New density diffusion term

RENATO VACONDIO
UNIVERSITY OF PARMA
Motivation

- Improve the stability of the scheme
- Long duration wave generation and propagation
- Costal engineering
- Numerical design of wave energy devices (WECs)
- Wave-structure interaction
Standard Weakly – Compressible Formulation

\[
\frac{d \rho_a}{dt} = \rho_a \sum_b (v_a - v_b) \cdot \nabla W_{ab} V_b
\]

\[
\frac{d v_a}{dt} = - \sum_b m_b \left( \frac{P_b + P_a}{\rho_a \rho_b} + \Pi_{ab} \right) \nabla_a W_{ab} + g
\]

\[
\frac{d x_a}{dt} = v_a
\]

Correct kinematic, but noisy pressure field!

Main sources of noise on the pressure field:
- Lagrangian character: particle position rearrangement
- physical model: acoustic waves
- numerical scheme: centred + collocated + explicit in time
Density diffusion term

\[
\frac{d \rho_a}{d t} = \rho_a \sum_b (v_a - v_b) \cdot \nabla W_{ab} V_b + h c_0 D_a
\]

\[
\frac{d v_a}{d t} = - \sum_b m_b \left( \frac{p_b + p_a}{\rho_a \rho_b} + \Pi_{ab} \right) \nabla_a W_{ab} + g
\]

\[
\frac{d x_a}{d t} = v_a
\]

\[
D_a = 2 \sum_b \psi_{ab} \cdot \nabla W_{ab} V_b \quad V_b = \frac{m_b}{\rho_b}
\]

\(\psi_{ab}\) depends on the specific diffusive scheme adopted (see later)
\[ \psi_{ab} = \delta \left[ (\rho_b - \rho_a) \right] \frac{x_b - x_a}{\|x_b - x_a\|^2} \]

- \( \delta \) is a dimensionless parameter (usually assumed equal to 0.1)
- Effective in improving the pressure field
- Almost no additional computational cost
- Non-consistent close to the free surface

2-D jet impinging an orthogonal plate (Molteni and Colagrossi 2009)

2-D still water (Antuono et al. 2010)
\[ \psi_{ab} = \delta \left[ (\rho_b - \rho_a) - \frac{1}{2} \left( \langle \nabla \rho \rangle^L_b + \langle \nabla \rho \rangle^L_a \right) \cdot (\mathbf{x}_b - \mathbf{x}_a) \right] \frac{\mathbf{x}_b - \mathbf{x}_a}{\| \mathbf{x}_b - \mathbf{x}_a \|^2} \]

\[ \langle \nabla \rho \rangle^L_a = \sum_b (\rho_a - \rho_b) \mathbb{I}_a \nabla W_{ab} V_b \]

\[ \mathbb{I}_a = \left[ \sum_b (\mathbf{x}_b - \mathbf{x}_a) \otimes W_{ab} V_b \right]^{-1} \]

- consistent close to the free surface
- Effective in reducing the pressure spurious oscillations
- Remarkable additional computational cost (2 – 3 times)
Other density diffusion terms

Ferrari et al., 2009: Based on Rusanov flux, not consistent close to the free surface

\[ \psi_{ab} = \left[ \frac{\rho_b - \rho_a}{2h} \right] \frac{x_b - x_a}{\| x_b - x_a \|} \]

cheap and no parameters, but **no free-surface consistency**

Green et al., 2019: Parameter-free diffusion based on Riemann solvers

\[ \psi_{ab} = B_{ab} \left[ (\rho_b - \rho_a) - \frac{1}{2} (\xi_{ba} (\nabla\rho)_b^L + \xi_{ab} (\nabla\rho)_a^L) \cdot (x_b - x_a) \right] \]

no parameters, free-surface consistency, but as **expensive** as Antuono et al. (2010)
New density diffusion terms (Fourtakas et al. 2019)

Efficient + consistent for free-surface

\[
\frac{d\rho_a}{dt} = \rho_a \sum_b (\mathbf{v}_{ab} \cdot \nabla W_{ab} V_b) + 2h c_a \sum_b \psi_{ab} \cdot \nabla W_{ab} V_b + 2 \frac{\rho_a}{\rho_H} \frac{\nabla \psi_{ab} \cdot \nabla V_b}{\rho_H}
\]

\[\psi_{ab} = \delta (\rho_a^D - \rho_b^D) \frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|^2}\]

\[\rho_a = \rho_a^H + \rho_a^D\]

\[\rho^H\] is the hydrostatic density

\[\rho^D\] is the dynamic density

Similar to Molteni & Colagrossi (2009) but with dynamic density

\[\psi_{ab} = 2 \left( \rho_{ab} - \rho_{ab}^H \right) \frac{\mathbf{x}_{ab}}{\|\mathbf{x}_{ab}\|^2}\]

If Tait’s EoS is adopted:

\[\rho_{ab}^H = \rho_0 \left( \sqrt{\frac{P_{ab}^H}{C_B} + 1} - 1 \right)\]

\[P_{ab}^H = \rho_0 g z_{ab}\]
What’s available in DualSPHysics?

