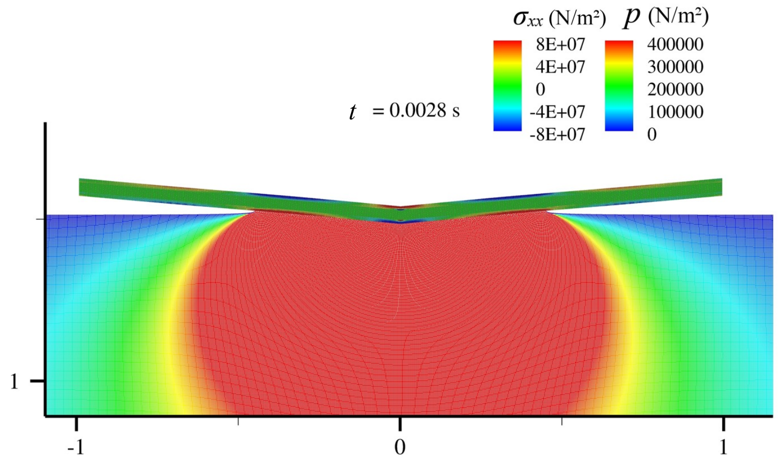
✓ Large density/Young's modulus ratios among laminates

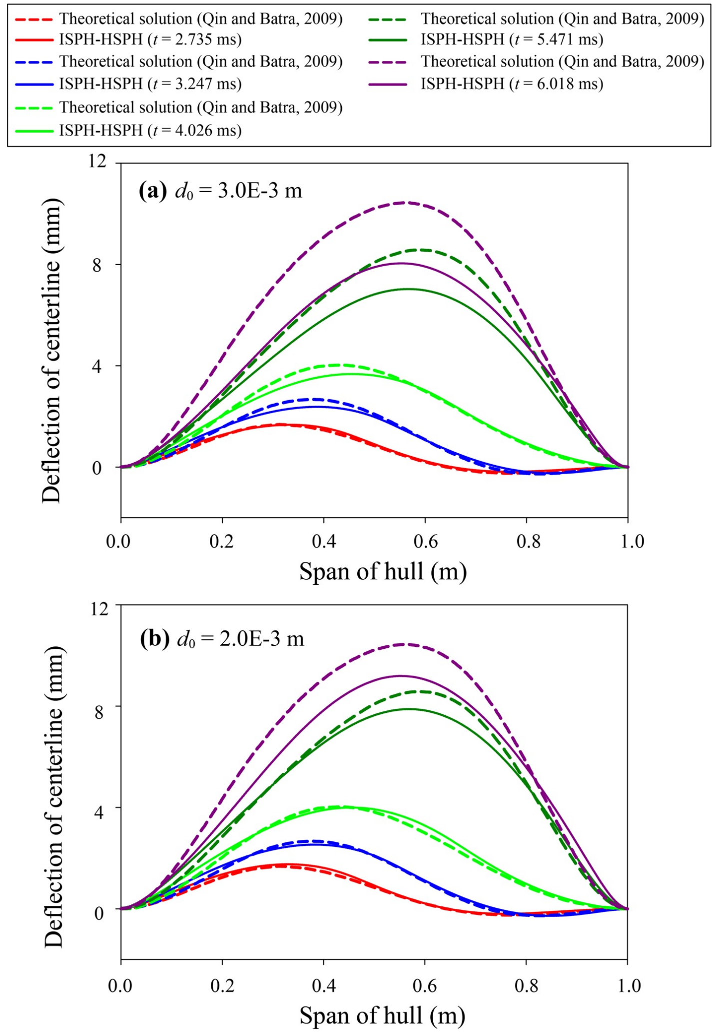
✓ Absence of artificial numerical stabilization or smoothing

Khayyer A., Shimizu Y., Gotoh H., Nagashima K., A coupled Incompressible SPH-Hamiltonian SPH solver for hydroelastic FSI corresponding to composite structures, *Applied Mathematical Modelling*, 2021.

