The University of Manchester

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SPH
now and in the future

Peter Stansby
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Manchester
Vigo
Parma
Lisbon
Gent
Barcelona

Why SPH ?

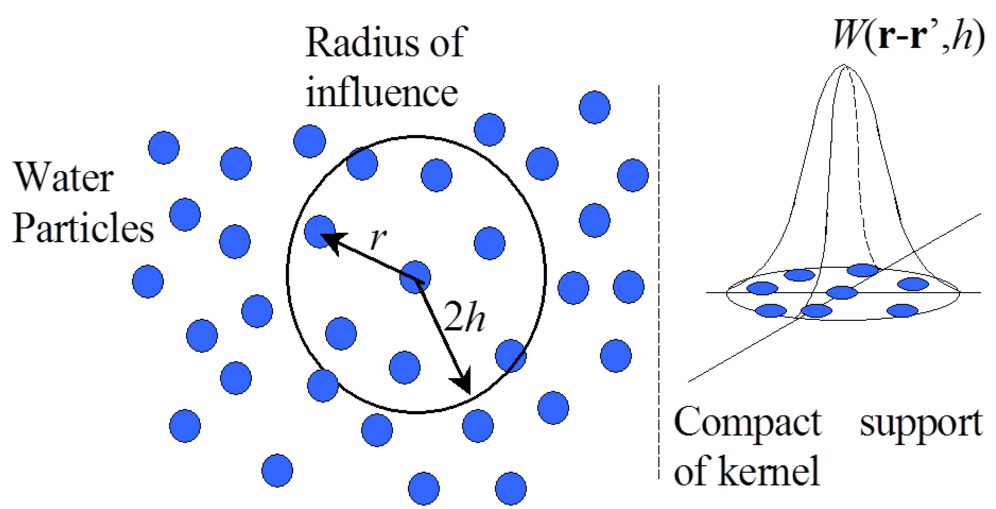
- We have commercial fluids and solids codes – STAR CCM, ANSYS Fluent, Mechanical, Structural, ABAQUS, NASTRAN, Delft3D, etc
- Open source codes – Code_SATURNE, Code_ASTER, OpenFOAM, Telemac, SWAN (coastal) etc
- So successful that more is wanted !
- So what are blockages – highly distorted free surfaces/interfaces, multi-phase, phase change, multi-physics, complex boundaries/meshing, computational times and cost

Smoothed Particle Hydrodynamics (SPH)

- SPH is a Lagrangian particle method
- Flow variables determined according to an interpolation over discrete interpolation points (fluid particles) with kernel *W*

$$\phi(r) \approx \int W(r - r')\phi(r')dr' \approx \sum_i W(r - r_i)\phi_i V_i$$

- Interpolation points flow with fluid
- Complex free-surface (including breaking wave) dynamics captured automatically



Typical operators

$$\phi(r_i) = \sum_j V_j \phi(r_j) W(r_{ij})$$

$$\nabla\phi_i = \sum_j - V_j\,(\phi_i - \phi_j) \nabla W_{ij}$$

$$(\mu\Delta\boldsymbol{u})_i = \sum_j \frac{m_j(\mu_i + \mu_j)\boldsymbol{r}_{ij}\,.\nabla\,W_{ij}}{\rho_j\,({r^2}_{ij} + \eta^2)}\,\boldsymbol{u}_{\,ij}$$

$$\Delta p_i = \sum_j \frac{m_j\,\boldsymbol{r}_{ij}\,.\nabla\,W_{ij}}{\rho_j\,({r^2}_{ij} + \eta^2)}\;p_{ij}$$

Basic form : weakly compressible equations (computationally simple: no solver)

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot u$$

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_j (\mathbf{u}_j - \mathbf{u}_i) \cdot \nabla_i W_j(\mathbf{r}_i)\, dV_j,$$

$$\frac{Du}{Dt} = -\frac{\nabla p}{\rho} + f$$

$$\frac{D\mathbf{u}_i}{Dt} = -\frac{1}{\rho_i} \sum_j (p_i + p_j) \nabla_i W_j(\mathbf{r}_i)\, dV_j + \mathbf{f}_i,$$

$$\frac{D\boldsymbol{r}_i}{Dt} = u_i.$$

$$p(\rho) = \frac{c_s^2 \rho_0}{\gamma} \left[\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right]$$

Speed of sound ~ 10 max velocity , so artificial pressure waves, noise

Stabilising options in WCSPH

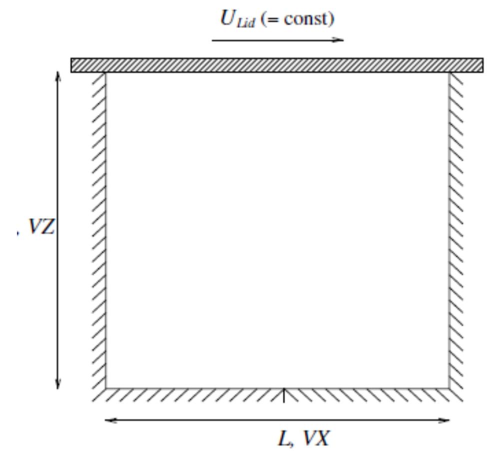
- Artificial viscosity (in momentum equation)
- Shepard filter – smooths particle distribution
- XSPH – extra diffusion term in momentum eq
- δ SPH – diffusion term in continuity eq
- Shifting – purely numerical device

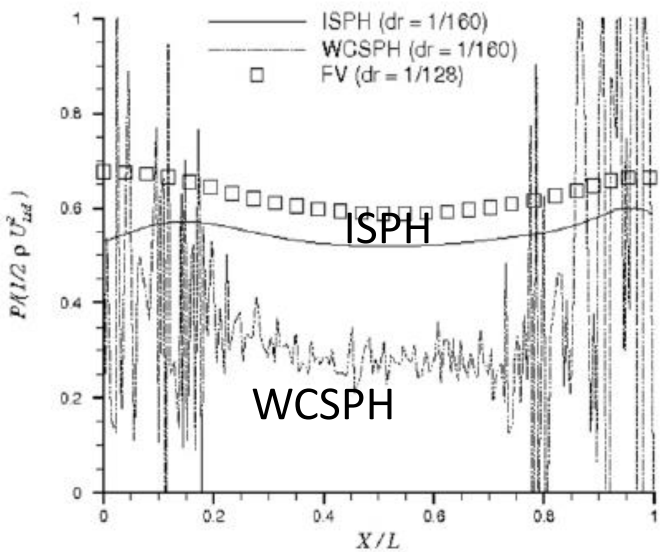
- Also Riemann solver formulation with artificial viscosity

Noise and error in 2008

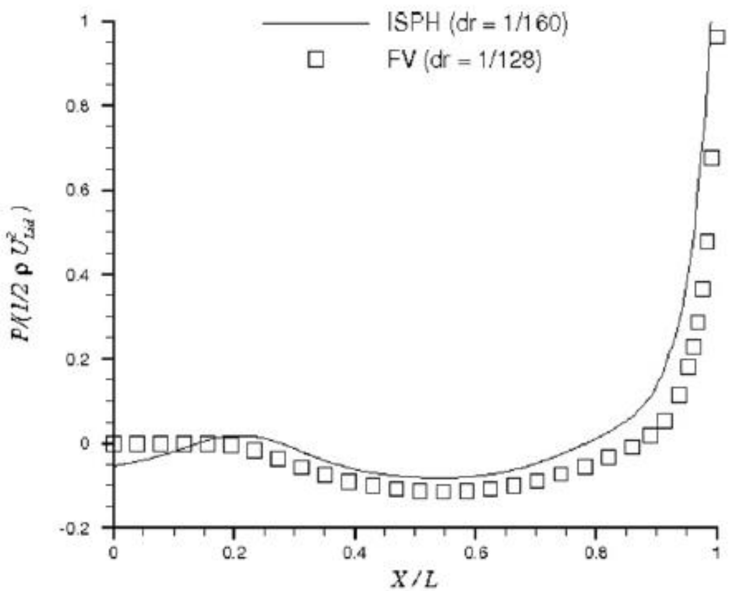
Lid driven cavity

Lee, E-S., Moulinec, C., Xu, R., Violeau, D., Laurence, D., Stansby, P., 2008 , JCP, 227.





(a) Comparisons of pressure profiles at $Z = L/2$.



(b) Comparisons of pressure profiles in diagonal direction.

Incompressible SPH now

- Noise free with numerical stabilisation (without contriving physics)
- Greater accuracy
- But requires Poisson solver for pressure so less ideal for GPUs but progress made there

Incompressible SPH (ISPH)

- Solves incompressible Navier-Stokes equations

$$\rho \frac{d\boldsymbol{u}}{dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{f}; \qquad \boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0}$$

- **Incompressible SPH** uses a **projection method** to enforce incompressibility and solves a Poisson equation for the pressure

$$\nabla^2 p = \frac{\rho}{\Delta t} \nabla \cdot \boldsymbol{u}$$

- Pressure field is smooth and accurate when used with particle regularisation or shifting

(Xu et al JCP, 2009, 228; Lind et al., JCP, 2012, 231)

ISPH Time-Stepping Algorithm

- Determine intermediate positions $\boldsymbol{r_i}^* = \boldsymbol{r_i}^n + \Delta t\, \boldsymbol{u}_i{}^n$
- Determine intermediate velocity from viscous and body force terms

$$\boldsymbol{u}_i^* = \boldsymbol{u}_i^n + (\mu \nabla^2 \boldsymbol{u}_i^n + \boldsymbol{f}_i)\Delta \boldsymbol{t}/\rho$$

- Pressure obtained from pressure Poisson equation for zero divergence

$$\nabla^2 p_i^{n+1} = \frac{\rho}{\Delta t} \nabla \cdot \boldsymbol{u}_i^*$$

- Intermediate velocity corrected with pressure gradient to obtain divergence-free velocity at time n+1

$$\boldsymbol{u}_i^{n+1} = \boldsymbol{u}_i^* - (\nabla p_i/\rho)\Delta t$$

- Particle positions updated with centred differencing

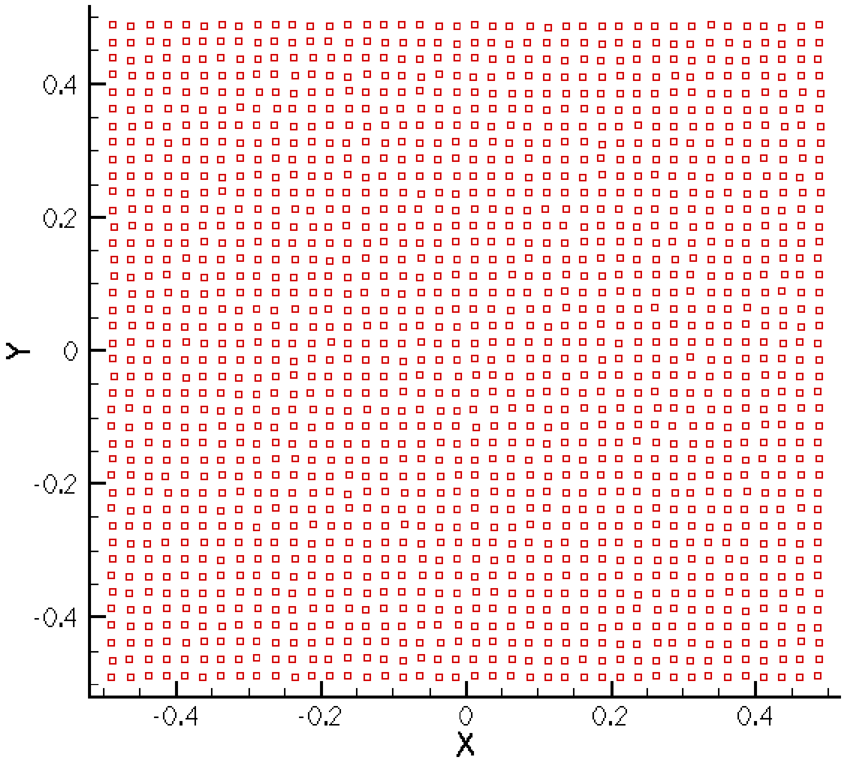
$$\boldsymbol{r}_i^{n+1} = \boldsymbol{r}_i^n + \frac{\left(\boldsymbol{u}_i^{n+1} + \boldsymbol{u}_i^n\right)\Delta t}{2}$$

- Particle distributions regularised according to local particle concentration (Fick's law, Lind et al. 2012)

$$\boldsymbol{r}_i^{n+1^*} = \boldsymbol{r}_i^{n+1} - D \nabla C_i^{n+1}$$

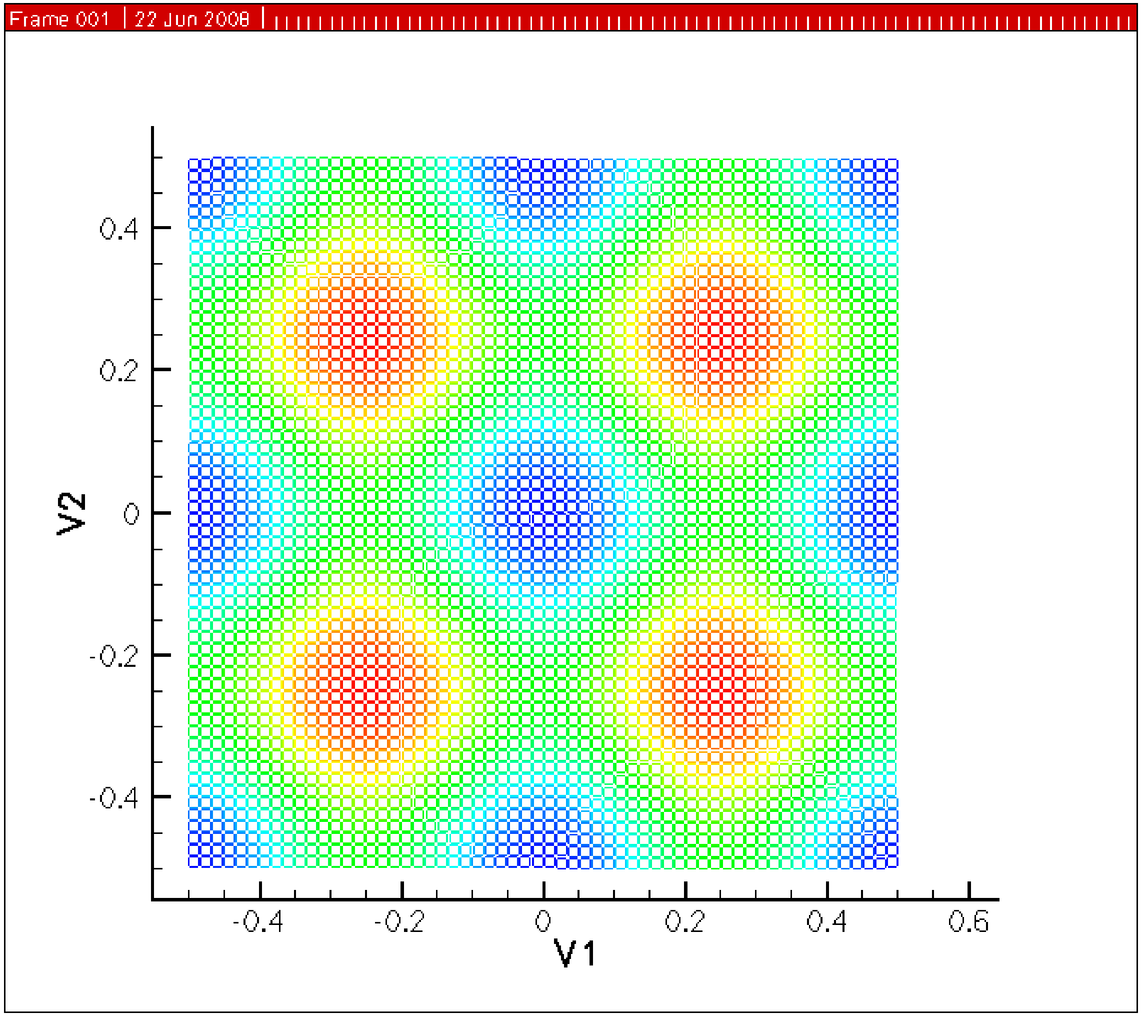
- Velocities, pressures corrected using interpolation - Taylor expansion

Taylor Green vortices – Stability Problem.



Taylor-Green vortices are simulated by ISPH_DF (Cummins & Rudman), with 4th order Runge-Kutta time marching scheme and random initial particle distribution.

