New density diffusion term

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Motivation

- Improve the stability of the scheme
- Long duration wave generation and propagation
- Costal engineering
- Numerical design of wave energy devices (WECs)
- Wave-structure interaction

Standard Weakly – Compressible Formulation

$$\frac{\mathrm{d}\,\rho_a}{\mathrm{d}\,t} = \rho_a \sum_b (\mathbf{v}_a - \mathbf{v}_b) \cdot \nabla W_{ab} V_b$$

$$\frac{\mathrm{d}\,\mathbf{v}_a}{\mathrm{d}\,t} = -\sum_b m_b \left(\frac{P_b + P_a}{\rho_a \rho_b} + \Pi_{ab} \right) \nabla_a W_{ab} + \mathbf{g}$$

$$\frac{\mathrm{d}\,\mathbf{x}_a}{\mathrm{d}\,t} = \mathbf{v}_a$$

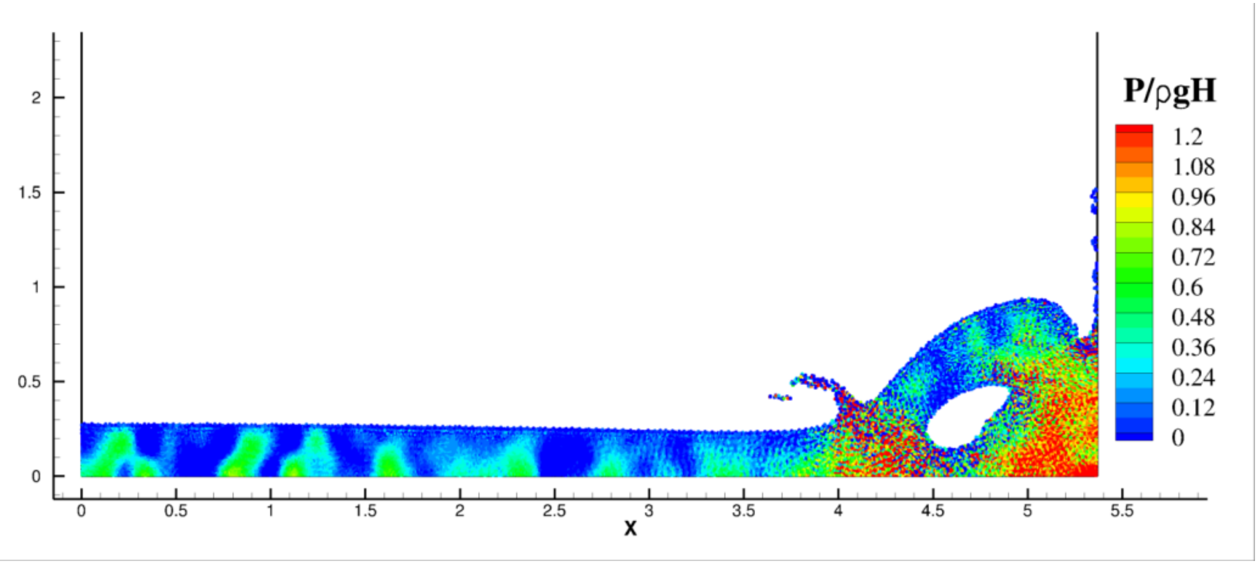
Correct kinematic, but noisy pressure field!

Main sources of noise on the pressure field:

- Lagrangian character: particle position rearrangement

- physical model: acoustic waves

- numerical scheme: centred + collocated + explicit in time



Density diffusion term

$$\frac{\mathrm{d}\,\rho_a}{\mathrm{d}\,t} = \rho_a \sum_b (\mathbf{v}_a - \mathbf{v}_b) \cdot \nabla W_{ab} V_b + hc_0 \mathcal{D}_a$$

$$\frac{\mathrm{d}\,\mathbf{v}_a}{\mathrm{d}\,t} = -\sum_b m_b \left(\frac{P_b + P_a}{\rho_a \rho_b} + \Pi_{ab} \right) \nabla_a W_{ab} + \mathbf{g}$$

$$\frac{\mathrm{d}\,\mathbf{x}_a}{\mathrm{d}\,t} = \mathbf{v}_a$$

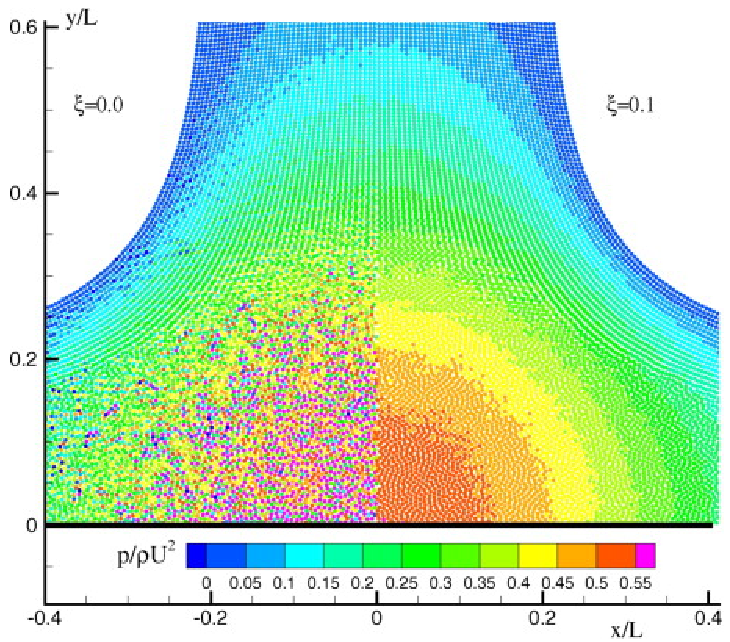
$$\mathcal{D}_a = 2 \sum_b \psi_{ab} \cdot \nabla W_{ab} V_b \qquad V_b = m_b / \rho_b$$

ψ_{ab} depends on the specific diffusive scheme adopted (see later)

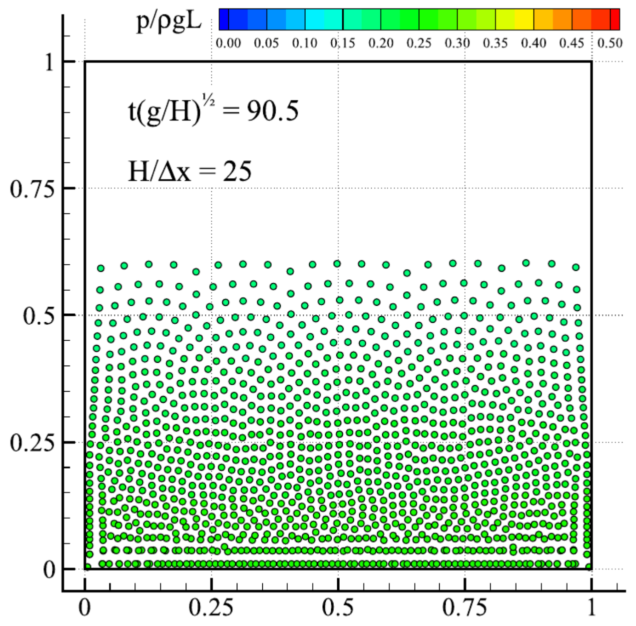
Molteni and Colagrossi 2009

$$\psi_{ab} = \delta[(\rho_b - \rho_a)] \frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|^2}$$