Key aspects – III. ***Generality*** – Hydroelastic FSI solvers for ***Composite materials***

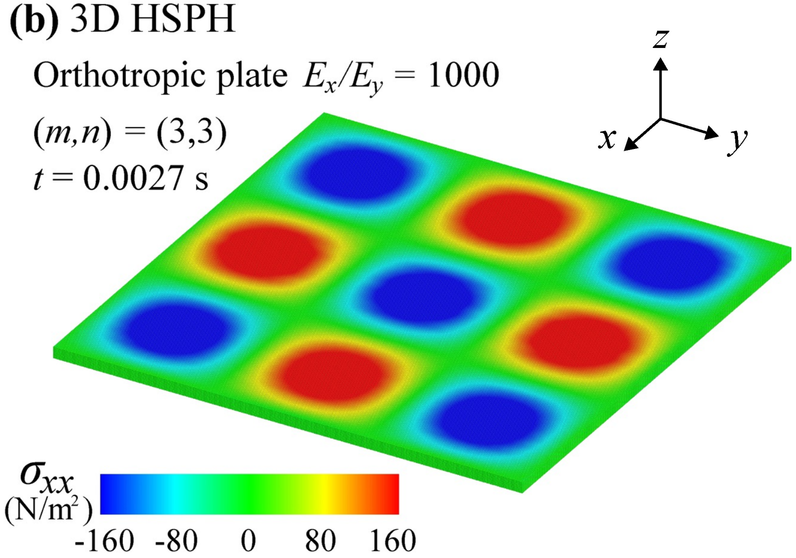


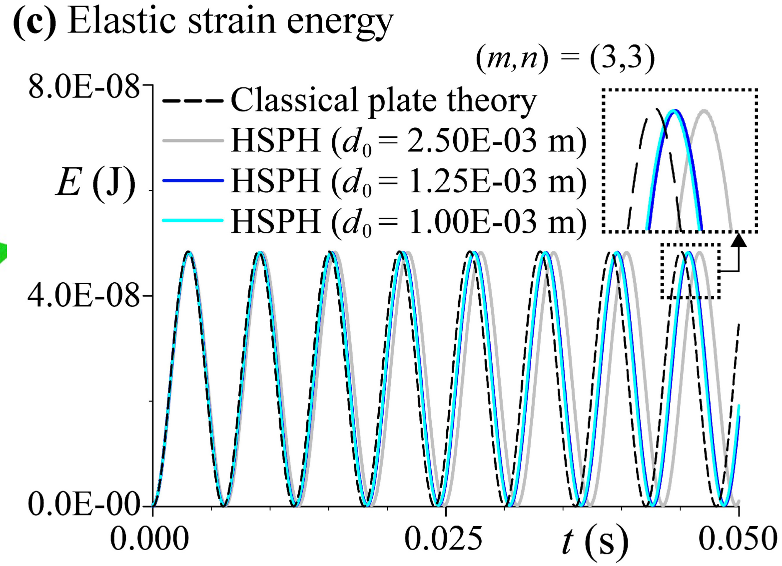




Key aspects – III. ***Generality*** – ***Anisotropic Structures***

Free vibration of a thin plate





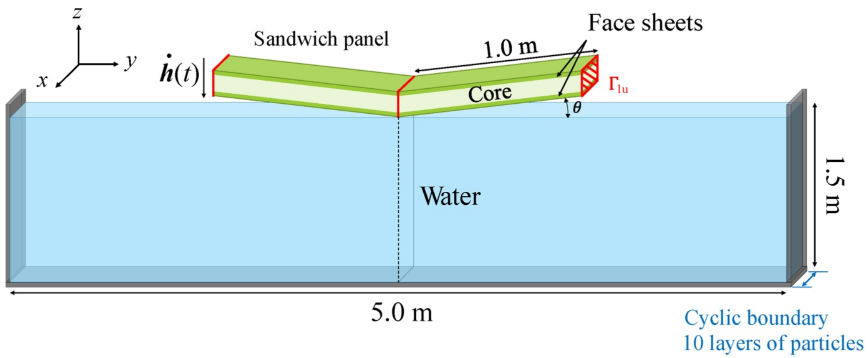
- ✓**Large material anisotropy: E_x / E_y = 1000**
- ✓Smooth stress field
- ✓Accuracy/robustness of the anisotropic HSPH structure model
- ✓Accurate estimation of elastic strain energy time history with respect to corresponding classical plate theory
- ✓Good convergence property