Stabilising with shifting to regularise gives highly accurate solutions



The development of pressure field in Taylor-Green Vortices, with ISPH_DFS, Re=1,000

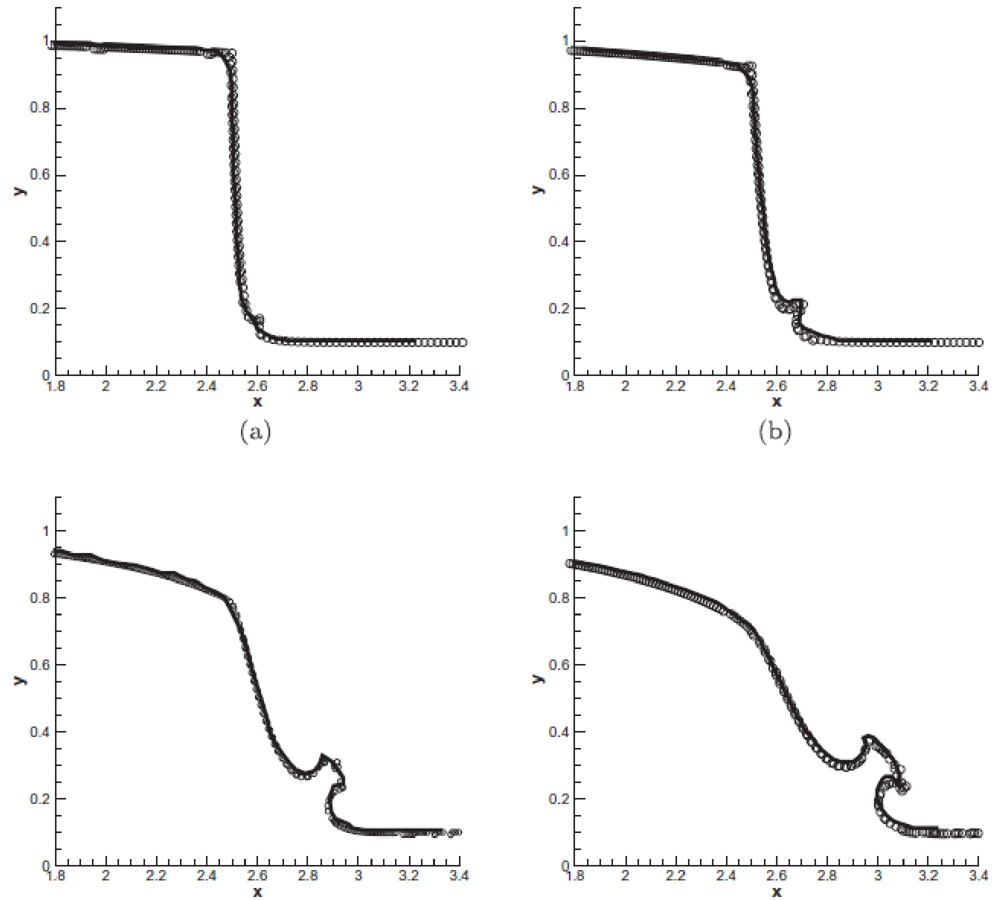
Accuracy and stability tests

- Taylor Green vortices – 2D periodic array,
- lid driven cavity,
- dam breaks,
- impulsive plate,
- wave propagation

Above with analytical or high accuracy solutions

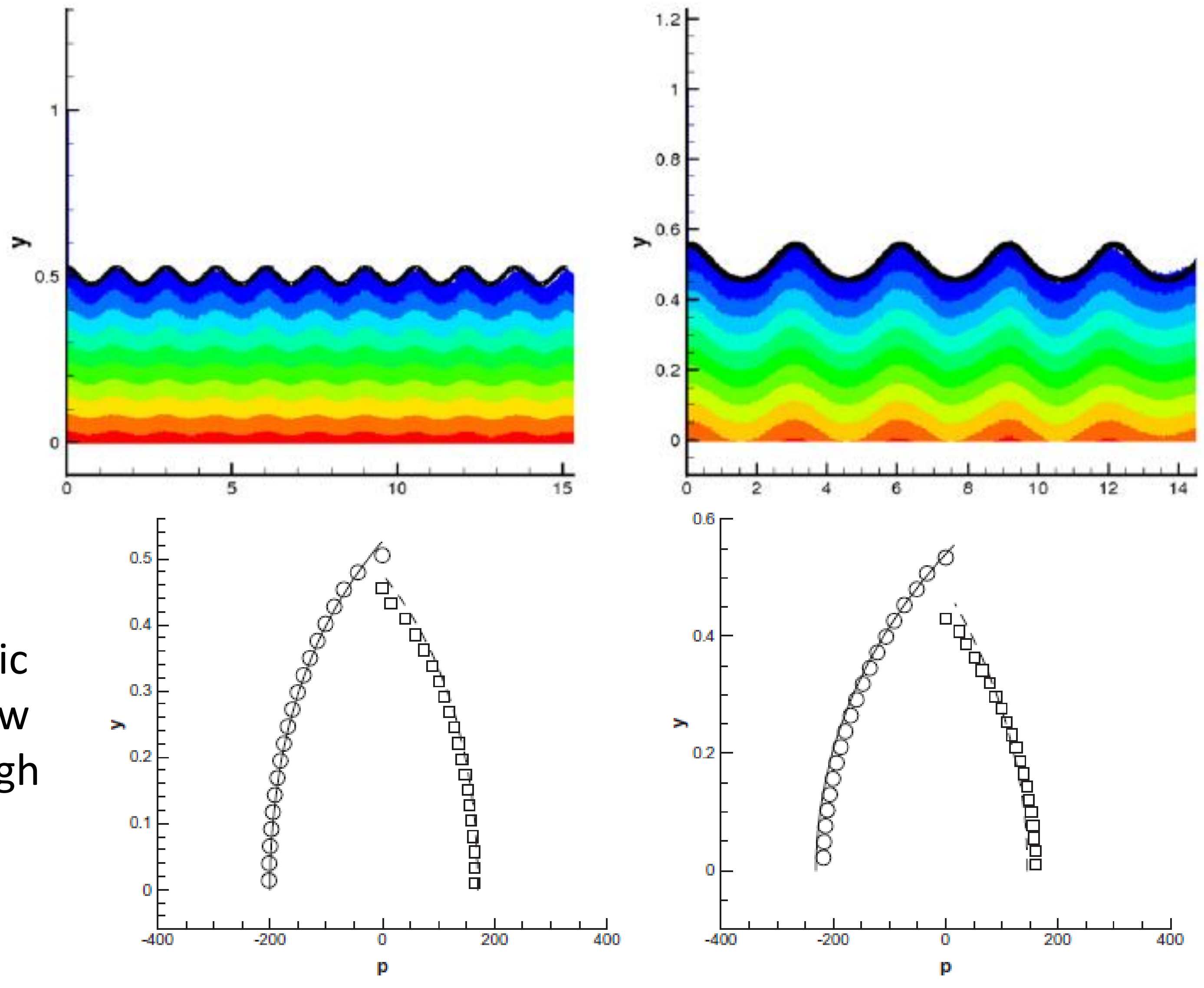
complex SPHERIC test cases

Dam break (wall of water problem)



(a) (b) (c) (d)

Wave propagation



(a) (b)

Non hydrostatic pressure below crest and trough

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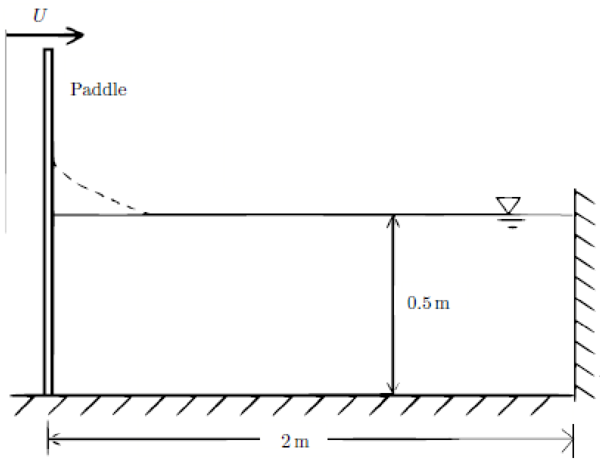
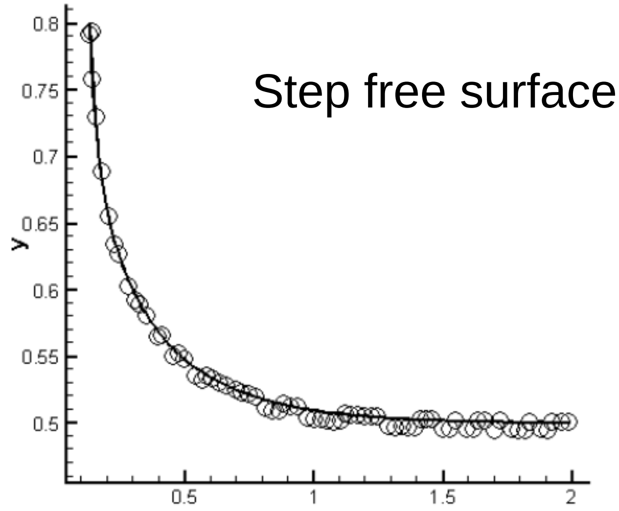
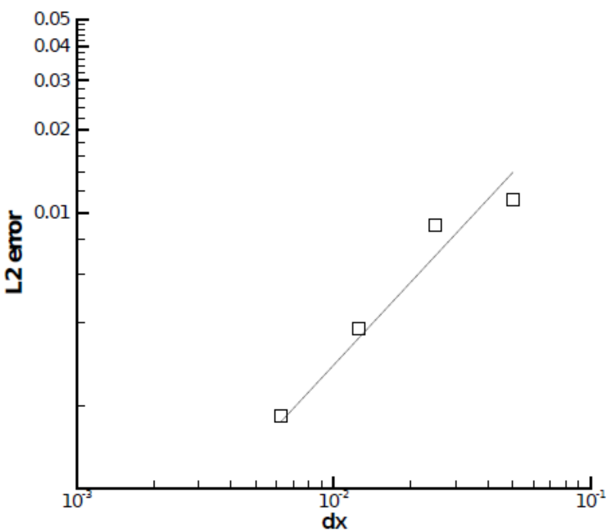
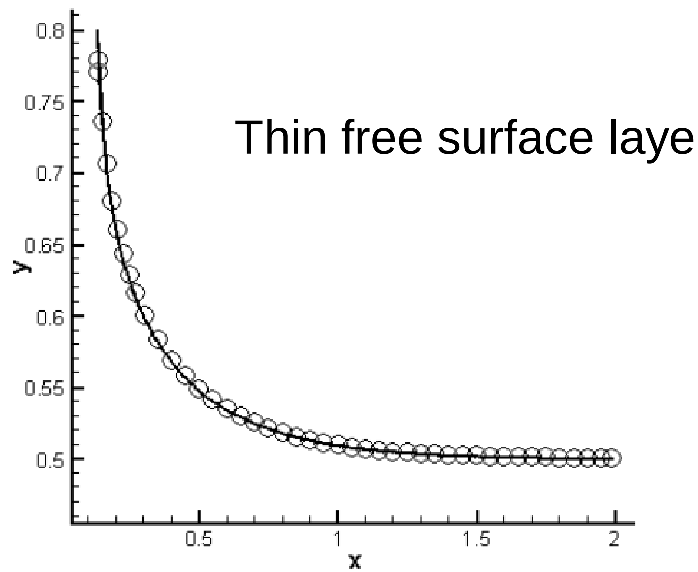
MANCHESTER 1824

IMPULSIVE PLATE (zero gravity analytical solution from Peregrine)



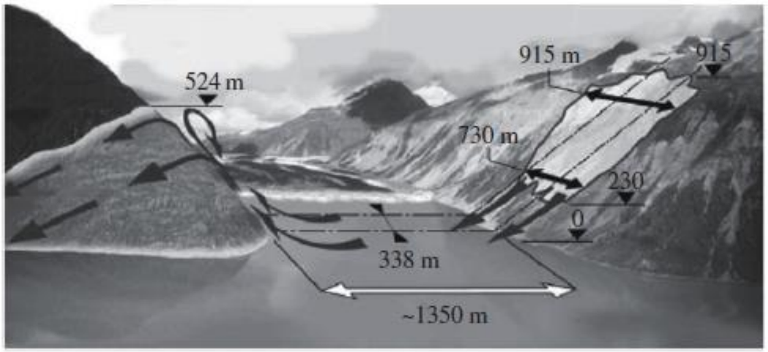


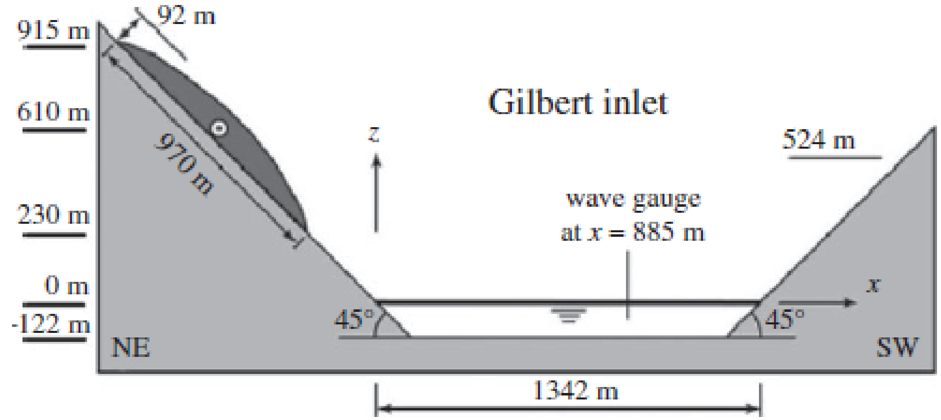




Skillen,A., Lind,S., Stansby,P.K., Rogers,B.D. 2013 J. CMAME, 265.

Lituya Bay landslide and tsunami 1958





ISPH

Non Newtonian flows

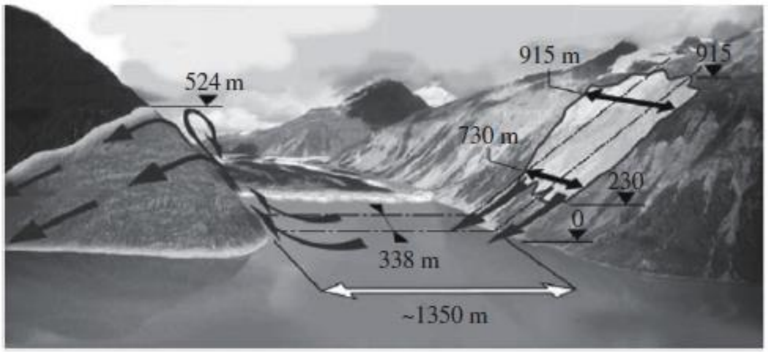
Herschel–Bulkley rheology model

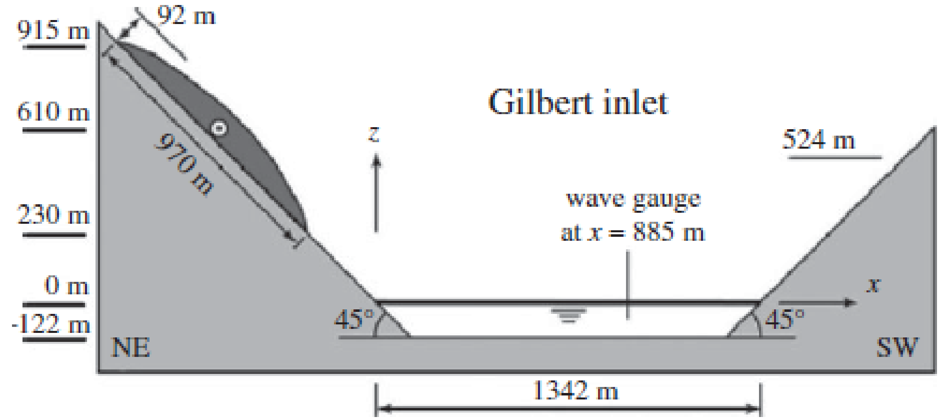
Saturated soil with Bingham model

k-ε turbulence model

Xenakis, A.M., et al. 2017 *Proc. Royal Soc. A: Math. Phys. Eng Sci*, 473 (2199), 20160674

Lituya Bay landslide and tsunami 1958





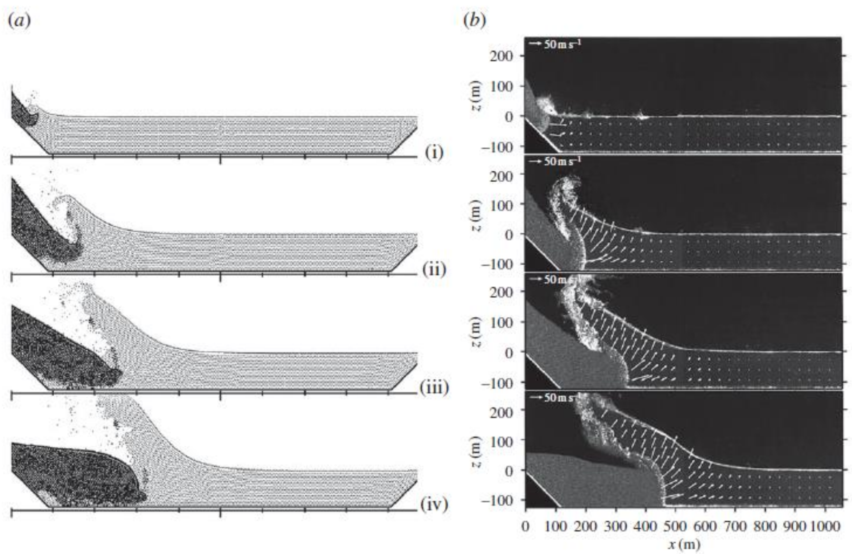
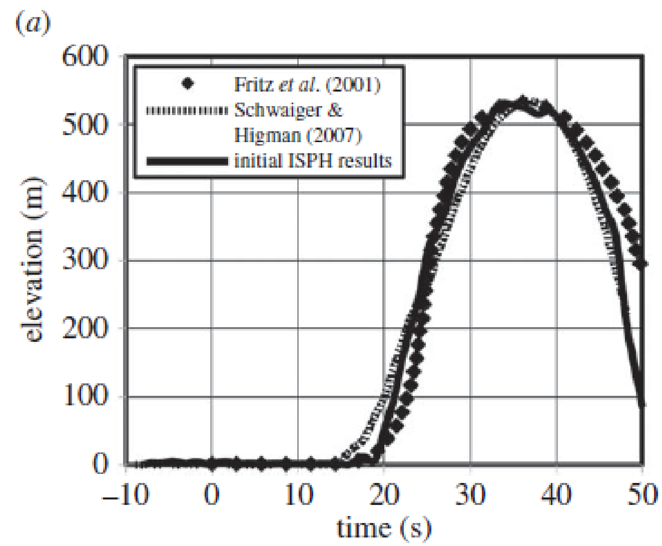


Figure 10. Impact of the two phases at 1.73 s intervals with the first image at time $t = 0.76$ s after impact: (*a*) the final ISPH results and (*b*) the experimental results of [2].