- δ is a dimensionless parameter (usually assumed equal to 0.1)
- Effective in improving the pressure field
- Almost no additional computational cost
- non – consistent close to the free surface



2-D jet impinging an orthogonal plate (Molteni and Colagrossi 2009)



2-D still water (Antuono et al. 2010)

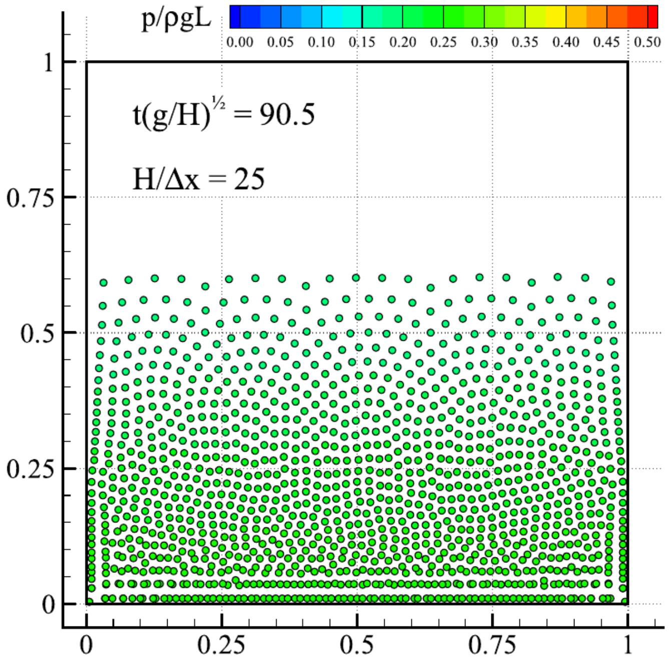
Antuono et al. (2010 - 2012)

$$\psi_{ab} = \delta\left[(\rho_b - \rho_a) - \frac{1}{2}\left(\langle\nabla\rho\rangle_b^L + \langle\nabla\rho\rangle_a^L\right)\cdot(\mathbf{x}_b - \mathbf{x}_a)\right]\frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|^2}$$

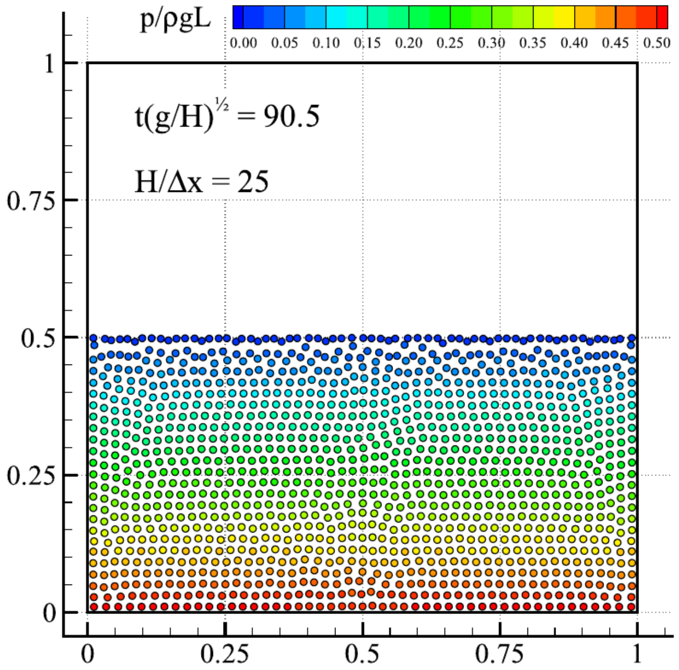
$$\langle\nabla\rho\rangle_a^L = \sum_b (\rho_a - \rho_b)\mathbb{L}_a\nabla W_{ab}V_b$$

$$\mathbb{L}_a = \left[\sum_b (\boldsymbol{x}_b - \boldsymbol{x}_a)\otimes W_{ab}V_b\right]^{-1}$$

- consistent close to the free surface
- Effective in reducing the pressure spurious oscillations
- Remarkable additional computational cost (2 – 3 times)



Molteni & Colagrossi (2009) CPC



Antuono et al. (2012) CPC

Other density diffusion terms

Ferrari *et al.*, 2009: Based on Rusanov flux, not consistent close to the free surface

$$\psi_{ab} = \left[\frac{(\rho_b - \rho_a)}{2h}\right] \frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|}$$

cheap and no parameters, but **no free-surface consistency**

Green *et al.*, 2019: Parameter-free diffusion based on Riemann solvers

$$\psi_{ab} = B_{ab}\left[(\rho_b - \rho_a) - \frac{1}{2}\left(\xi_{ba}\langle\nabla\rho\rangle_b^L + \xi_{ab}\langle\nabla\rho\rangle_a^L\right)\cdot(\mathbf{x}_b - \mathbf{x}_a)\right]$$

no parameters, free-surface consistency, but as **expensive** as Antuono et al. (2010)

New density diffusion terms (Fourtakas et al. 2019)

Efficient + consistent for free-surface

$$\frac{d\rho_a}{dt} = \rho_a \sum_b (\mathbf{v}_{ab} \cdot \nabla W_{ab} V_b) + 2hc_a \sum_b^N \psi_{ab} \cdot \nabla W_{ab}\, V_b$$

$$\rho_a = \rho_a^H + \rho_a^D$$

ρ^H is the hydrostatic density

ρ^D is the dynamic density

$$\psi_{ab} = \delta\left(\rho_a^D - \rho_b^D\right) \frac{\mathbf{x}_b - \mathbf{x}_a}{\|\mathbf{x}_b - \mathbf{x}_a\|^2}$$

Similar to Molteni & Colagrossi (2009) but with dynamic density

$$\psi_{ab} = 2\left(\rho_{ab} - \rho_{ab}^H\right) \frac{\mathbf{x}_{ab}}{\|\mathbf{x}_{ab}\|^2}$$