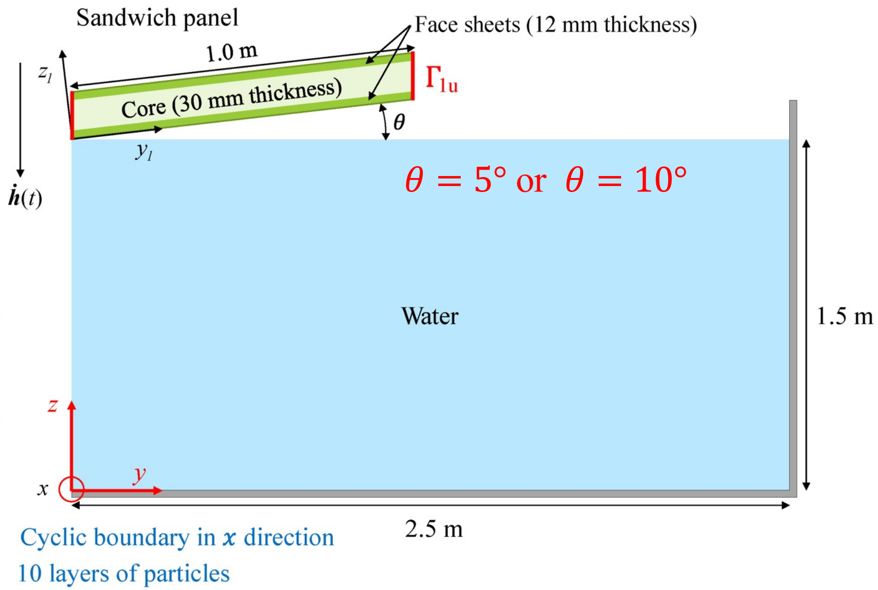
Key aspects – III. *Generality* – 3D Hydroelastic FSI corresponding to ***Anisotropic Composite Structures***

Water density	1000 (kg/m^3)
Face sheet density	31400 (kg/m^3)
Core density	150 (kg/m^3)
Face sheet Young's modulus along length	1.38E+11(Pa)
Face sheet Young's modulus along thickness	8.96E+9 (Pa)
Face sheet shear modulus	7.10E+9 (Pa)
Core Young's modulus	2.8E+9 (Pa)
Poisson's ratio	0.30

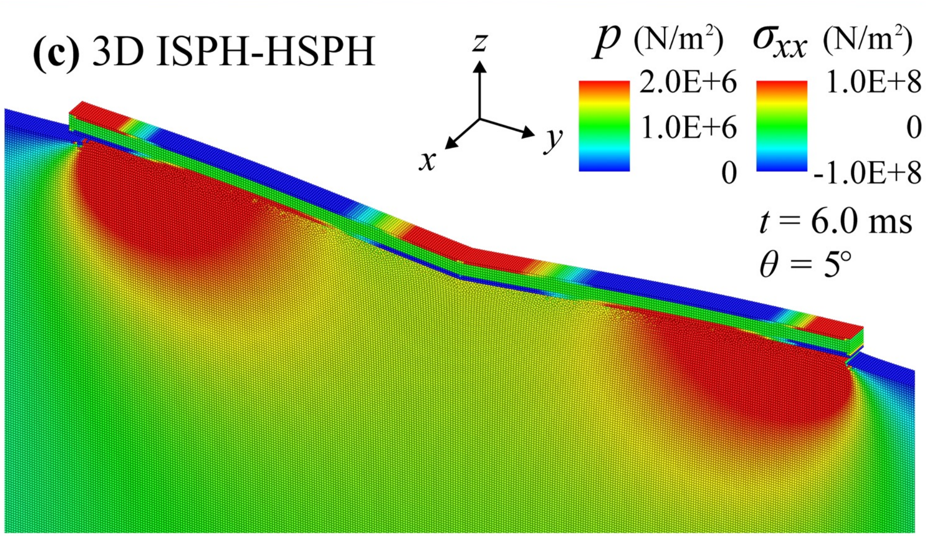
(A) Face sheets: transversely isotropic $E_{A,y1}/E_B \approx 50$
(B) Core: isotropic $\rho_A/\rho_B \approx 200$