Xenakis, A.M., et al. 2017 *Proc. Royal Soc. A: Math. Phys. Eng Sci*, 473 (2199), 20160674

WCSPH improves
δ SPH with artificial viscosity

$$\frac{D\rho_i}{Dt} = -\rho_i \sum_i (\boldsymbol{u}_j - \boldsymbol{u}_i) \cdot \nabla_i W(\mathbf{r}_j) V_j + \delta h c_0 \sum_i \psi_{ij} \cdot \nabla_i W(\mathbf{r}_j) V_j$$

$$\rho_i \frac{D\boldsymbol{u}_i}{Dt} = -\sum_i (p_j + p_i) \nabla_i W(\mathbf{r}_j) V_j + \rho_i \boldsymbol{f}_i + \alpha h c_0 \rho_0 \sum_i \pi_{ij} \nabla_i W(\mathbf{r}_j) V_j$$

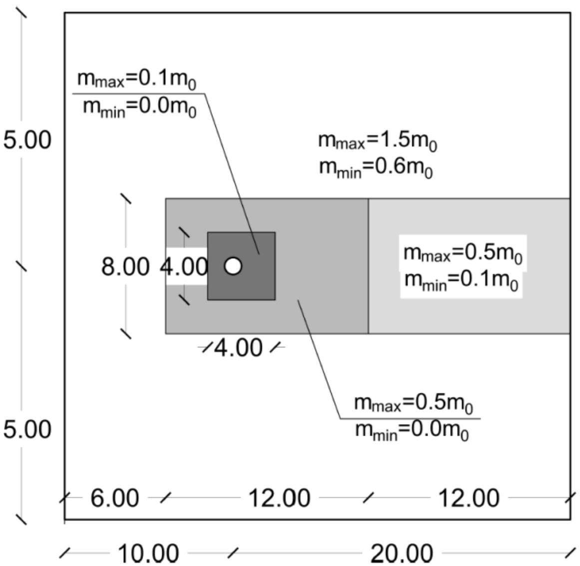
$$p_i = c_0^2(\rho_i - \rho_0)$$

Note both artificial diffusion in continuity and viscosity → 0 as h → 0

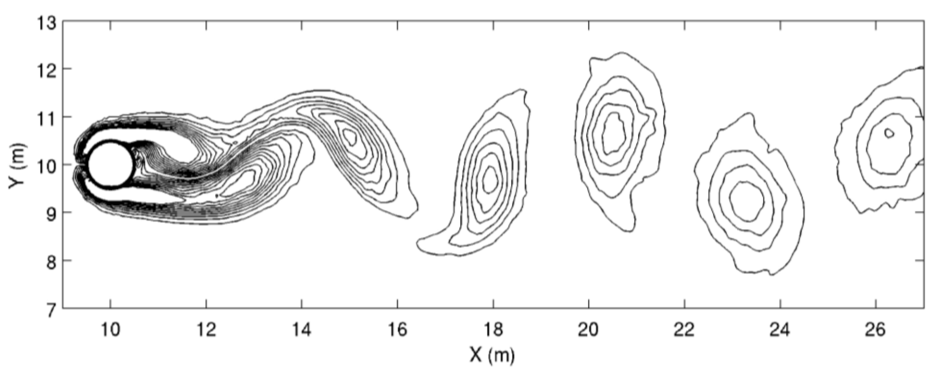
Molteni,D. and Colagrossi, A. 2009 Computer Physics Communications 180 , 861–872
Marrone,S. et al 2011 CMAME 2010, 1526-1542

δ SPH + shifting

Adaptivity to reduce number of particles



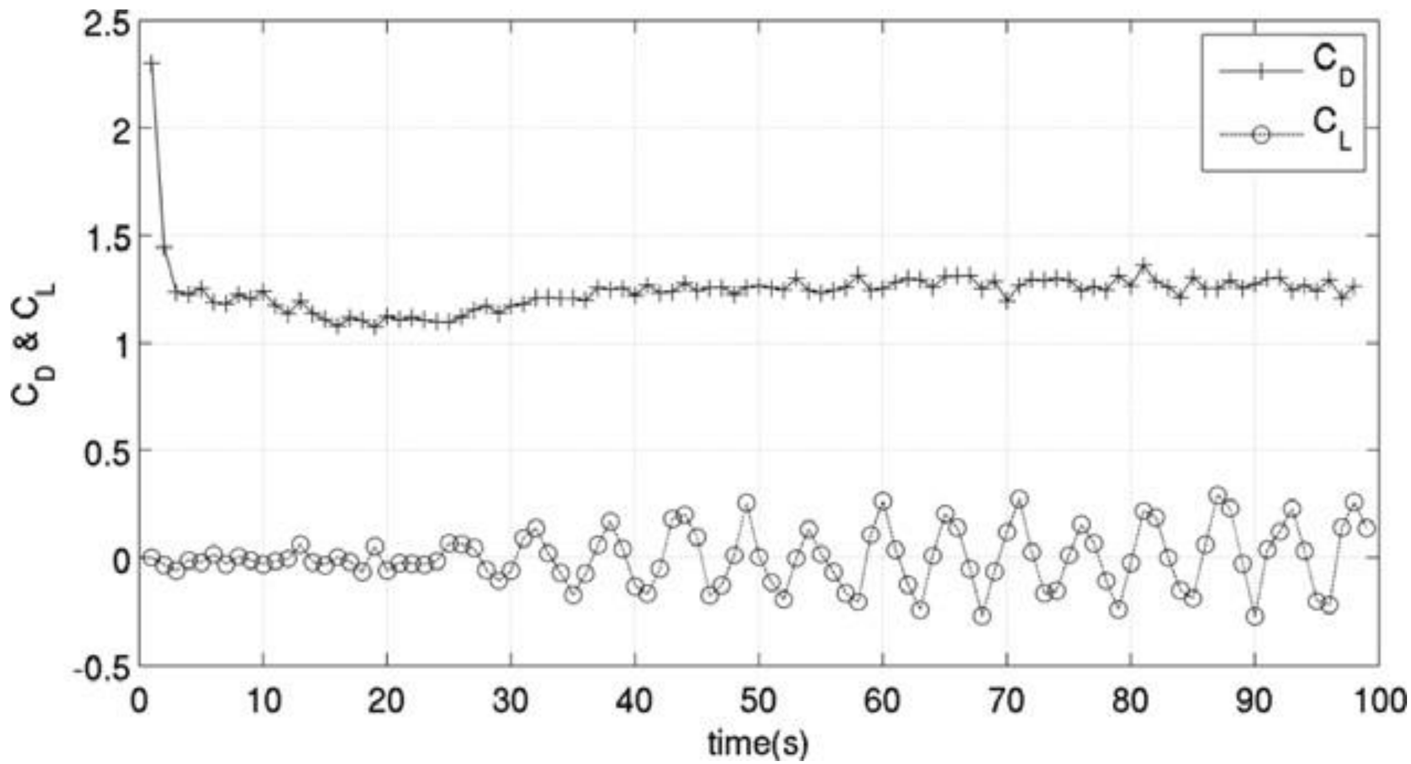
Zones of resolution



Extended to 3D

Vacondio,R., Rogers,B.D., Stansby,P.K, Mignosa,P., Feldman,J. 2013 J. CMAME, 256.

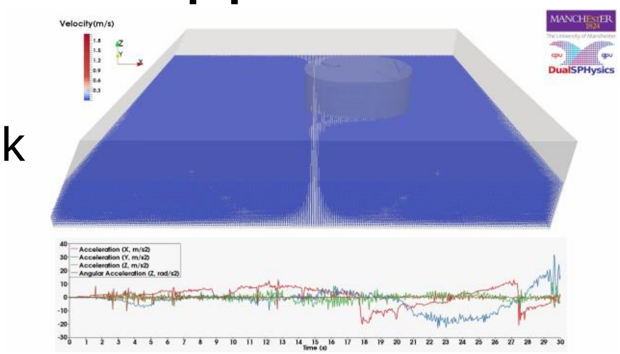
C_D and C_L Re=100



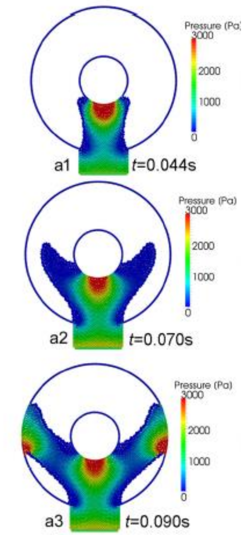
Little noise

Diverse applications for WCSPH

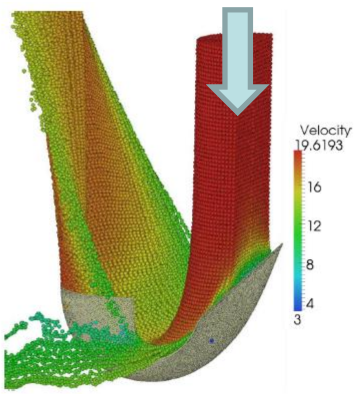
F1 fuel tank sloshing



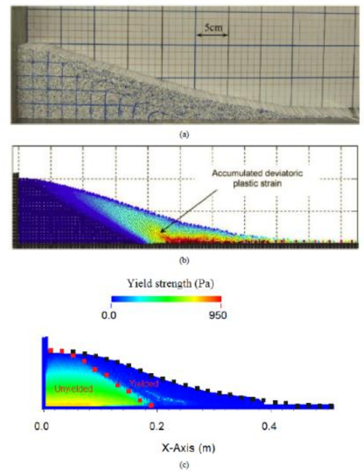
Moulding



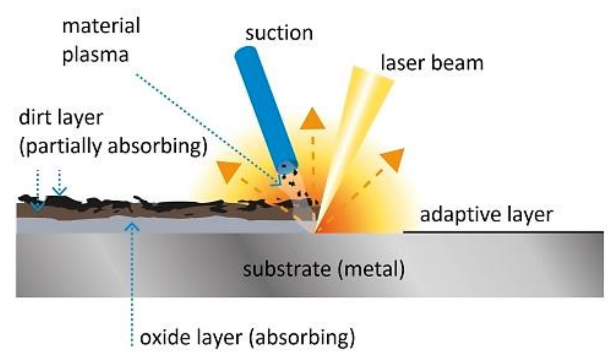
Pelton wheel



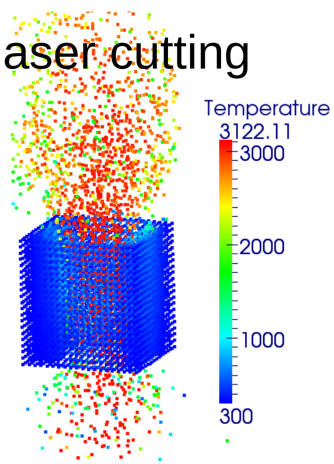
Slope collapse



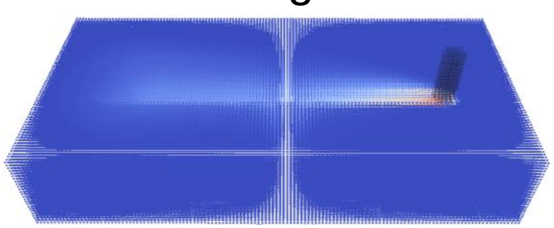
Laser cleaning



Dry laser cutting



Welding



Numerical wave basin

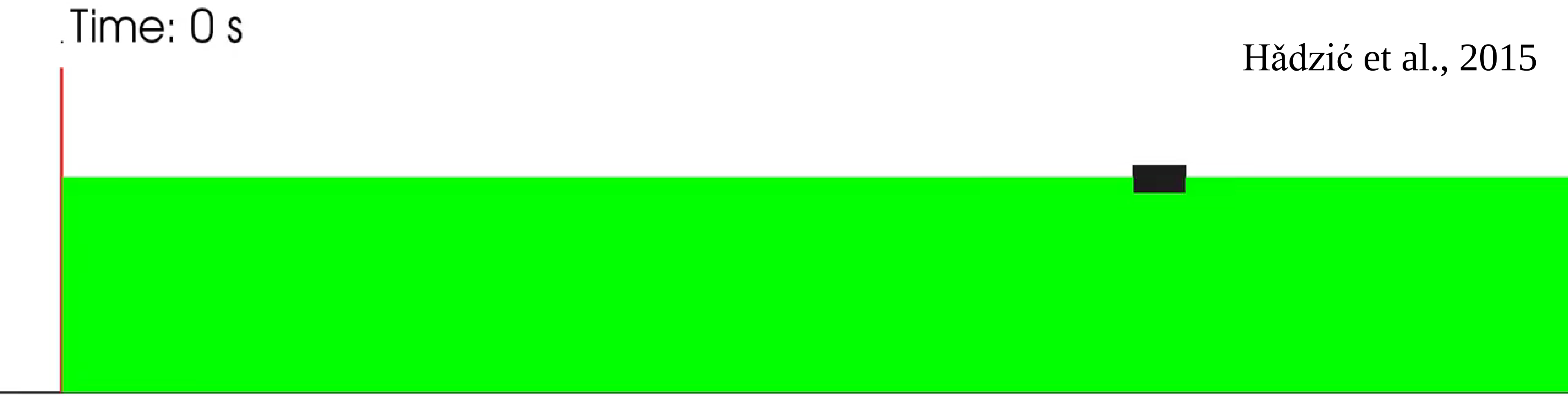
- 3D
- Progressive waves, focussed waves, directional waves
- Breaking waves
- Two phase, aeration
- Slam, wave on deck, green water
- Complex bodies, multi bodies
- Dynamics, moorings
- Extreme wave definition, storm, freak, tsunami
- Validation – experimental uncertainty
- Accessible computation time

Available options

- Linear diffraction, frequency and time domains – WAMIT, Nemoh, WECsim
- 2nd order diffraction, WAMIT - drift forces, BEM - time domain
- Nonlinear potential flow – QALE, HOBEM
- FV / VOF - OpenFOAM, Fluent, STAR CCM
- WCSPH - DualSPHysics
- PICIN
- ISPH
- Hybrids

δ SPH with artificial viscosity

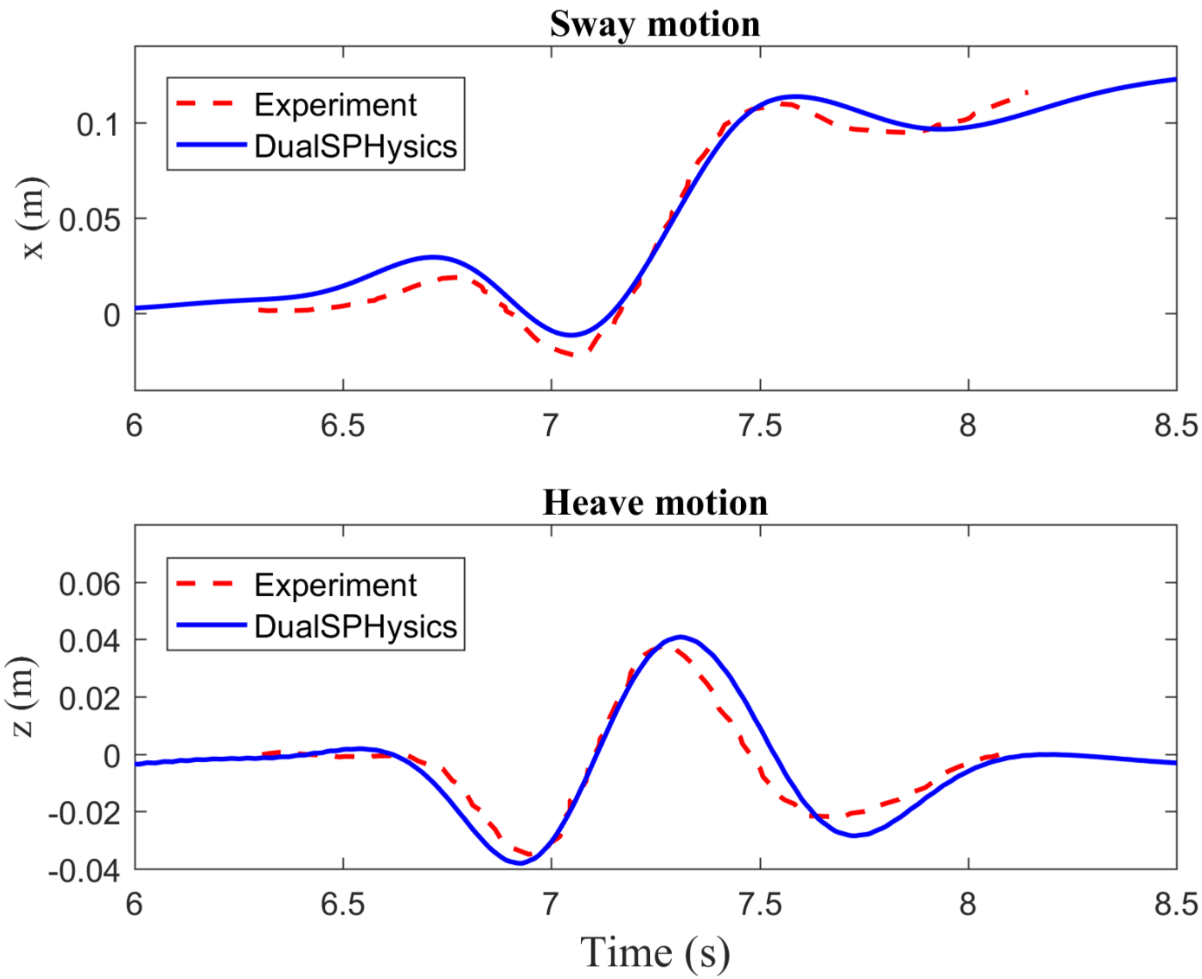
Wave interaction with floating bodies , represented by particles (dummy/dynamic) moving with body