<parameter key="DensityDT" value="2" comment="0:None, 1:Molteni, 2:Fourtakas, 3:Fourtakas(full) />
<parameter key="DensityDTvalue" value="0.1" comment="DDT value (default=0.1)" />

<table>
<thead>
<tr>
<th>DensityDT</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No density diffusion term</td>
</tr>
<tr>
<td>1</td>
<td>Molteni &amp; Colagrossi 2009, only fluid – fluid interaction</td>
</tr>
<tr>
<td>2</td>
<td>Fourtakas et al. 2019, only fluid – fluid interaction</td>
</tr>
<tr>
<td>3</td>
<td>Fourtakas et al. 2019 also for Fluid – Boundary interaction</td>
</tr>
</tbody>
</table>

Doubts? DSPH forum

email George Fourtakas!
Hydrostatic test case

\[ t = 300 \text{ s} \]
\[ \Delta x/H = 0.01 \]
\[ H = 1 \text{ m} \]
\[ \alpha_\pi = 0.05 \]
\[ \delta = 0.1 \]

Normalised velocity field at \( t = 300 \text{ sec} \) for a) Molteni and Colagrossi DDT scheme and b) Fourtakas \textit{et al.} DDT scheme.

Normalised pressure field at \( t = 300 \text{ sec} \) for a) Molteni and Colagrossi DDT scheme and b) Fourtakas \textit{et al.} DDT scheme.
Hydrostatic test case
Wave propagation for 150 periods

Wave characteristics:
- depth = 0.66 m
- height = 0.15 m
- period = 2 s
- length = 4.523

SPH model
- $\Delta x/H = 0.133$
- $\alpha_\pi = 0.05$
- $\delta = 0.1$

Velocity field at 150 periods using the Fourtakas et al. DDT scheme (300 sec).
Wave propagation for 150 periods
Comparison of the velocity profile at location $(x, y) = (6, 0.4)$ m with the analytical results using Fourtakas et al. DDT scheme

Normalised free-surface elevation profile using Fourtakas et al. DDT scheme at $x = 6$ m
Long-duration simulation of a sloshing tank

Normalised pressure field ($t = 60$ sec).

Fourtakas et al. DDT scheme

Molteni and Colagrossi DDT scheme
Long-duration simulation of a sloshing tank

Normalised pressure field ($t = 60$ sec).

Fourtakas et al. DDT scheme

Molteni and Colagrossi DDT scheme
Long-duration simulation of a sloshing tank

Normalised pressure at $t = 60$ sec

[Graph showing normalised pressure over time with different schemes and analytical hydrostatic pressure.]
Long-duration simulation of a sloshing tank

Normalized kinetic energy vs. time (sec)
Conclusions

• Long duration wave generation and propagation
• Stable and accurate free surface flows and wave-structure interaction

• Density diffusion term
  • Applicable to DBC and mDBC
  • Hydrostatic pressure is calculated locally
  • Dependence on gravity (applicable to gravity driven flows)

• Negligible computational cost
Acknowledgements

Ben Rogers, George Fourtakas, Peter K. Stansby, Paolo Mignosa, Mashy Green, Jose M. Domínguez, M. Gómez-Gesteira, A J. Crespo
Thank you
renato.vacondio@unipr.it
Acknowledgements

- EPSRC, UK grant number EP/L014890/1
- Royal Academy of Engineering and Leverhulme Trust Senior Research Fellowship (LTSRF1718\14\37)
- Ministry of Education, Universities and Research (Italian government) under the Scientific Independence of young Researchers project, grant number RBSI14R1GP
- Xunta de Galicia (Spain) under project ED431C 2017/64-GRC, WELCOME ENE2016-75074-C2-1-R and project IJCI- 2017-32592 "Ayudas Juan de la Cierva-incorporación 2017"

- The University of Manchester SPH group
Proposed density diffusion term

Pressure field WCSPH

• Density diffusion term – Fourtakas et al., 2019:

\[
\frac{du_i}{dt} = -\sum_{j=1}^{N} m_j \left( \frac{p_i + p_j}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla W_{ij} + g_i
\]

\[
\frac{dp_i}{dt} = \sum_{j} m_j u_{ij} \cdot \nabla W_{ij} + \delta h c_i \sum_{j} \psi_{ij} \cdot \nabla W_{ij} V_j
\]

With,

\[
\psi_{ij} = 2 \left( \rho_i^D - \rho_j^D \right) \frac{x_{ij}}{||x_{ij}||^2}
\]

\[
P_{ij}^H = \rho_0 g z_{ij}
\]

\[
\Pi_{ij} = \begin{cases} 
-\frac{\alpha_s c_{ij}^2}{\rho_i} u_{ij} \cdot x_{ij} & u_{ij} \cdot x_{ij} < 0 \\
0 & u_{ij} \cdot x_{ij} \geq 0
\end{cases}
\]
Proposed density diffusion term

Pressure field WCSPH

• Density diffusion term – Fourtakas et al., 2019:

\[ \psi_{ij} = 2\left(\rho_T - \rho_H\right) \frac{x_{ij}}{||x_{ij}||^2} \]

\[ \rho_{ij}^H = \rho_0 \left( \frac{\frac{P_{ij}^H + 1}{C_B}}{C_B} - 1 \right) \]

\[ P_{ij}^H = \rho_0 g z_{ij} \]

• Hydrostatic pressure is calculated locally
• Dependence on gravity (applicable to gravity driven flows)

• No costly gradient correction
• Applicable to BCs with severe kernel truncation
Proposed density diffusion term

- Density diffusion term – Fourtakas et al., 2019
- DualSPHysics (https://dual.sphysics.org)
  - CPU and
  - GPU implementation
- Release v 5.0

Beta version: 5th DualSPHysics Users Workshop
Universitat Politecnica de Catalunya - BarcelonaTech (UPC)