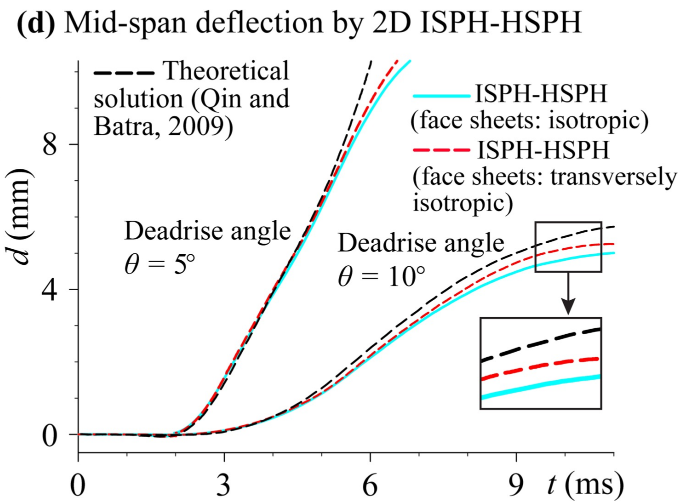
Z. Qin, R.C. Batra, Local slamming impact of sandwich composite hulls, Int. J. Solids Struct. Vol. 46, 2009, pp. 2011-2035.





Key aspects – III. *Generality* – 3D Hydroelastic FSI corresponding to ***Anisotropic Composite Structures***





- ✓ Water slamming on **anisotropic composite** sandwich hull
- ✓ The **first 3D hydroelastic FSI solver** corresponding to **composite/anisotropic** material.
- ✓ Large density/Young's modulus ratios among laminates
- ✓ Large ratios of density and Young's modulus between laminates ($\rho_A^S / \rho_B^S \approx 200$ and $E_{A,y1}^S / E_B^S \approx 50$)

Khayyer A., Shimizu Y., Gotoh H., Hattori S., A 3D SPH-based entirely Lagrangian meshfree hydroelastic FSI solver for anisotropic composite structures, *Applied Mathematical Modelling*, 2022.

Concluding remarks

- Three aspects of ***Reliability***, ***Adaptivity*** & ***Generality*** must be rigorously considered in development of FSI solvers for Coastal & Offshore Engineering.
- The first ***entirely Lagrangian meshfree FSI solvers*** for simulation of incompressible fluid flow interacting with ***isotropic/anisotropic composite elastic structures*** in both 2D & 3D
- In terms of thorough consideration of stability, accuracy, conservation & convergence, ISPH-HSPH shows superior performance.
- The ISPH-HSPH serves as an excellent candidate for FSIs related to ***extreme events in Coastal & Offshore Engineering***.

Future works:

- Comprehensive structural responses including viscoelasticity, elastoplasticity, damage/fracture
- Multi-phase FSI including air-water-structure interactions
- HPC implementations for large scale practical engineering simulations

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Journal of Fluids and Structures

Volume 105, August 2021, 103342

A 3D Lagrangian meshfree projection-based solver for hydroelastic Fluid–Structure Interactions

Abbas Khayyer, Hitoshi Gotoh, Yuma Shimizu, Yusuke Nishijima

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Applied Mathematical Modelling

Volume 94, June 2021, Pages 242-271

A coupled incompressible SPH-Hamiltonian SPH solver for hydroelastic FSI corresponding to composite structures

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Computer Physics Communications

Volume 232, November 2018, Pages 139-164

An enhanced ISPH–SPH coupled method for simulation of incompressible fluid–elastic structure interactions

Abbas Khayyer, Hitoshi Gotoh, Hosein Falahaty, Yuma Shimizu

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Development of a projection-based SPH method for numerical wave flume with porous media of variable porosity

Abbas Khayyer [a], Hitoshi Gotoh [a], Yuma Shimizu [a], Kohji Gotoh [a], Hosein Falahaty [a], Songdong Shao [b, c]

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Applied Ocean Research
Volume 115, October 2021, 102822

Entirely Lagrangian meshfree computational methods for hydroelastic fluid-structure interactions in ocean engineering—Reliability, adaptivity and generality

Hitoshi Gotoh, Abbas Khayyer, Yuma Shimizu

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Applied Ocean Research
Volume 114, September 2021, 102734

Particle methods in ocean and coastal engineering

Min Luo [a], Abbas Khayyer [b], Pengzhi Lin [c]

Thank you for your kind attention!