Hădzić et al., 2015

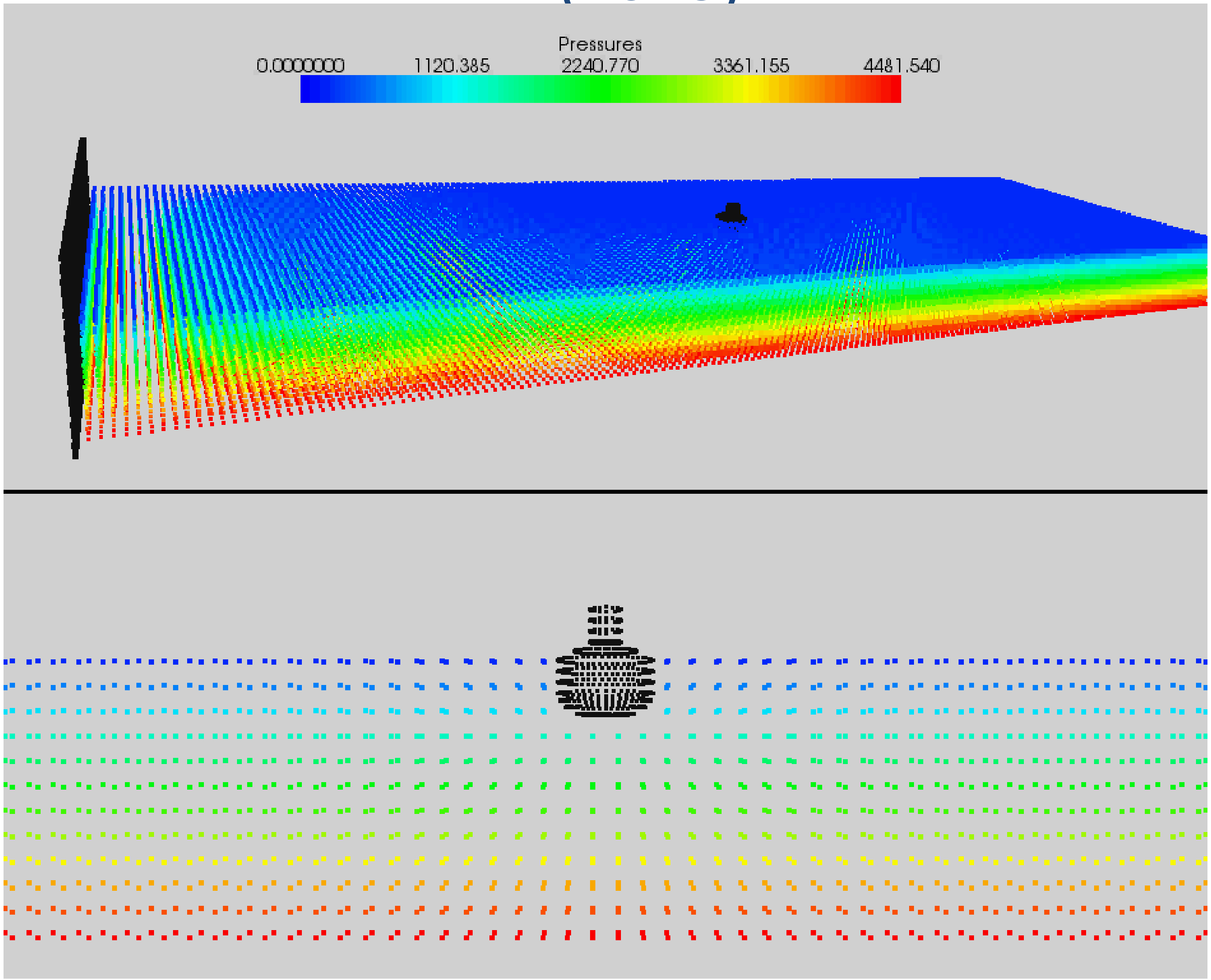
From Alex Crespo, Vigo

Wave interaction with floating bodies



From Alex Crespo, Vigo

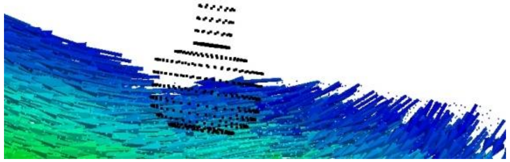
3-D Numerical Wave Basin using Riemann solvers (2013)



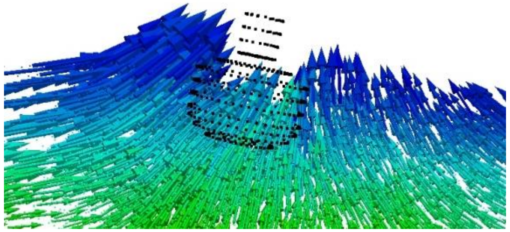
Omidvar,P., Stansby,P.K. and Rogers,B.D. 2013 Int. J. Numer. Methods Fluids, 72, 427-452

3-D Float Simulation

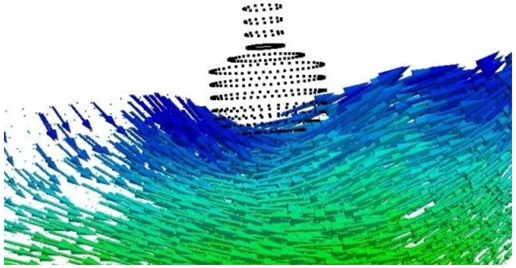
t = 3.8 s



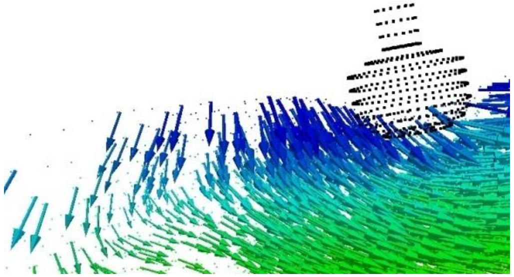
t = 4.2 s



t = 4.4 s



t = 4.6 s

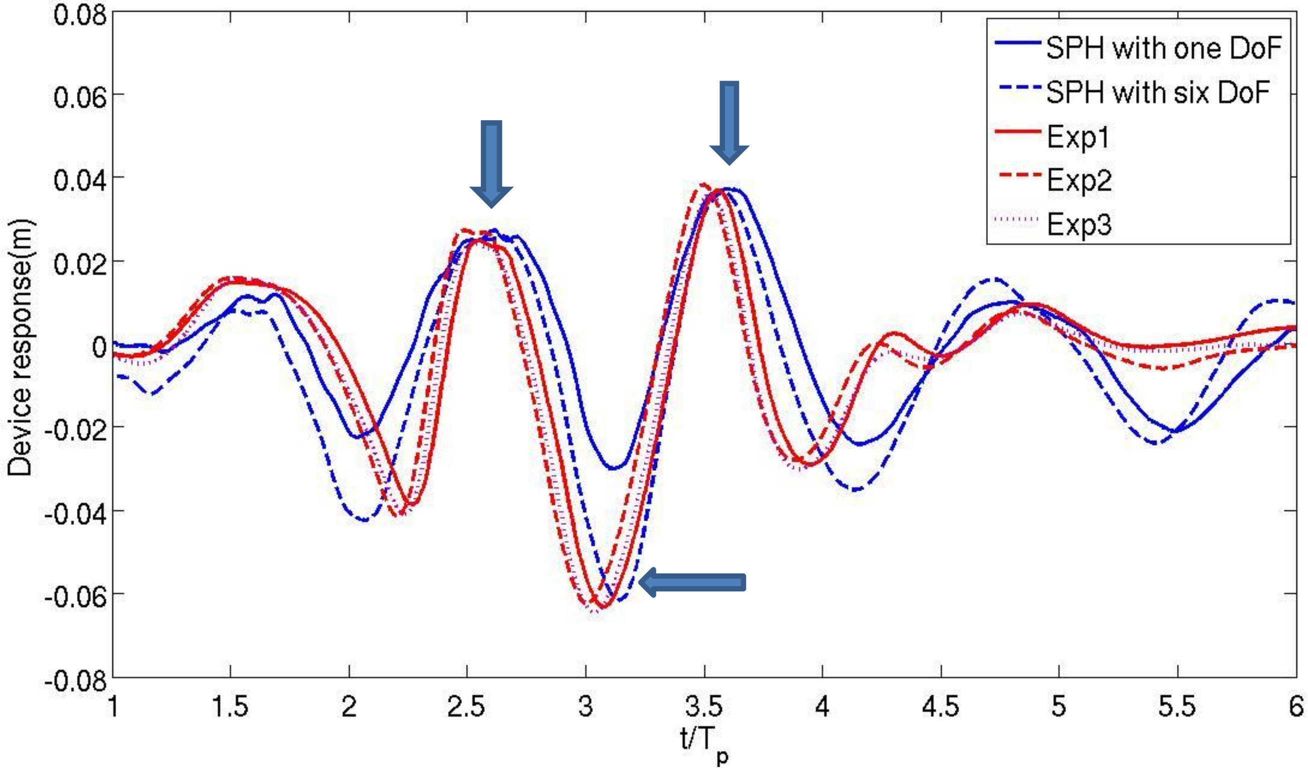


VALIDATION VITAL

MANCHESTER 1824

The University of Manchester

3-D Float Response



For a **full degrees-of-freedom system**, the results are very promising

Back to ISPH

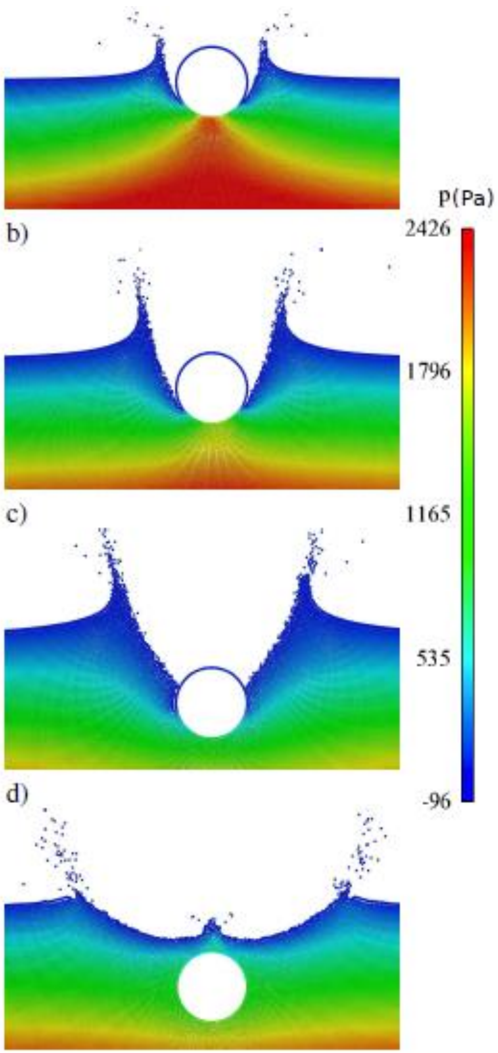
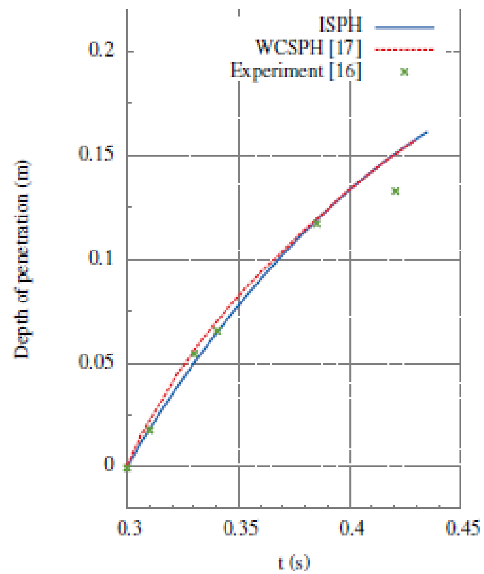
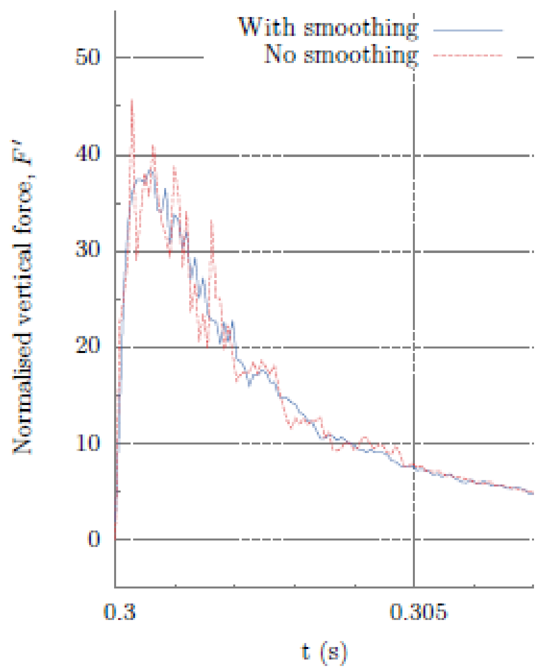


Figure 9: Cylinder of half buoyant mass dropping into initially still water. a) $t = 0.332s$. b) $t = 0.365s$. c) $t = 0.44s$. d) $t = 0.56s$. Time $t = 0$ is at the cylinder's release, 0.3s prior to impact.



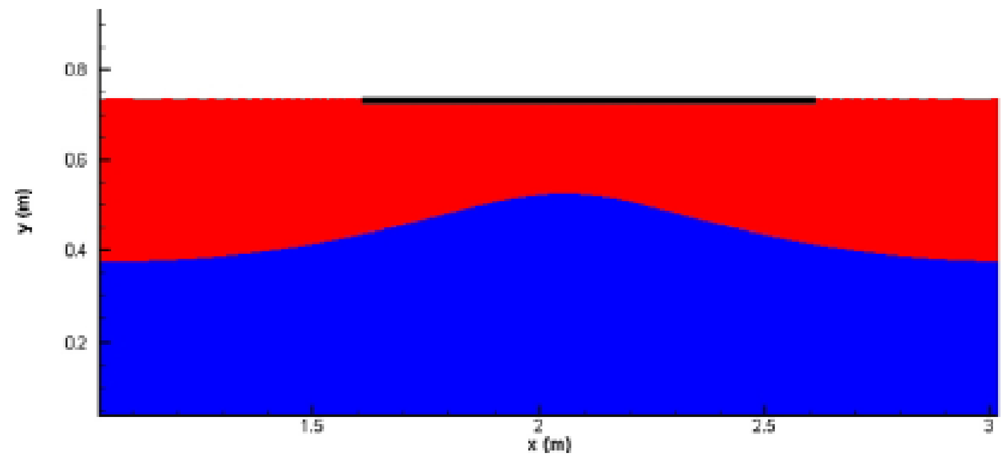


Effect thin free surface layer

Importance of air in slam force

Experiment (1998)