If Tait's EoS is adopted: $\rho_{ab}^{\mathrm{H}} = \rho_0 \left(\sqrt[\gamma]{\frac{P_{ab}^{\mathrm{H}} + 1}{C_B}} - 1 \right)$ $\quad P_{ab}^{\mathrm{H}} = \rho_0 g z_{ab}$

What's available in DualSPHYSICS?

```
 <parameter key="DensityDT" value="2" comment="0:None, 1:Molteni, 2:Fourtakas,
3:Fourtakas(full) />
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```

DensityDT	description
0	No density diffusion term
1	Molteni & Colagrossi 2009, only fluid – fluid interaction
2	Fourtakas et al. 2019, only fluid – fluid interaction
3	Fourtakas et al. 2019 also for Fluid – Boundary interaction

DSPH forum

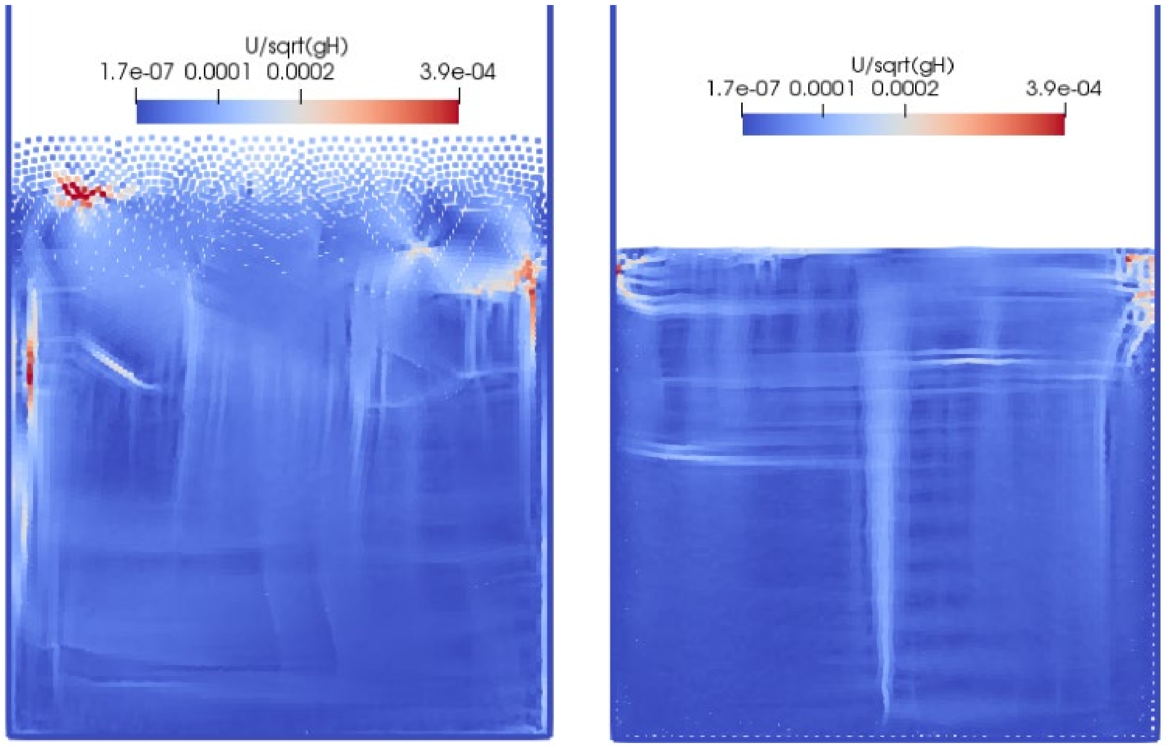
Doubts?

email George Fourtakas!

t =300 s
$\Delta x/H$ = 0.01
H = 1 m
α_π = 0.05
δ = 0.1

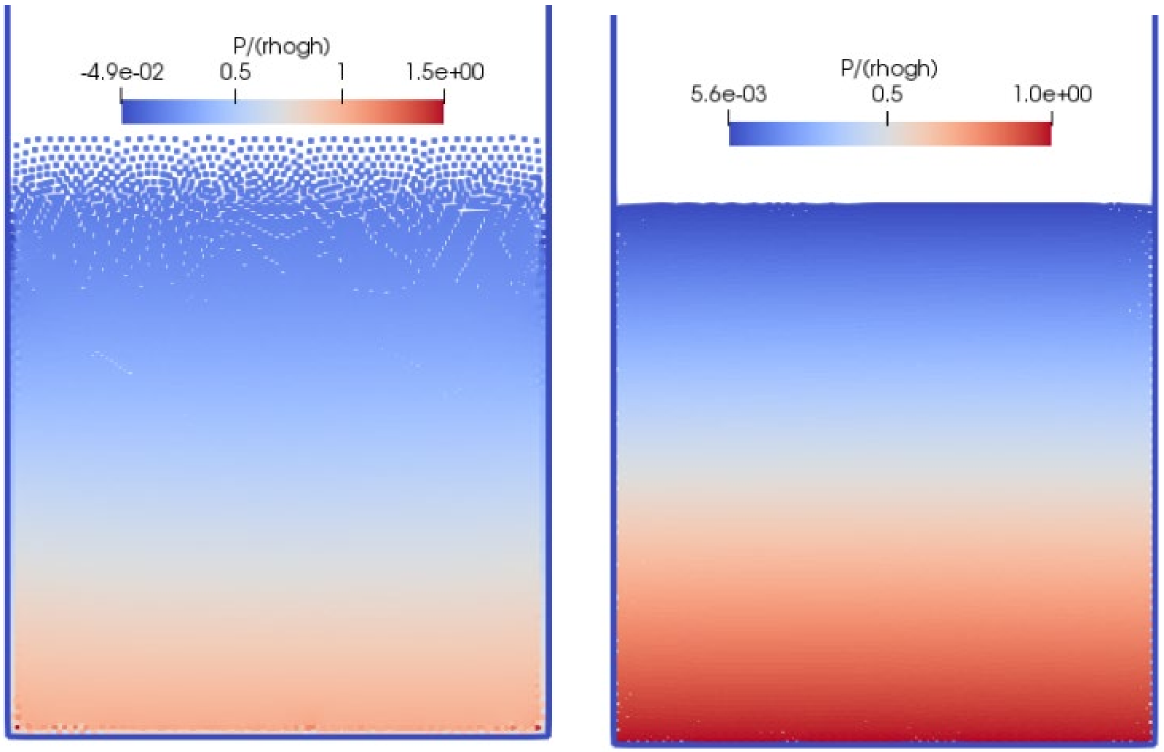
Hydrostatic test case

velocity