Slam force on a plate



SPH domain

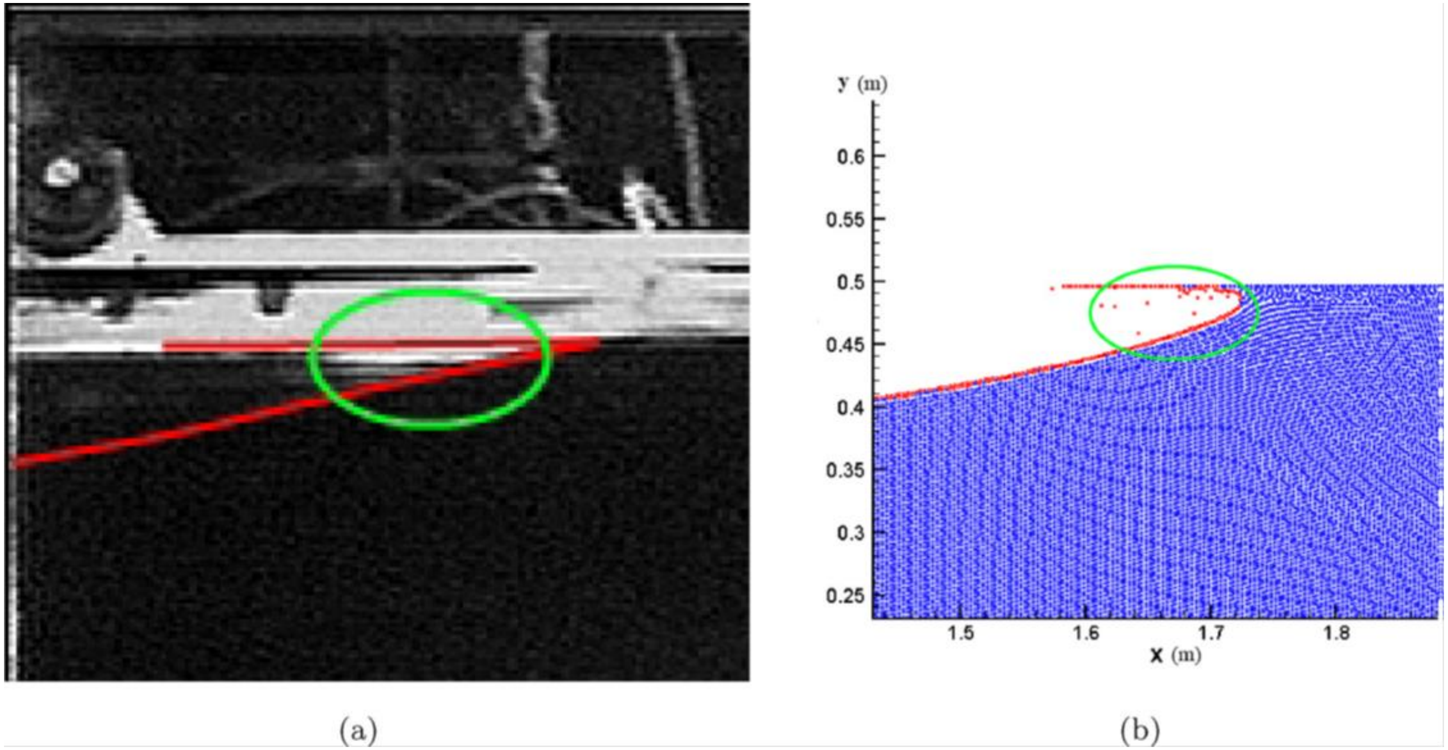
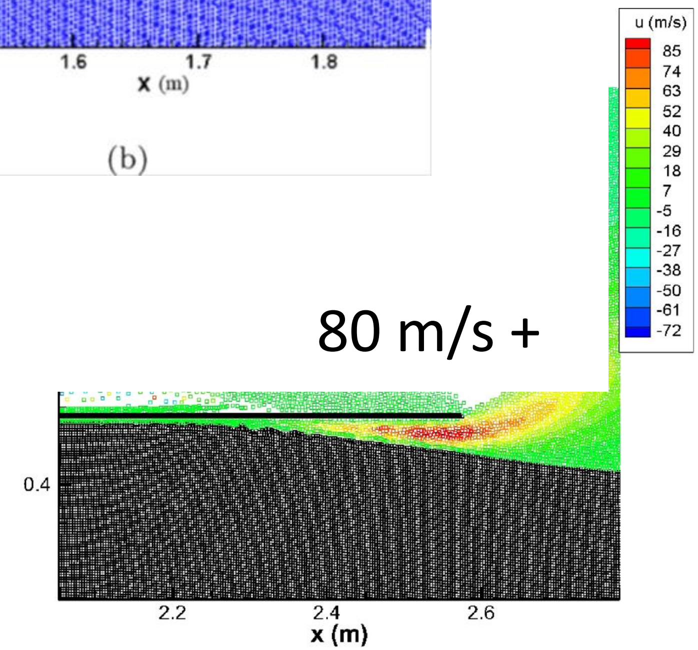
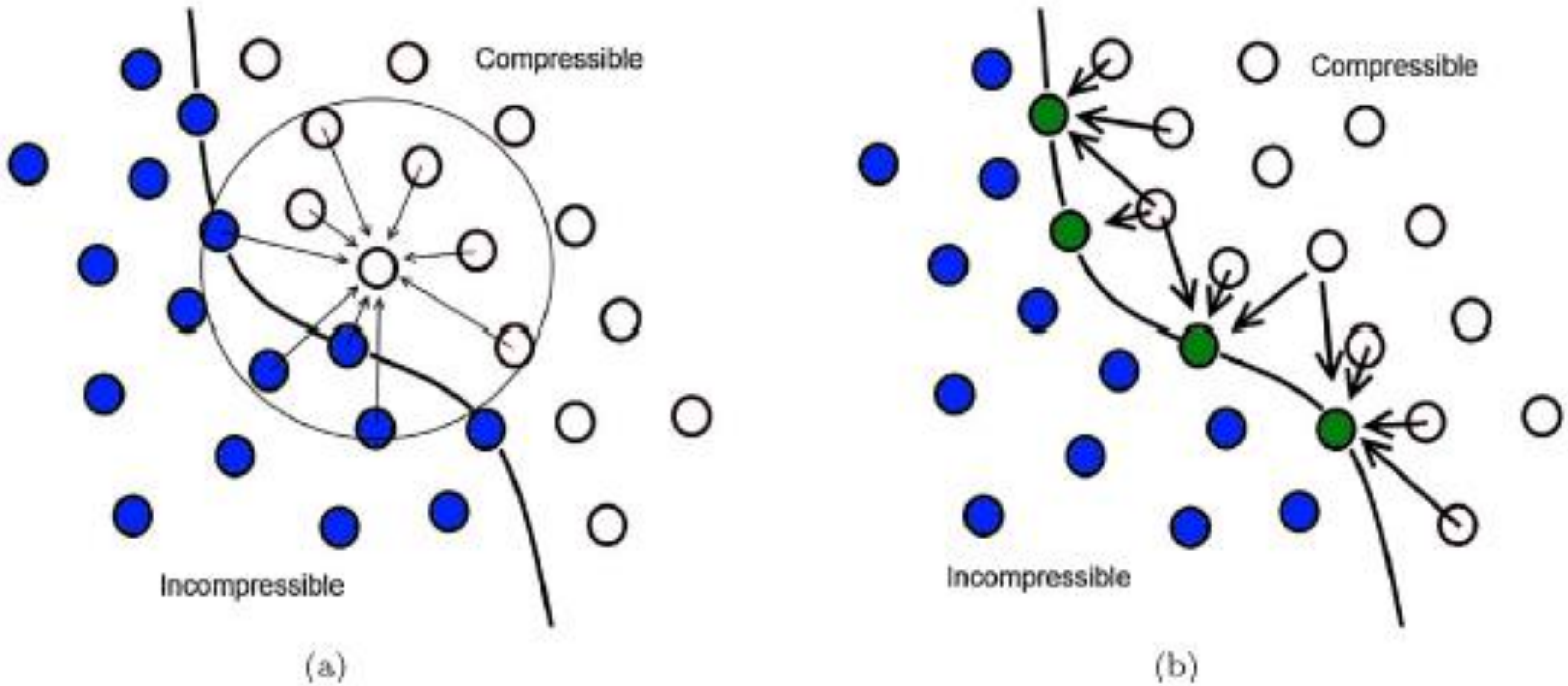


Plate impact on wave (5.4 m/s)
(represent wave impact on plate
- wave on deck)

80 m/s +



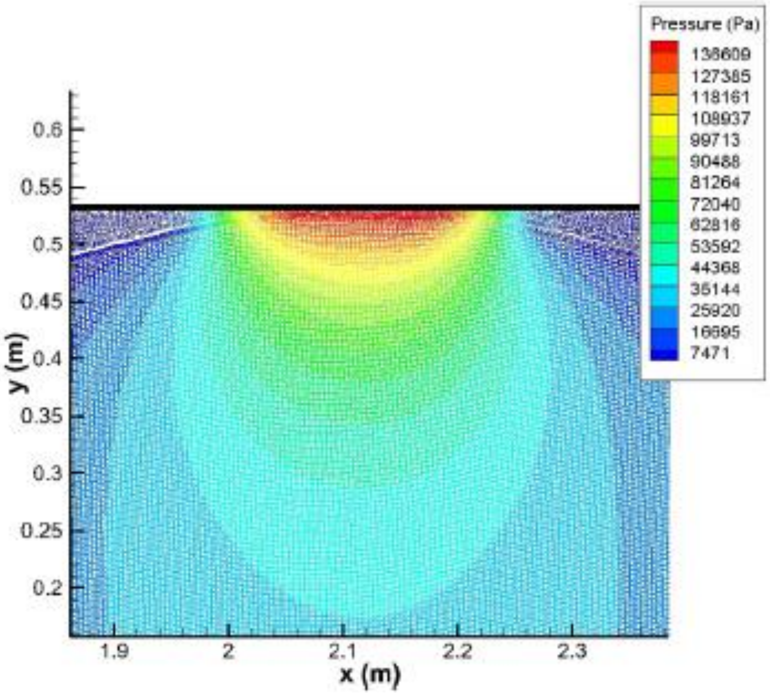
Air – water coupling (ICSPH)

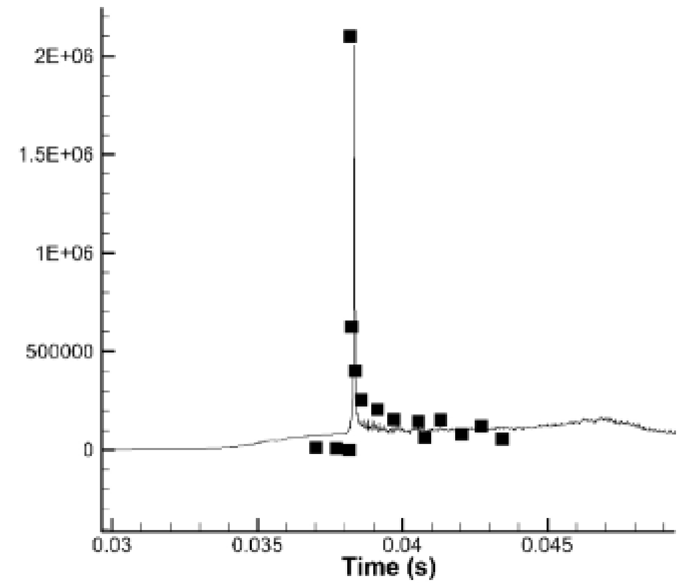


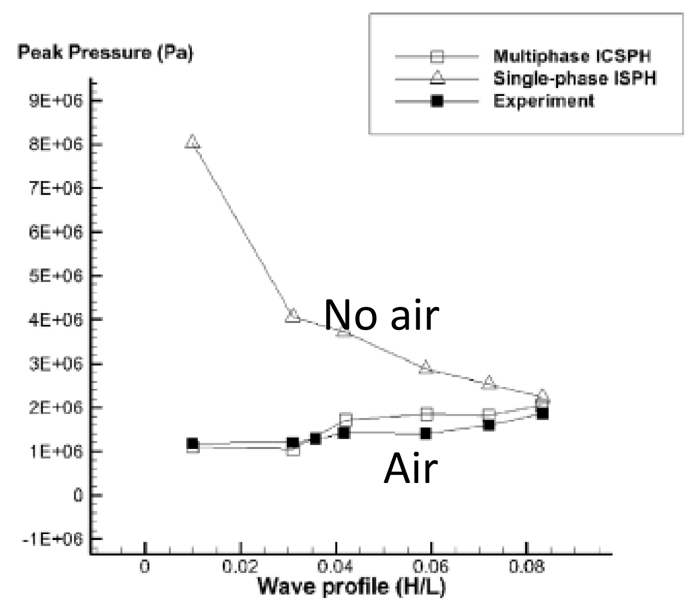
velocity pressure

Lind,S.J., Stansby,P.K., Rogers,B.D., Lloyd, P.M. 2015, Applied Ocean Research, 49, 57-71.