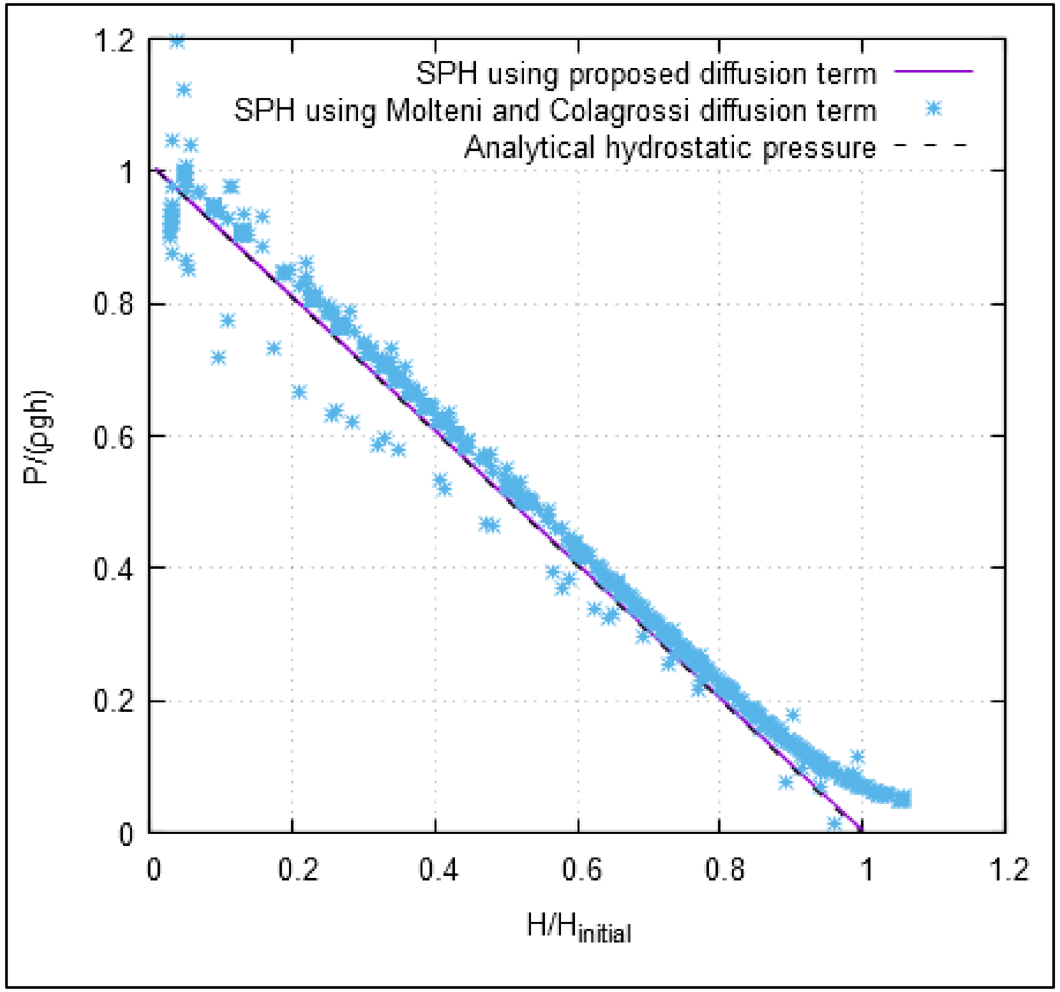
Normalised velocity field at t = 300 sec for a) Molteni and Colagrossi DDT scheme and b) Fourtakas *et al*. DDT scheme.

pressure



Normalised pressure field at t = 300 sec for a) Molteni and Colagrossi DDT scheme and b) Fourtakas *et al*. DDT scheme.

Hydrostatic test case



Wave propagation for 150 periods

Wave characteristics:

depth = 0.66 m

height = 0.15 m

period = 2 s

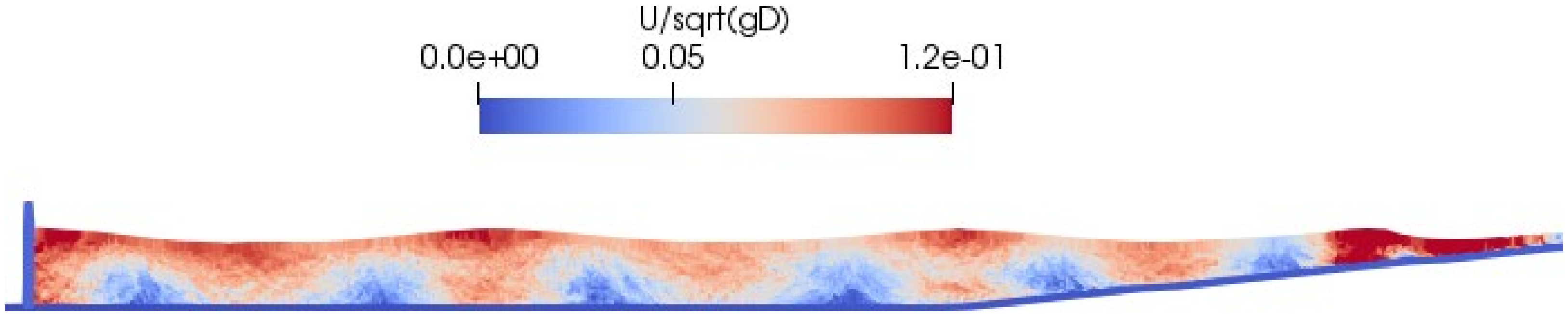
length = 4.523

SPH model

$\Delta x/H = 0.133$

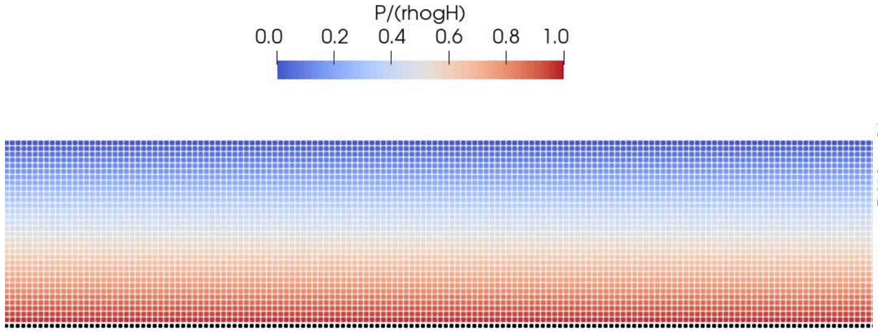
$\alpha_\pi = 0.05$

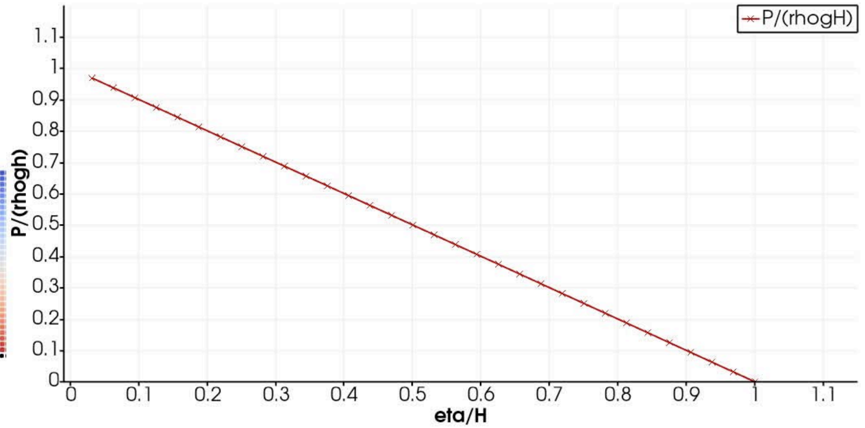
$\delta = 0.1$

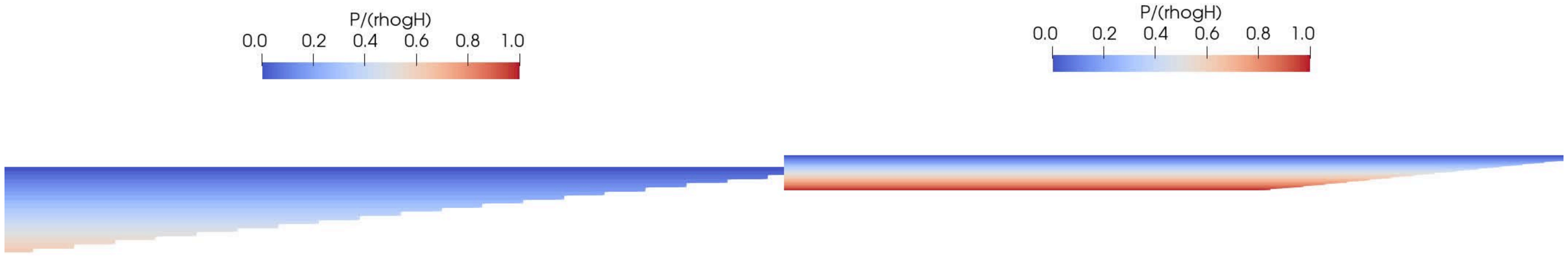


Velocity field at 150 periods using the Fourtakas *et al*. DDT scheme (300 sec).

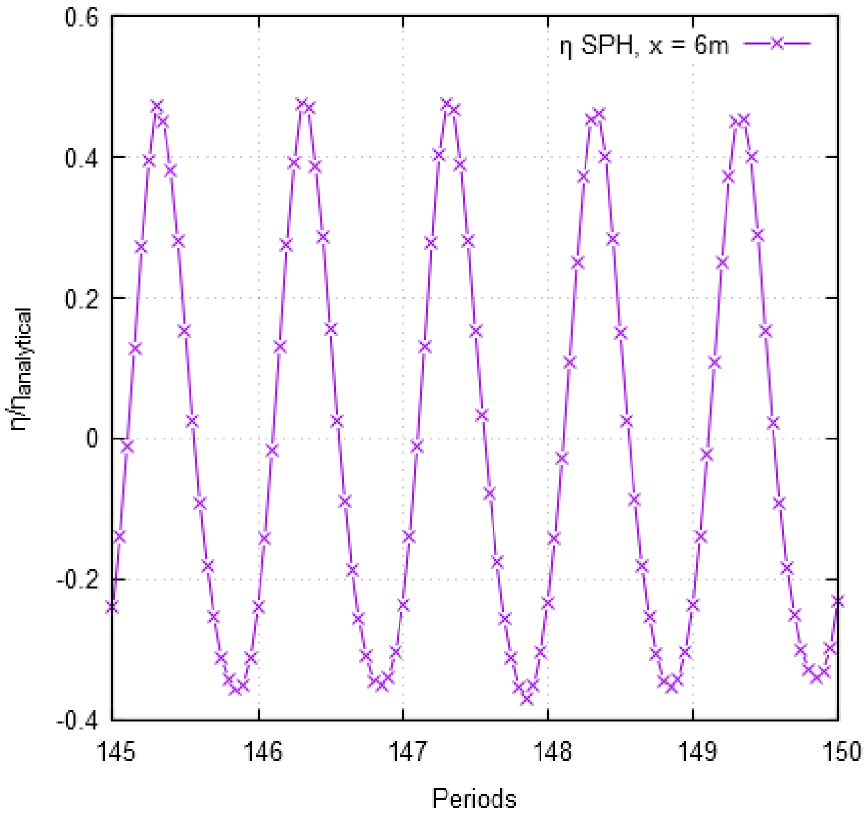
Wave propagation for 150 periods



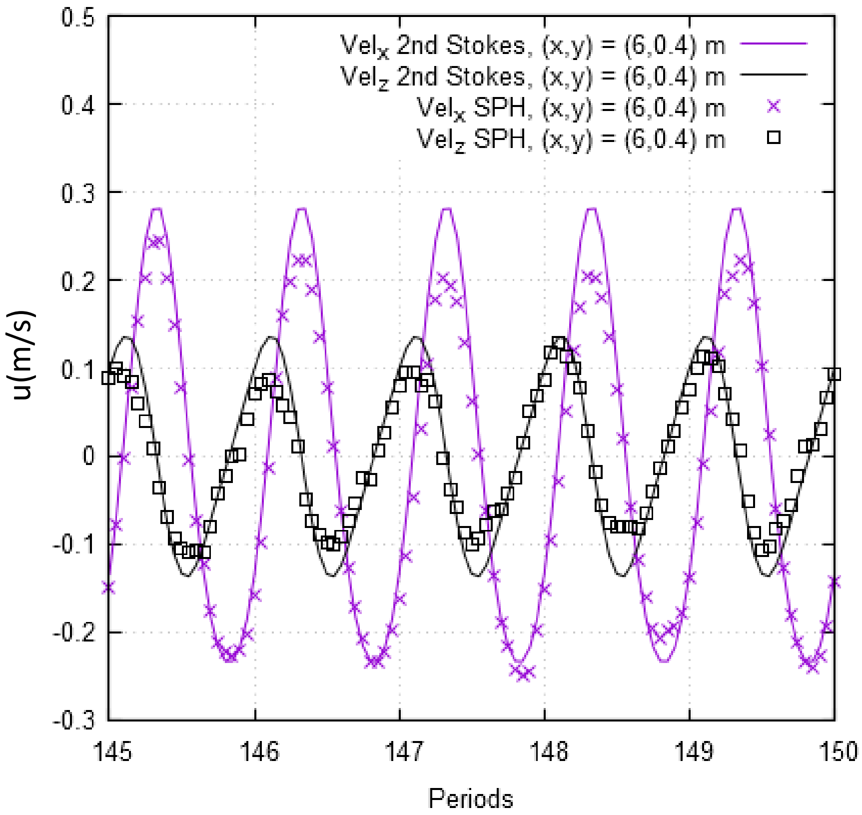




Wave propagation for 150 periods



Normalised free-surface elevation profile using Fourtakas *et al*. DDT scheme at x = 6 m

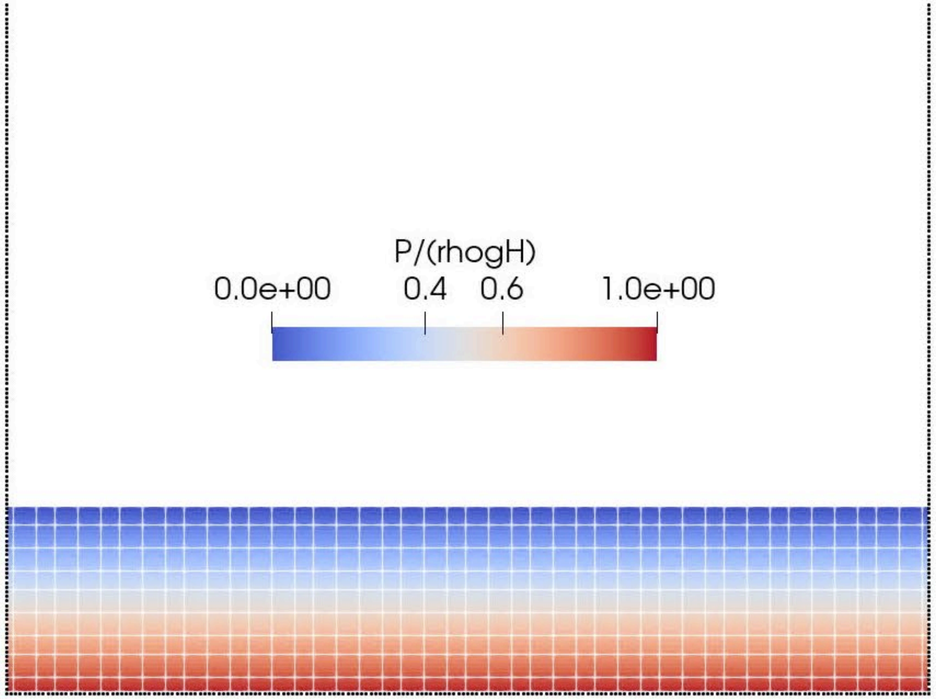


Comparison of the velocity profile at location *(x, y)* = (6, 0.4) m with the analytical results using Fourtakas *et al*. DDT scheme

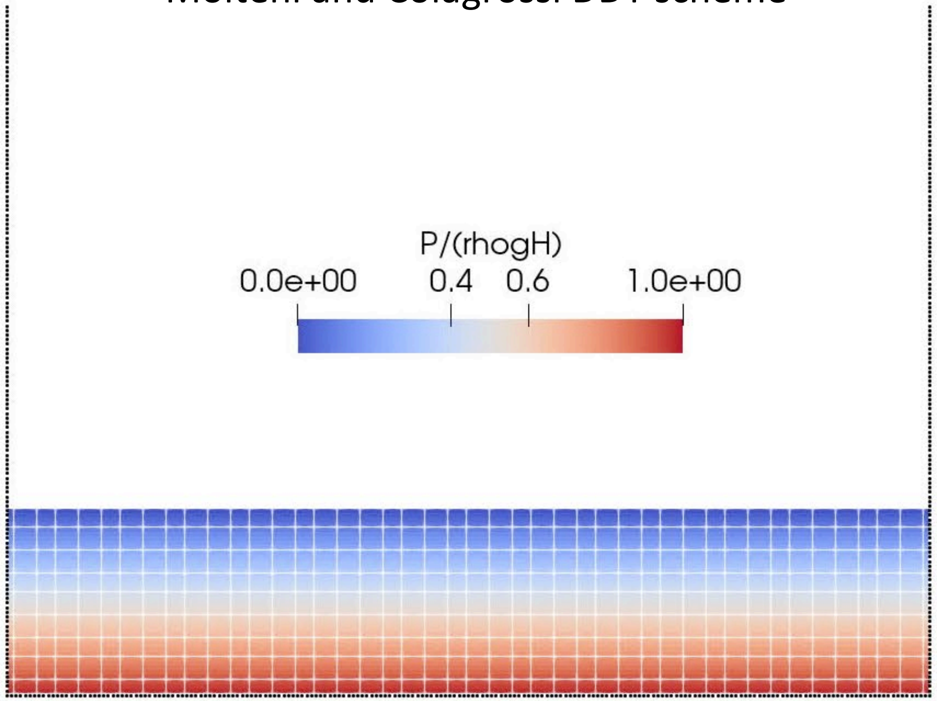
Long-duration simulation of a sloshing tank

Normalised pressure field (t = 60 sec).

Fourtakas *et al*. DDT scheme



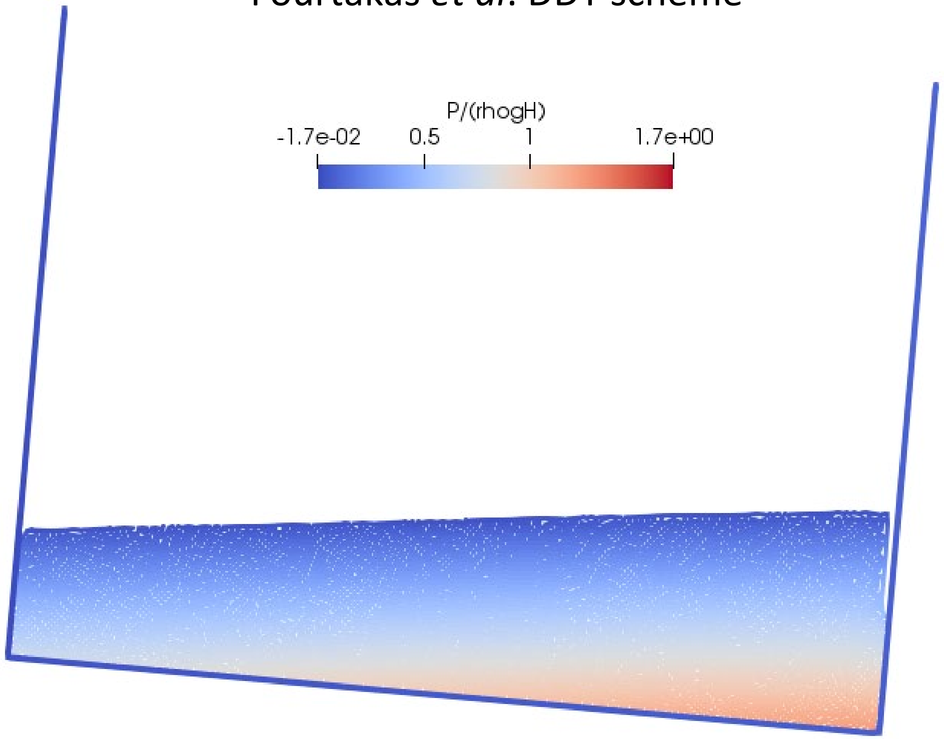
Molteni and Colagrossi DDT scheme



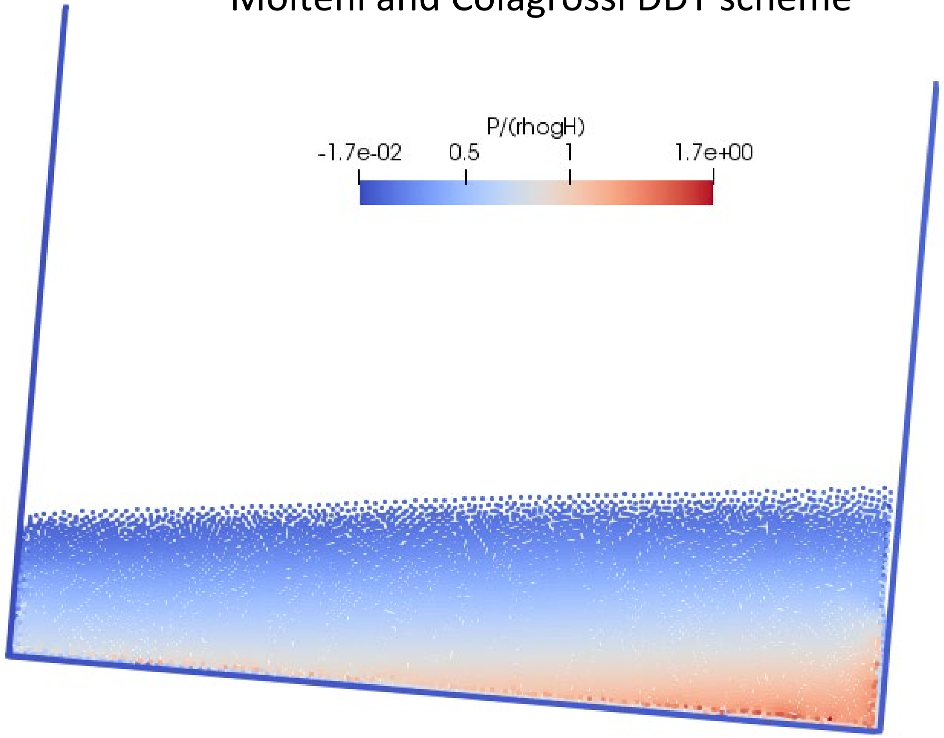
Long-duration simulation of a sloshing tank

Normalised pressure field (t = 60 sec).

Fourtakas *et al*. DDT scheme

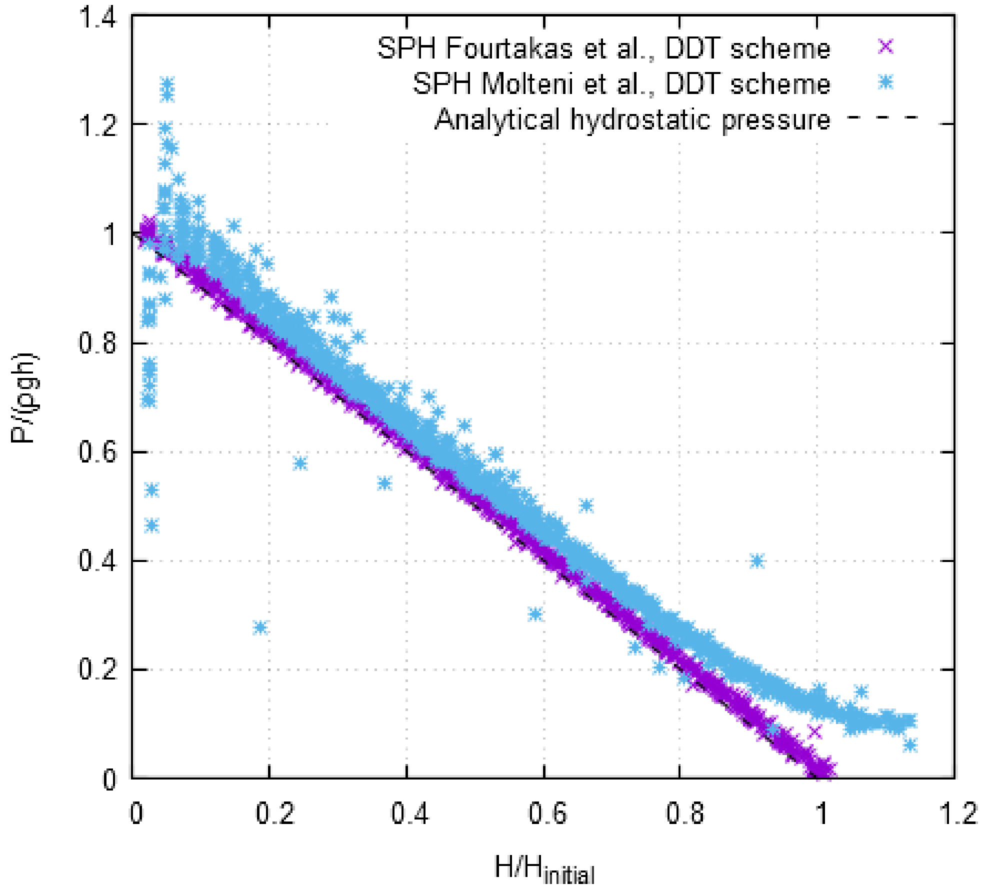


Molteni and Colagrossi DDT scheme