Pressures during slam

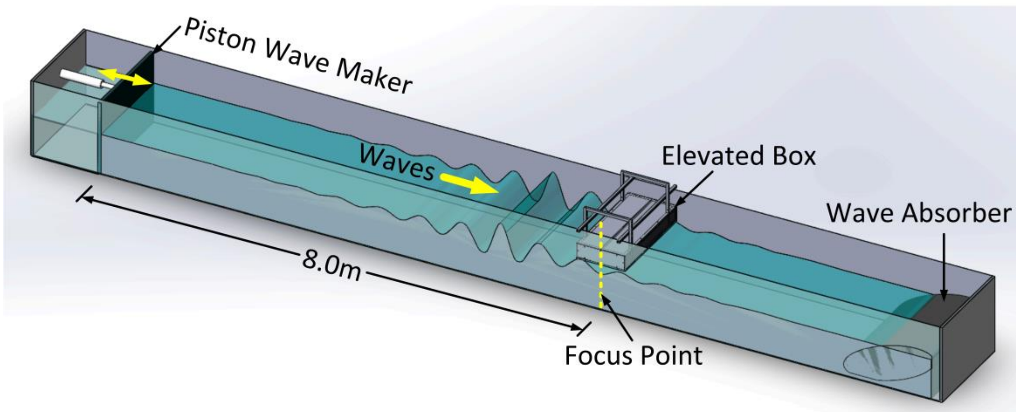




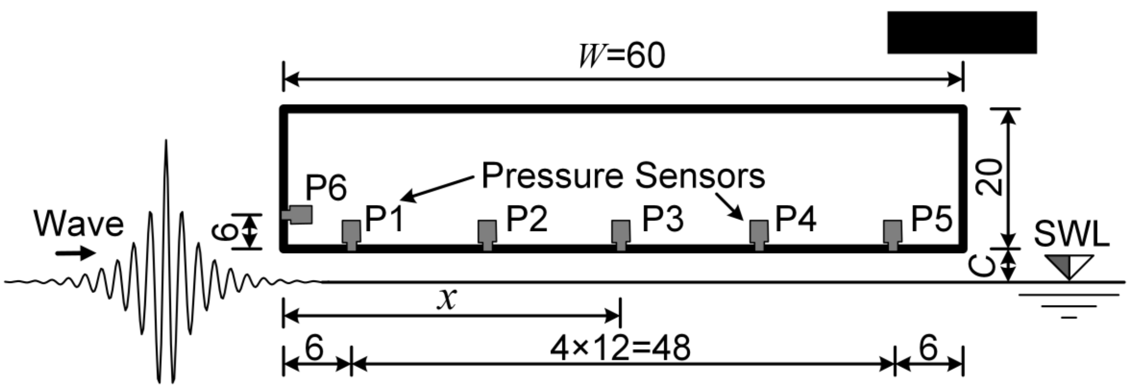


But new wave in deck experiments

undertaken by Qinghe Fang

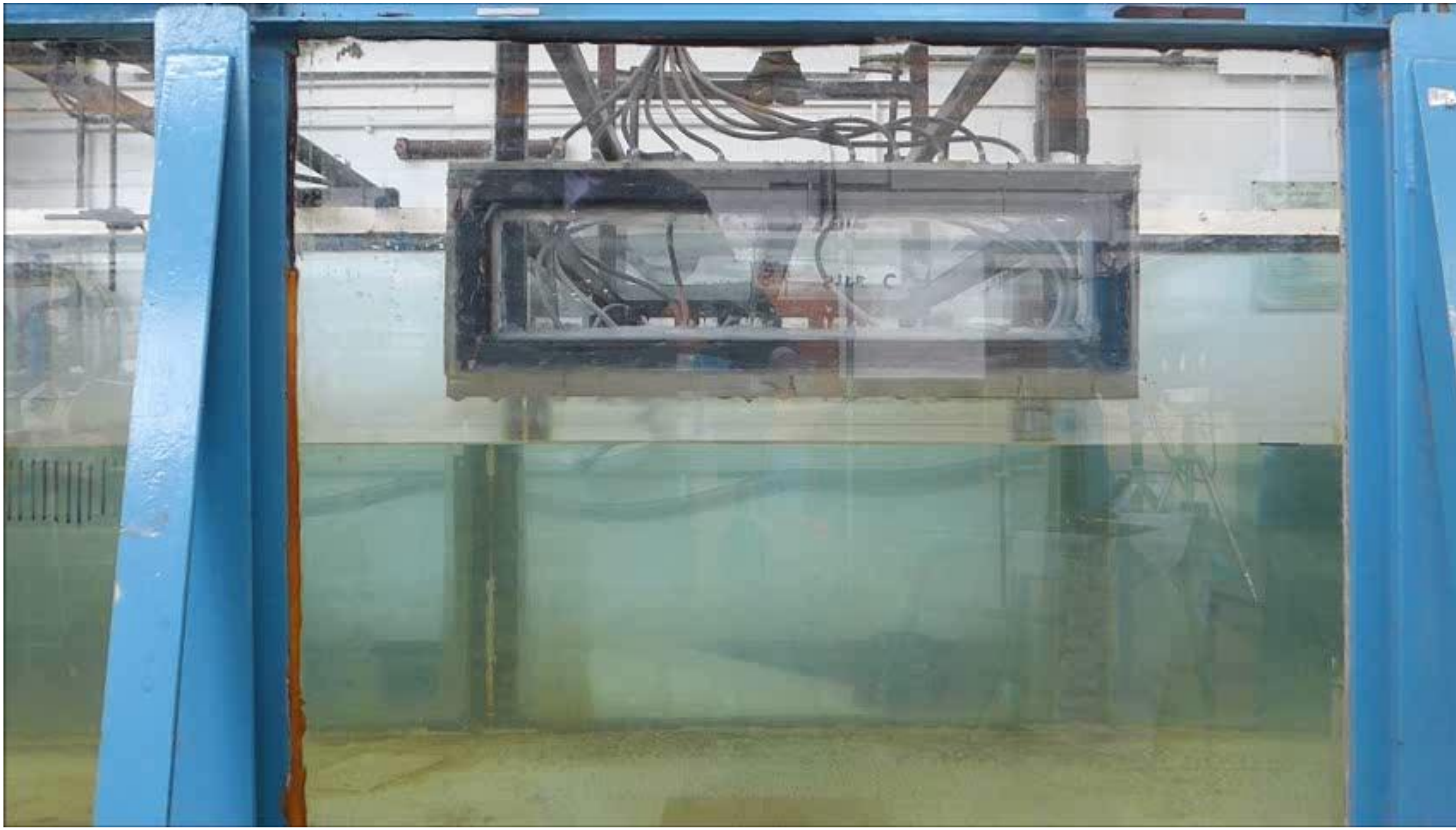


2D as possible



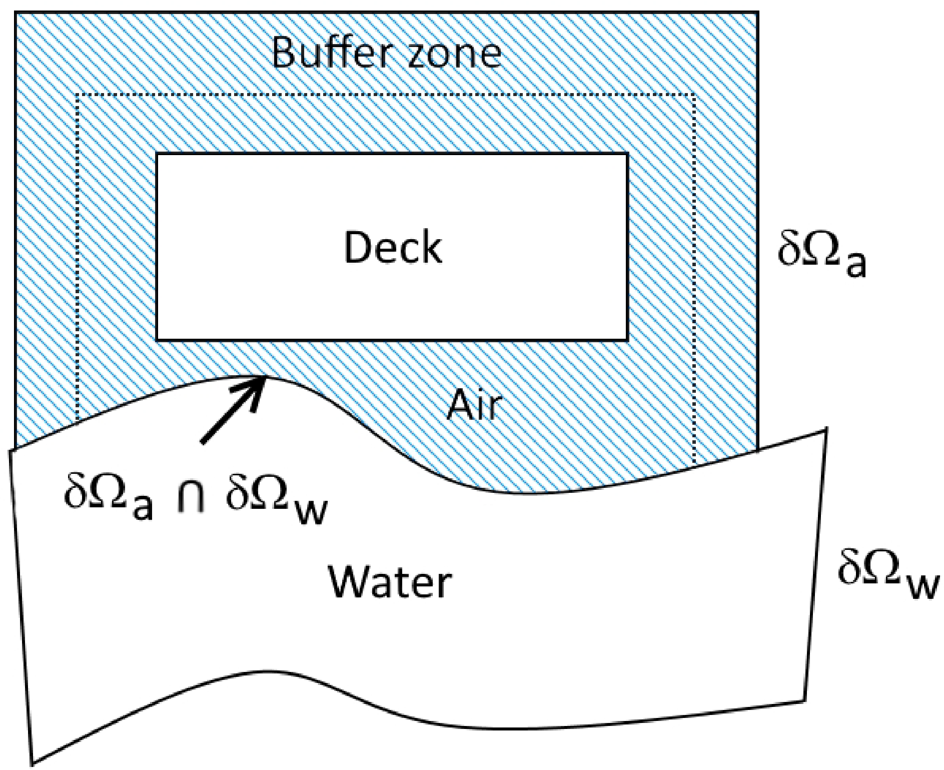
Focussed NewWave
JONSWAP

Lind, Fang, Stansby, Rogers, Fourtakas ISOPE Paper No. 2017-SQY-01



Computational SPH domains local to deck only

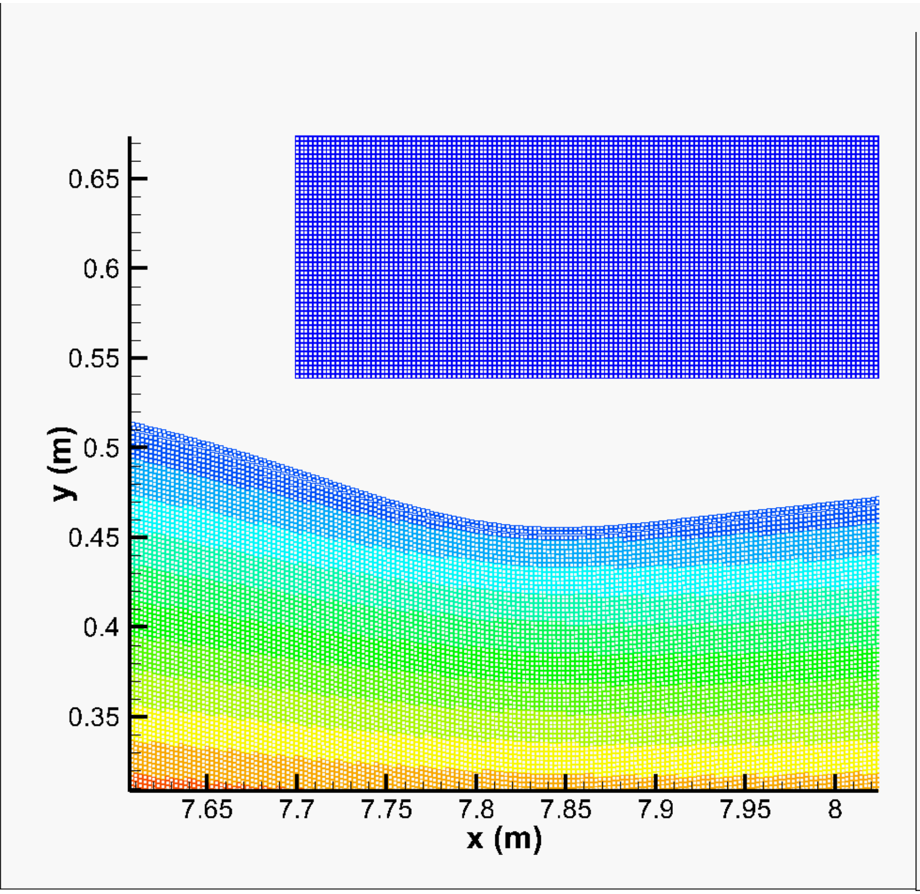
Air velocities damped to zero in buffer zone



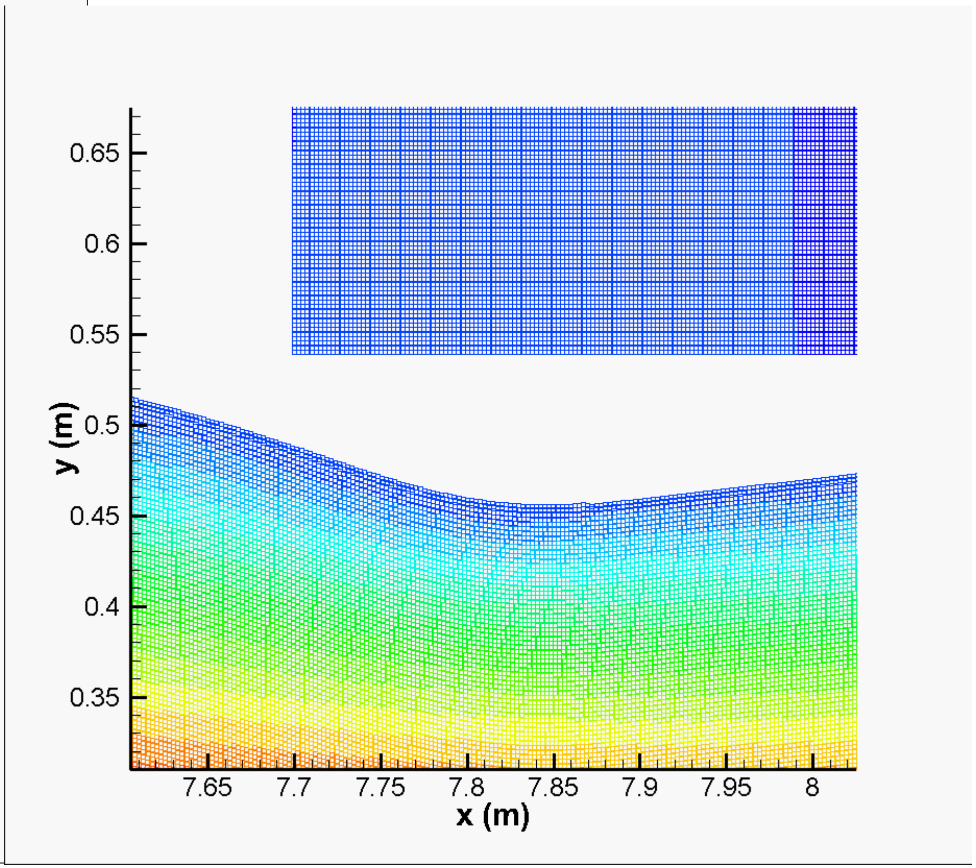
Focussed NewWave JONSWAP waves defined by linear theory

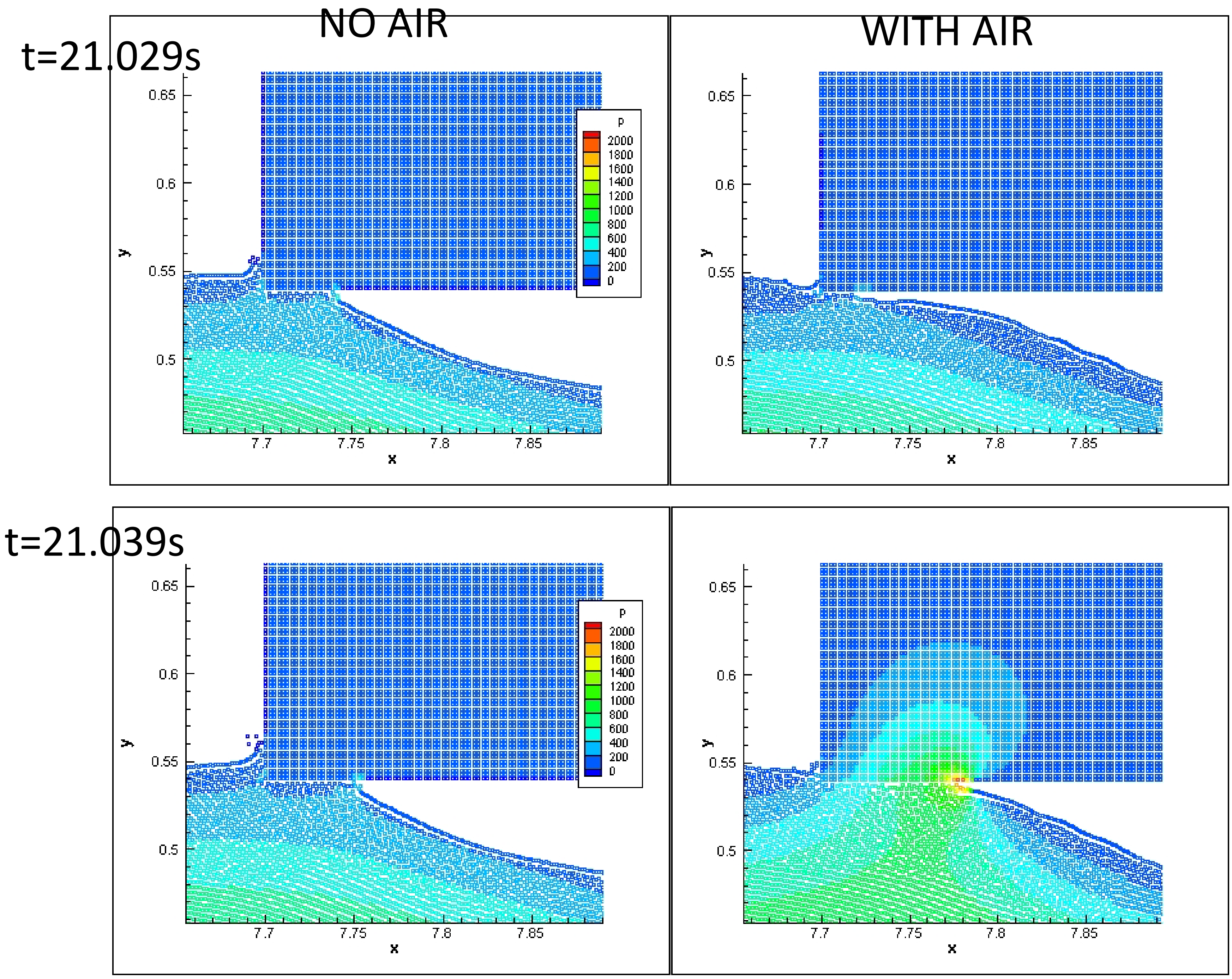
Boundary pressure in water from linear theory

Without Air

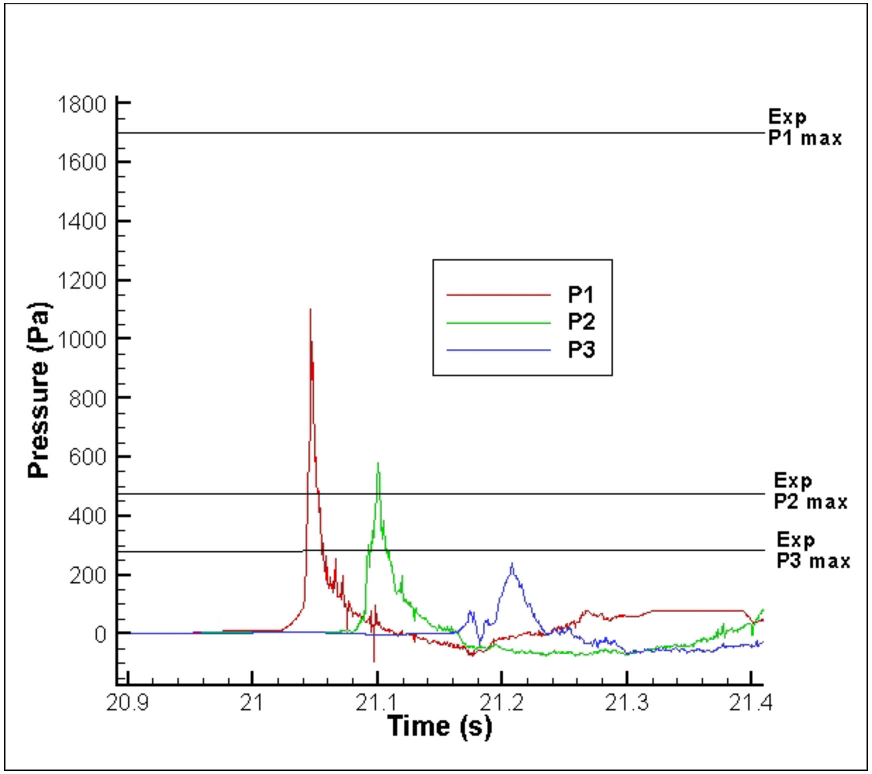


With Air

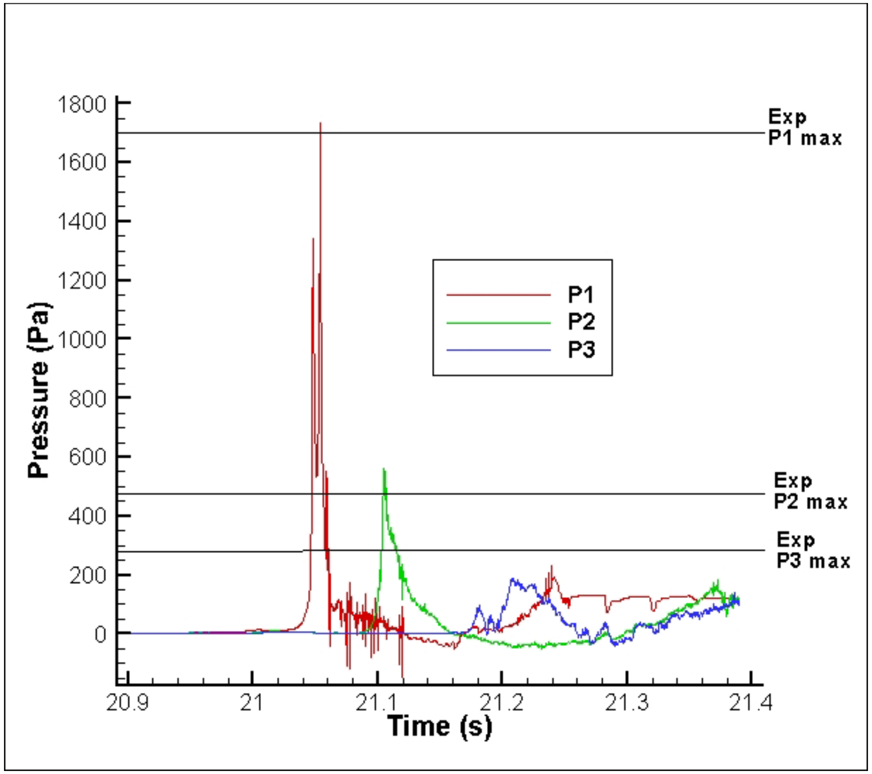



NO AIR
WITH AIR
t=21.029s
t=21.039s
p
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0.65
0.6
0.55
0.5
y
7.7
7.75
7.8
7.85
x