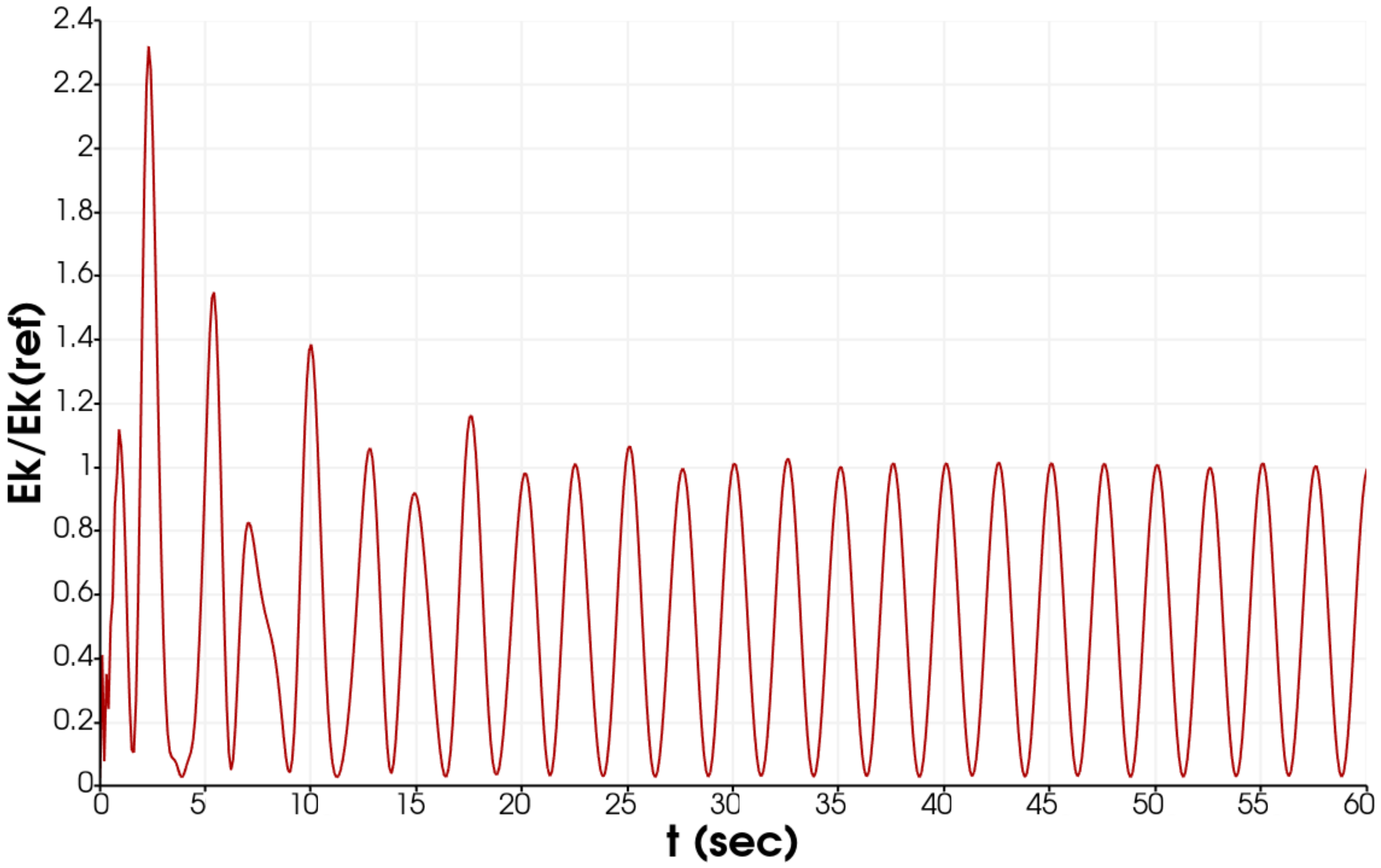
Long-duration simulation of a sloshing tank

Normalised pressure at *t* = 60 sec



Long-duration simulation of a sloshing tank

Normalised kinetic energy



Conclusions

- Long duration wave generation and propagation
- Stable and accurate free surface flows and wave-structure interaction

- Density diffusion term
  - Applicable to DBC and mDBC
  - Hydrostatic pressure is calculated locally
  - Dependence on gravity (applicable to gravity driven flows)

- Negligible computational cost

Acknowledgements

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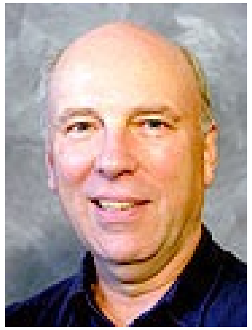


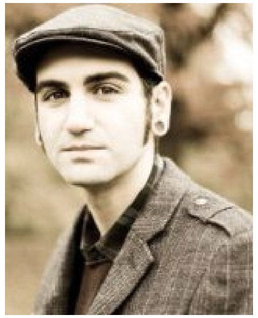


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Proposed density diffusion term

Pressure field WCSPH

- Density diffusion term – Fourtakas *et al.*, 2019:

$$\frac{d\mathbf{u}_i}{dt} = -\sum_{j=1}^{N} m_j \left(\frac{P_i + P_j}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla W_{ij} + \mathbf{g}_i$$

$$\frac{d\rho_i}{dt} = \sum_j^N m_j \mathbf{u}_{ij} \cdot \nabla W_{ij} + \delta h c_i \sum_j^N \psi_{ij} \cdot \nabla W_{ij} V_j$$

$$P_{ij}^{\mathrm{H}} = \rho_0 g z_{ij}$$

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha_\pi \overline{c_{ij}}}{\overline{\rho_{ij}}} \dfrac{h \mathbf{u}_{ij} \cdot \mathbf{x}_{ij}}{\mathbf{x}_{ij}^2} & \mathbf{u}_{ij} \cdot \mathbf{x}_{ij} < 0 \\ 0 & \mathbf{u}_{ij} \cdot \mathbf{x}_{ij} \geq 0 \end{cases}$$

With,

$$\psi_{ij} = 2\left(\rho_i^D - \rho_j^D\right) \frac{\mathbf{x}_{ij}}{\left\|\mathbf{x}_{ij}\right\|^2}$$

Proposed density diffusion term

Pressure field WCSPH

- Density diffusion term – Fourtakas *et al.*, 2019:

$$\psi_{ij} = 2\left(\rho_{ji}^{\mathrm{T}} - \rho_{ij}^{\mathrm{H}}\right)\frac{\mathbf{x}_{ij}}{\left\|\mathbf{x}_{ij}\right\|^2} \qquad \rho_{ij}^{\mathrm{H}} = \rho_0\left(\sqrt[\gamma]{\frac{P_{ij}^{\mathrm{H}}+1}{C_B}} - 1\right) \longrightarrow P_{ij}^{\mathrm{H}} = \rho_0 g z_{ij}$$

- Hydrostatic pressure is calculated locally
- Dependence on gravity (applicable to gravity driven flows)

- No costly gradient correction
- Applicable to BCs with severe kernel truncation

Proposed density diffusion term

- Density diffusion term – Fourtakas *et al.*, 2019
- DualSPHysics (https://dual.sphysics.org)
  - CPU and
  - GPU implementation
- Release v 5.0

  Beta version: 5th DualSPHysics Users Workshop

  Universitat Politecnica de Catalunya - BarcelonaTech (UPC)