Preliminary SPH results

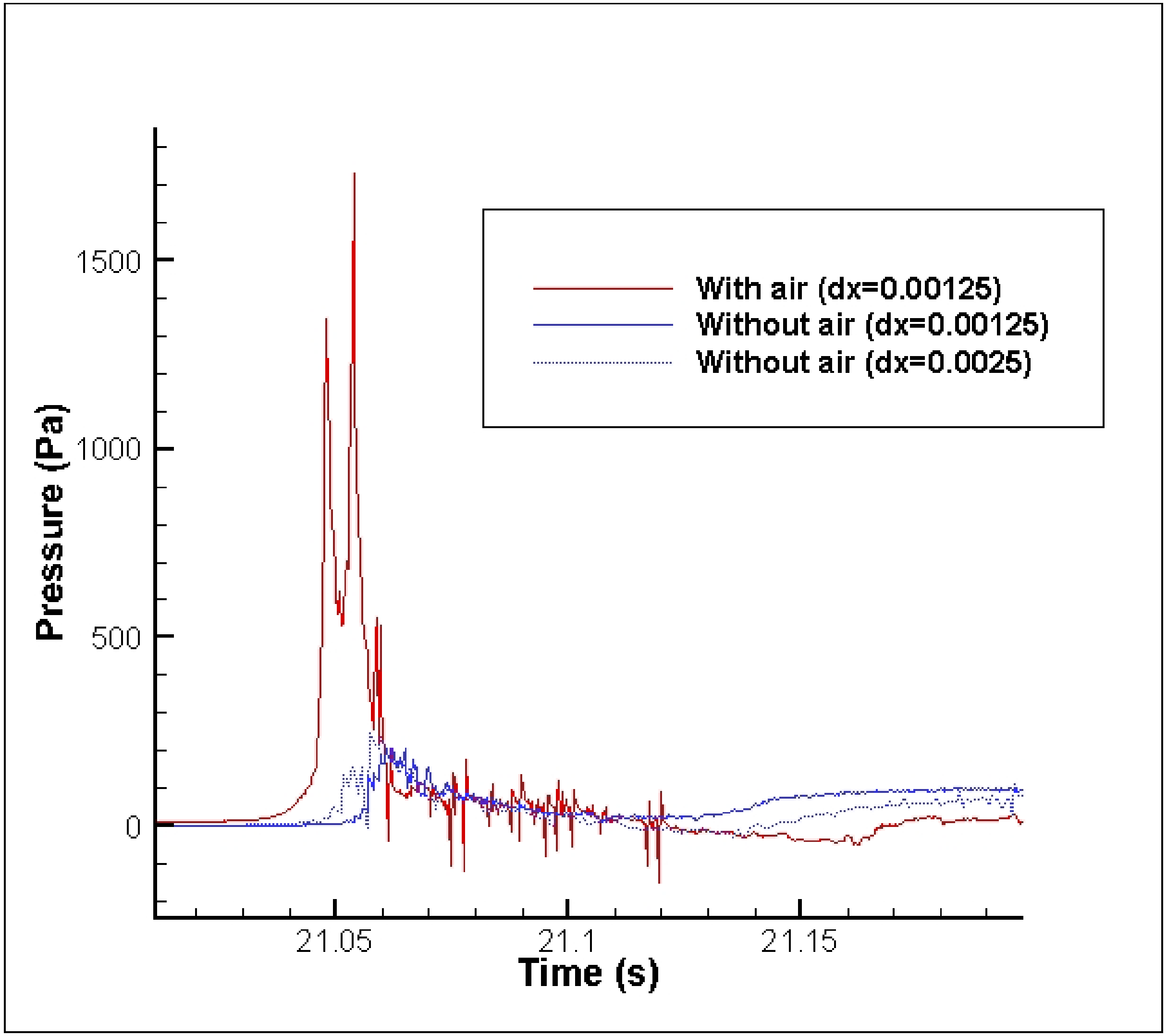


dx = 0.025m



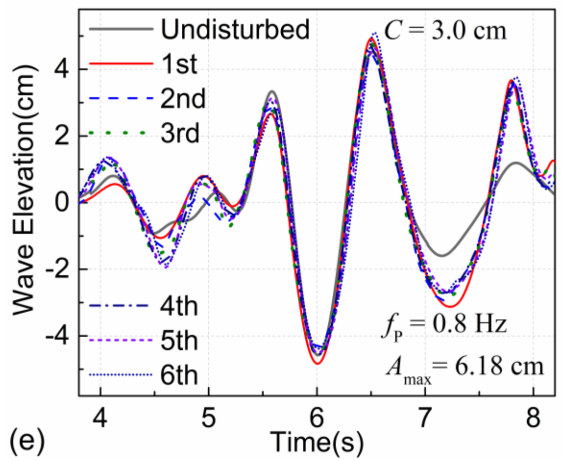
dx = 0.00125m

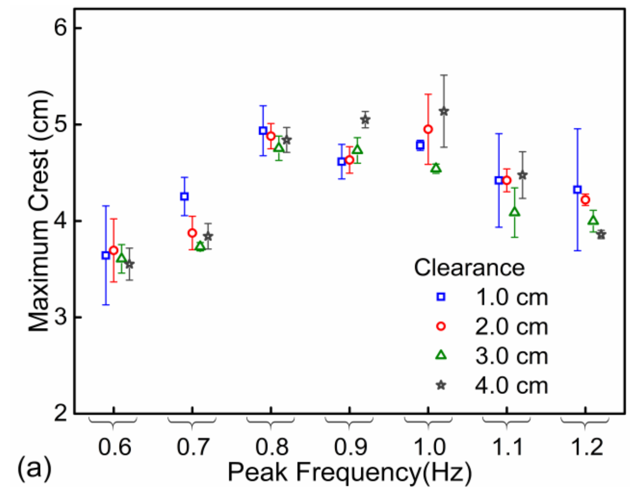
Results with/without air



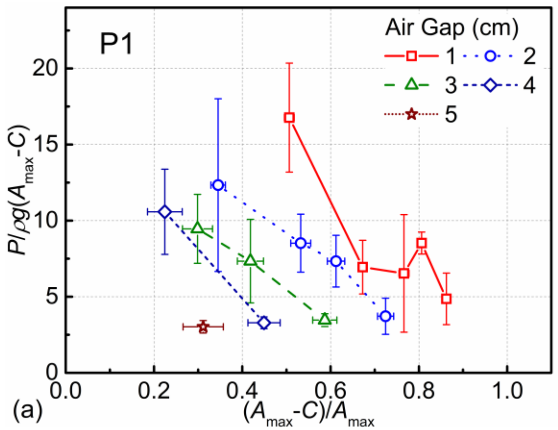
Uncertainty tracking

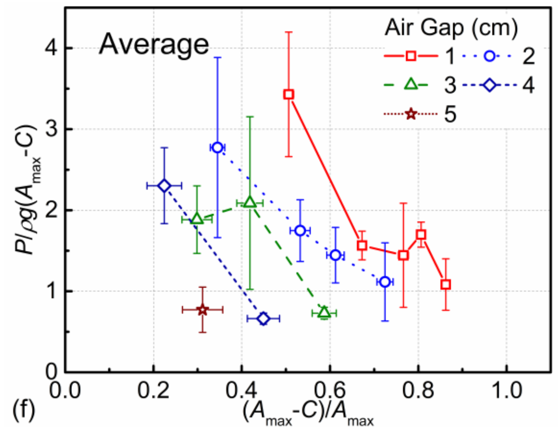
In front of box surface elevation uncertainty due to reflection





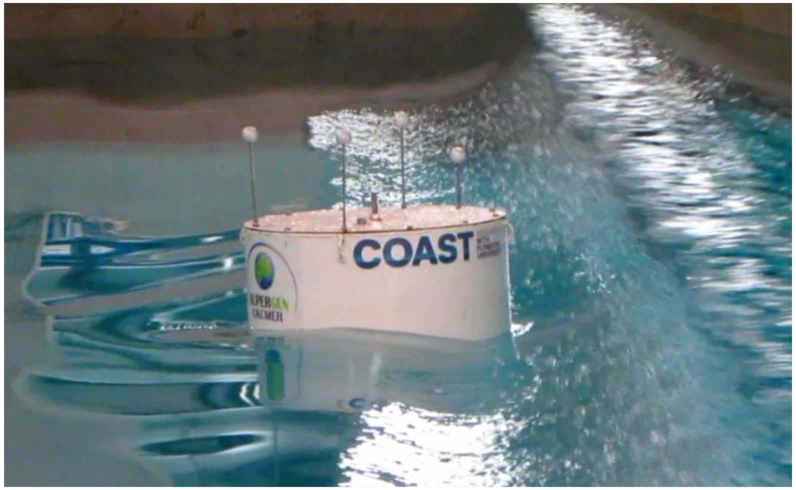
Pressures measured at front and average along deck

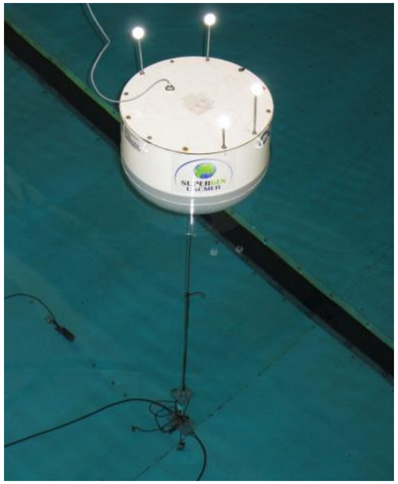




Froude Krylov approximate method for extreme inertia loading : useful fast solution

- Froude Krylov force may be accurately modelled , including breaking waves
- Added mass approximated from potential flow

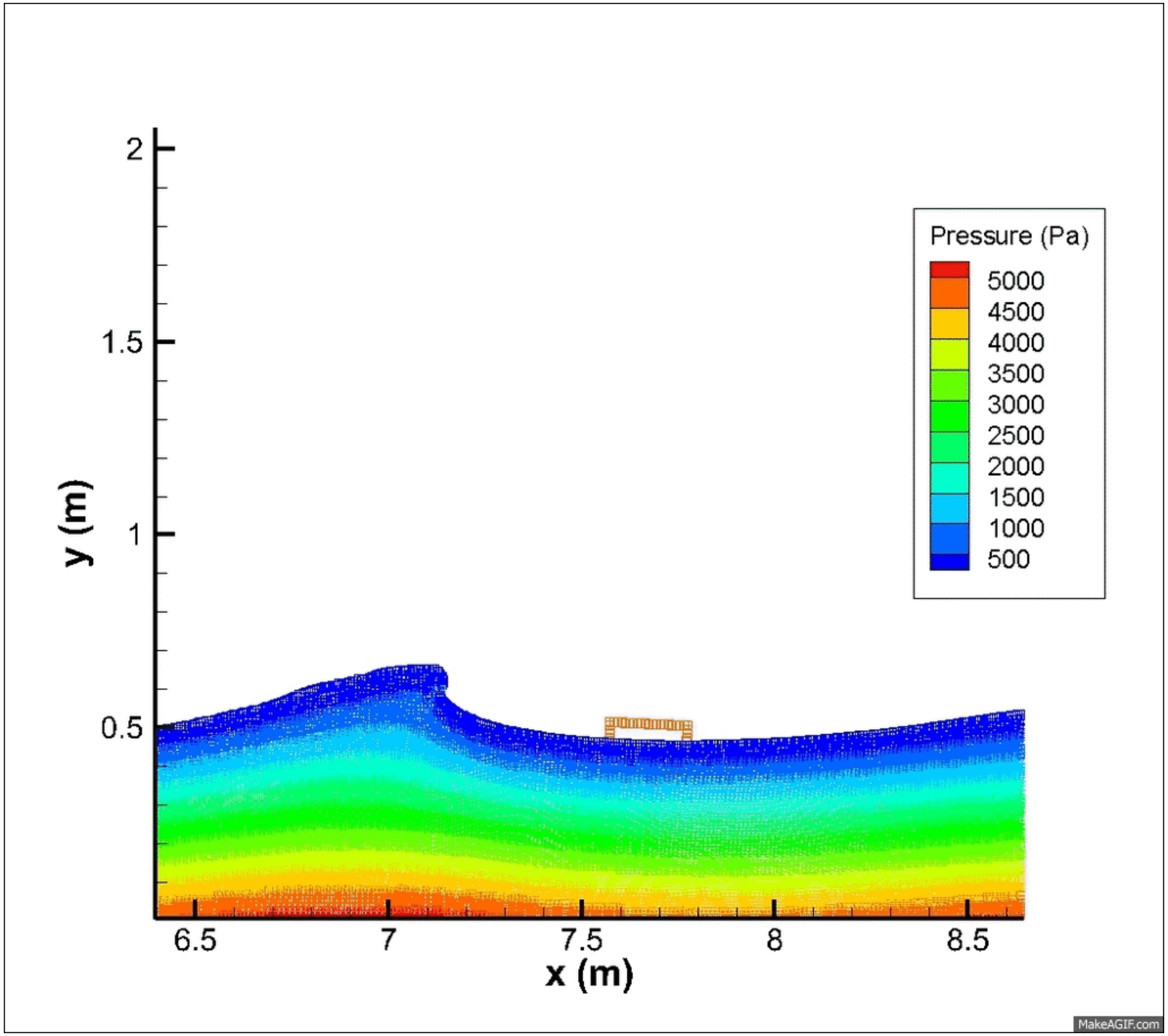




Taut moored buoy in COAST basin – inertia regime

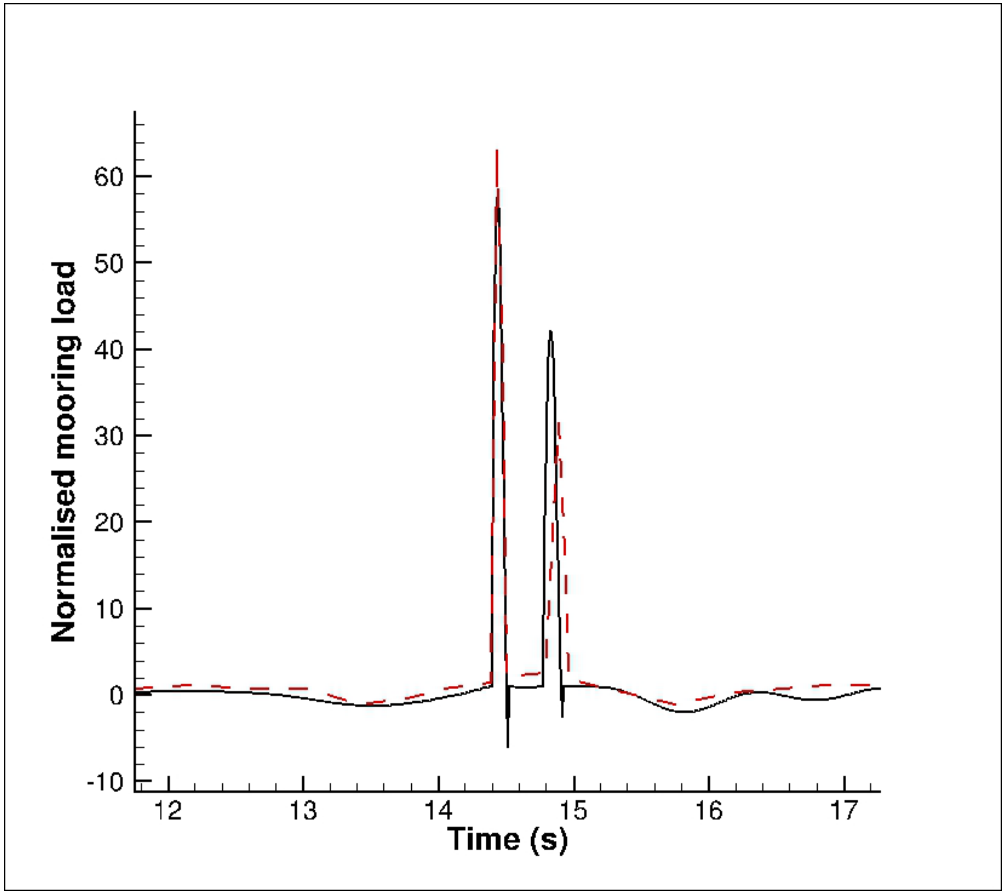
Hann, M., Greaves, D., Raby, A. 2015 'Snatch loading of a single taut moored floating wave energy converter due to focussed wave groups'
Ocean Engineering,2015, 96, 258–271

ISPH with FK forcing and empirical added mass



Lind SJ, Stansby PK, Rogers BD 2016 Fixed and moored bodies in steep and breaking waves using SPH with the Froude Krylov approximation. J Ocean Eng Mar Energy (special issue)

Snatch loads, non breaking waves



With breaking waves snatch loads overestimated , initially by 30%

Hybrid coupled schemes to reduce computation time

- Adaptivity (Parma, UoM)
- FV – SPH (Nantes/Rome)
- Eulerian – Lagrangian SPH (UoM)
- Boussinesq – SPH (UoM, Paris)
- QALE-SPH (UoM, City)

On the coupling of Incompressible SPH with a Finite Element potential flow solver for nonlinear free surface flows

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ISOPE-2017 San Francisco Conference

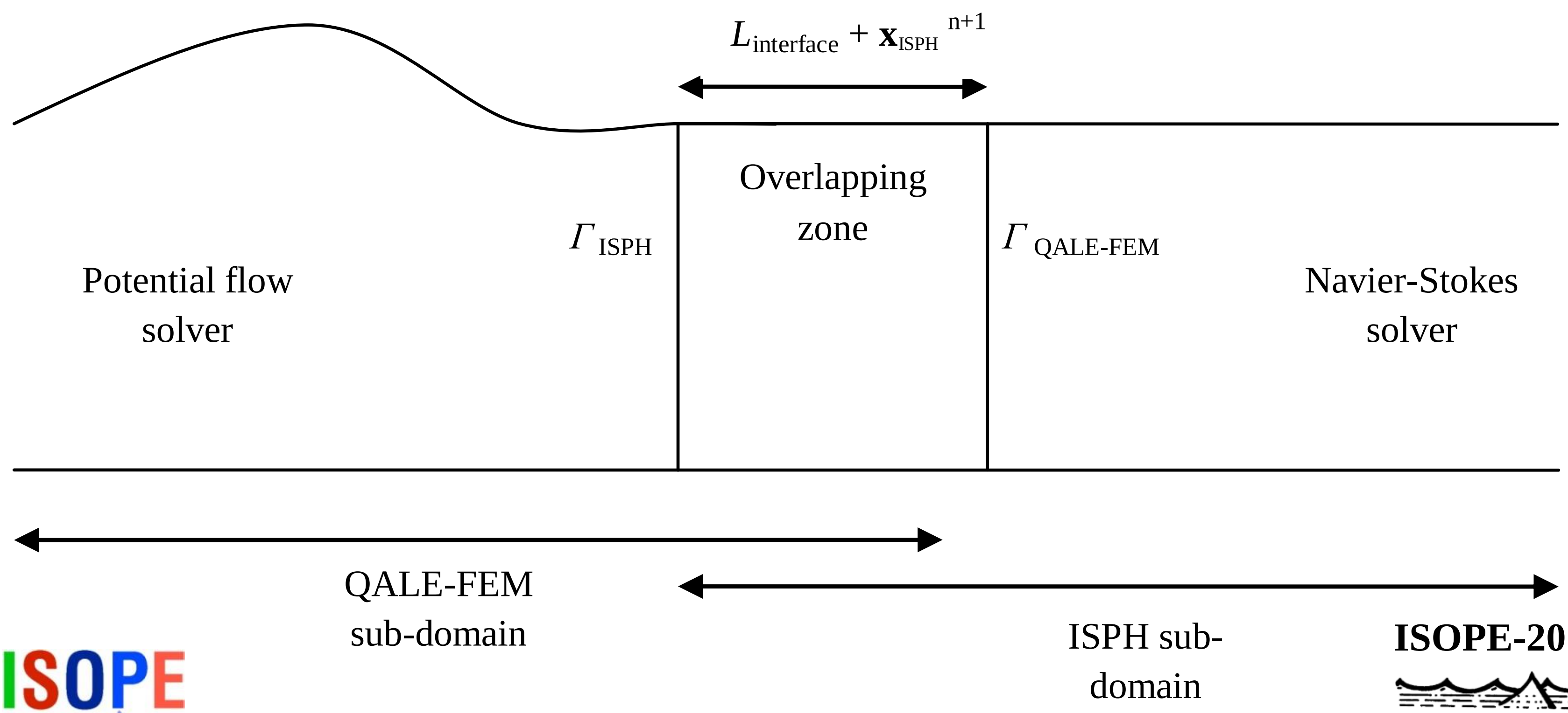
The 27th International Ocean and Polar Engineering Conference

San Francisco, California, June 25–30, 2017: www.isope.org

Coupling methodology

The domain is decomposed into two sub-domains

- The QALE-FEM and ISPH methodology applies respectively
  - With ISPH denoted with Γ_{ISPH} and QALE-FEM $\Gamma_{QALE\text{-}FEM}$
- Between the two sub-domains an overlapping zone exists



Coupling methodology

Overlapping zone:

–Linear weighted velocity

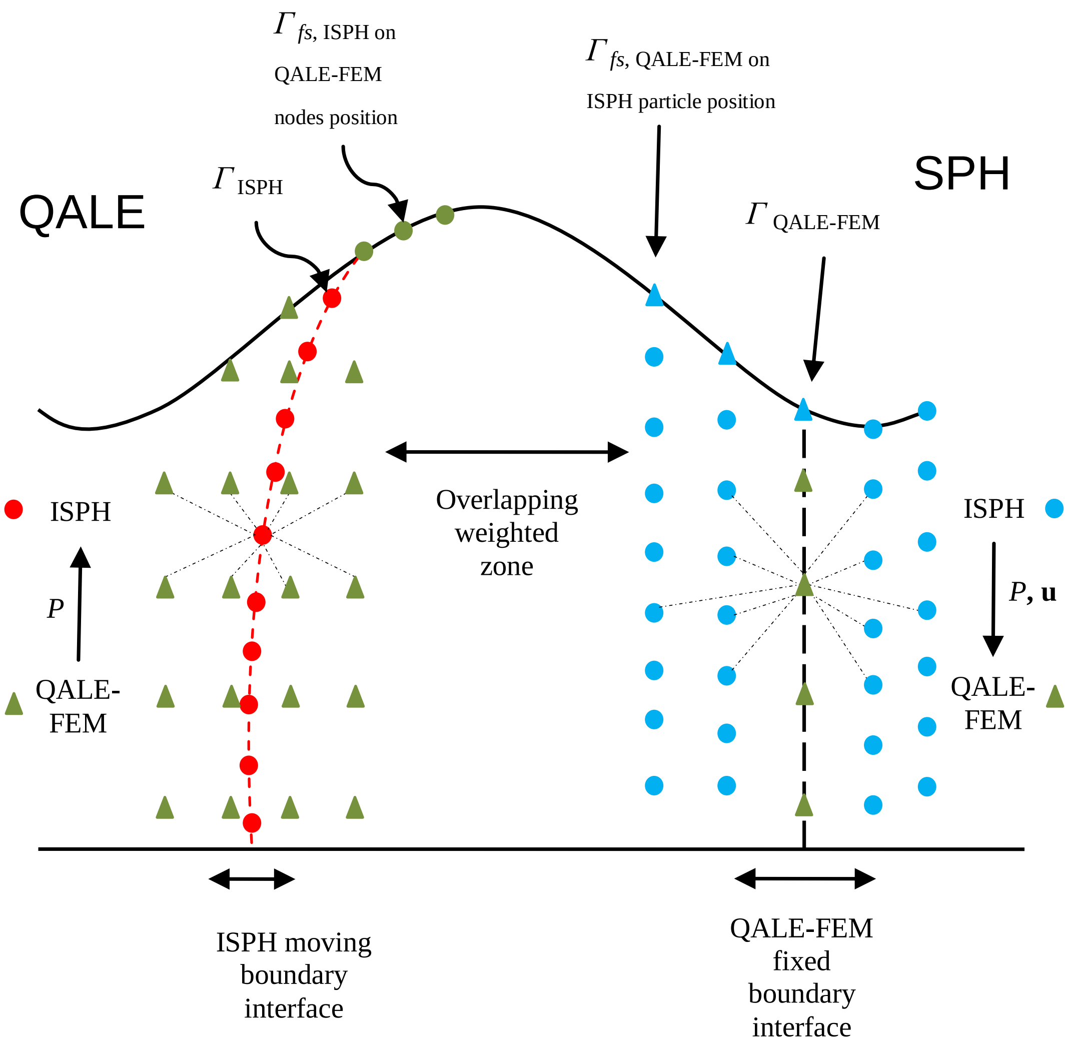
$$\mathbf{u} = a\mathbf{u}_{ISPH} + (1-a)\mathbf{u}_{QALE-FEM}$$

–Free-surface matching from

$$\mathbf{x}_{QALE-FEM} \rightarrow \mathbf{x}_{ISPH} \qquad at\,\Gamma_{ISPH}$$

and

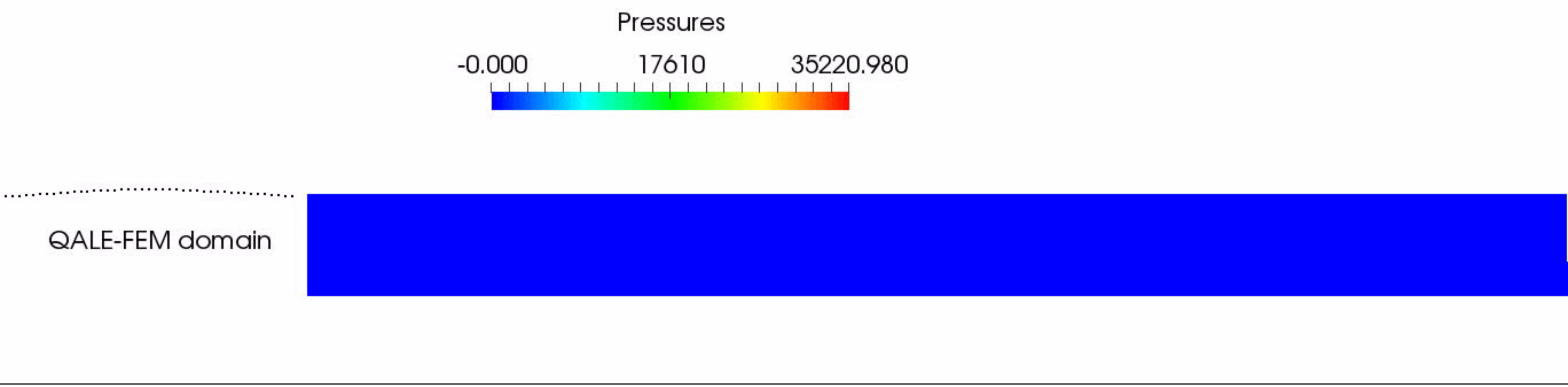
$$\mathbf{x}_{ISPH} \rightarrow \mathbf{x}_{QALE-FEM} \qquad at\,\Gamma_{QALE-FEM}$$



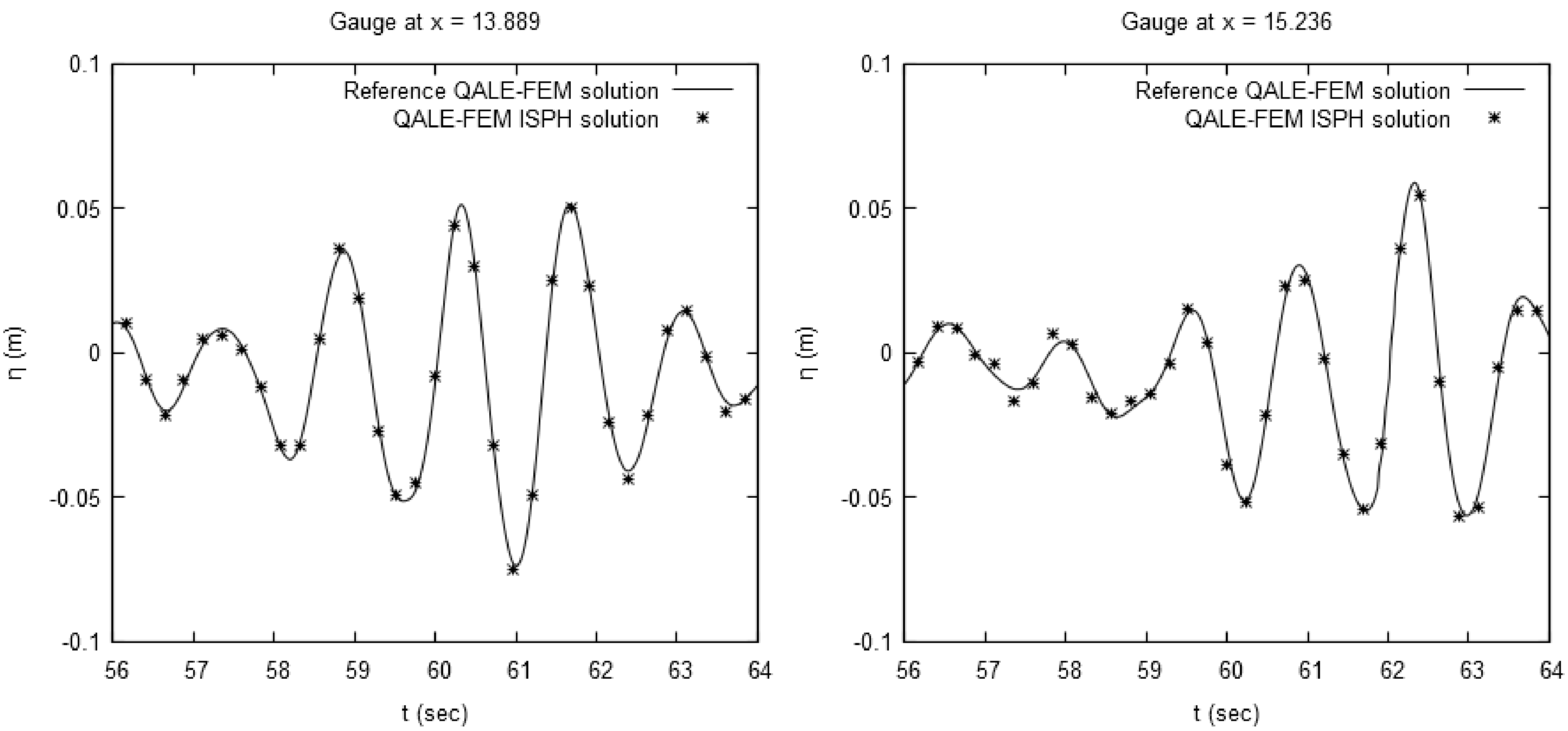
ISOPE

Regular wave

Hybrid solver simulation



Focused wave



Time history of the wave elevations comparison between QALE-FEM ISPH and reference solution

Grand challenges in SPH - SPHERIC

- Boundaries – various approaches, good progress
- Weakly compressible or incompressible
- Adaptivity –ongoing
- Convergence (and boundaries) – good progress

And

- Computing hardware – big effort
- Turbulence – progress needed

Solid boundaries

- Dynamic/dummy particles – layers fixed in solid otherwise as fluid
- Mirror particles to give zero wall velocity
- Renormalised boundary particle kernels - γ method
- Marrone – fixed mirror particles with interpolation
- Adami – fixed particles with pressure from fluid
- Eulerian layer in flow with interface to Lagrangian

Adami, S., et al. (2012). JCP, *231*(21), 7057–7075.
S. Marrone, et al, CMAME 200 (2011) 1526{1542
Ferrand, Met al. (2012). IJNMF *71*(4), 446–472.
Fourtakas, G., et al., 2018 CMAME, 329, 532-552.

Accuracy for general SPH solver

Is higher order possible? Presently ~ 1.5

Affected by

- Interpolation error (kernel)
- Discretisation error (particle spacing)
- Particle distribution (uniform is best)
- Boundary condition (requires zero velocity)

Eulerian SPH (sounds like a contradiction)

- Use Gaussian kernel – decreasing Fourier transform and exponential convergence
- Fix particle distributions , retain advection , control particle position

$$\frac{\partial \mathbf{u}}{\partial t} + \boxed{\mathbf{u} \cdot \nabla \mathbf{u}} = -\nabla p + \mu \nabla^2 \mathbf{u},$$

Lind SJ, Stansby PK 2016 High-order Eulerian incompressible smoothed particle hydrodynamics with transition to Lagrangian free-surface motion, J. Comp. Phys., 326, 290–311

Gaussian kernels

Fourth order kernel $O(h^4)$:

$$\omega_4 = \left(\frac{2}{\pi h^2}\right)\left(1 - \frac{r^2}{2h^2}\right)\exp(-r^2/h^2)$$

Sixth order $O(h^6)$:

$$\omega_6 = \left(\frac{3}{\pi h^2}\right)\left(1 - \frac{r^2}{h^2} + \frac{r^4}{6h^4}\right)\exp(-r^2/h^2)$$

Can be unstable in Lagrangian schemes with non-uniform/regular particle distributions

Time stepping to 2$^{\text{nd}}$ order

1. *The 1st step*

$$\frac{3\mathbf{u}_i^* - 4\mathbf{u}_i^n + \mathbf{u}_i^{n-1}}{2\Delta} = -\sum_j V_j(p_j^n - p_i^n)\nabla\omega_{ij} + \sum_j V_j \frac{2\mathbf{r}_{ij}\cdot\nabla\omega_{ij}}{r_{ij}^2}\mathbf{u}_{ij}^{n+1} + \mathbf{f}_i^{n+1}$$

2. *The Poisson Equation*

$$\sum_j 2V_j \frac{(q_i^{n+1} - q_j^{n+1})\mathbf{r}_{ij}\cdot\nabla\omega_{ij}}{r_{ij}^2} = \frac{3}{2\Delta t}\sum_j V_j(\mathbf{u}_j^* - \mathbf{u}_i^*)\cdot\nabla\omega_{ij}.$$

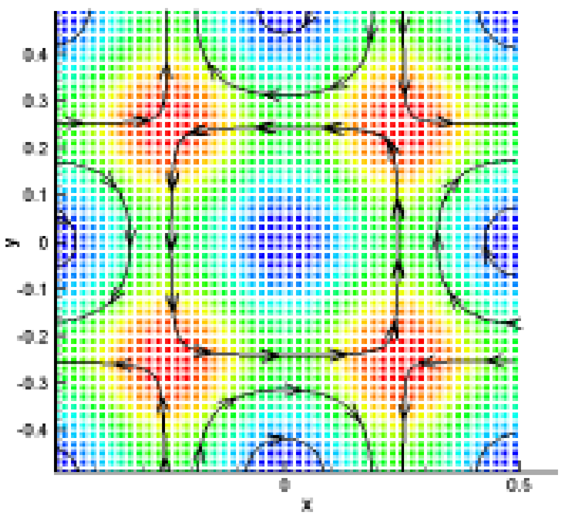
3. *The 2nd (velocity correction) step*

$$\frac{3\mathbf{u}_i^{n+1} - 3\mathbf{u}_i^*}{2\Delta} = -\sum_j V_j(q_j^{n+1} - q_i^{n+1})\nabla\omega_{ij}$$

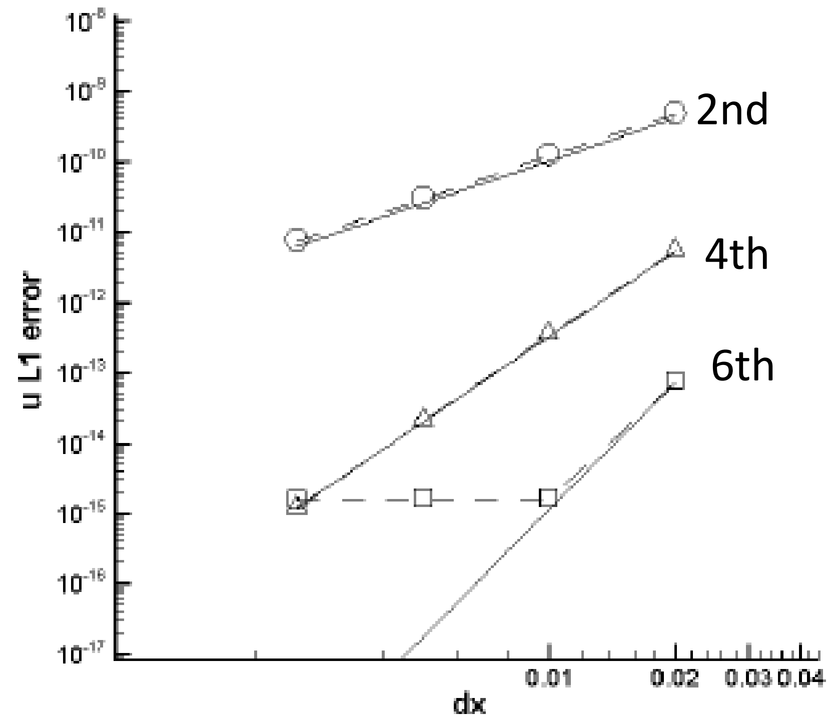
Taylor Green vortices

$$
\begin{aligned}
p &= e^{2bt}(\cos(4\pi x) + \cos(4\pi y)) \\
u &= -e^{bt}\cos(2\pi x)\sin(2\pi y) \\
v &= e^{bt}\sin(2\pi x)\cos(2\pi y)
\end{aligned}
$$

Kernel support is large, and $h = 2dx$



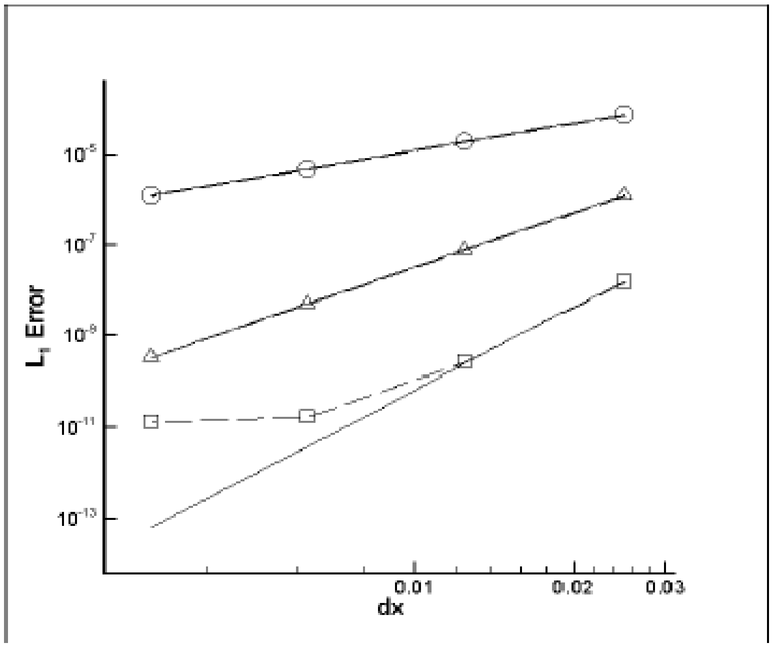
One time step test



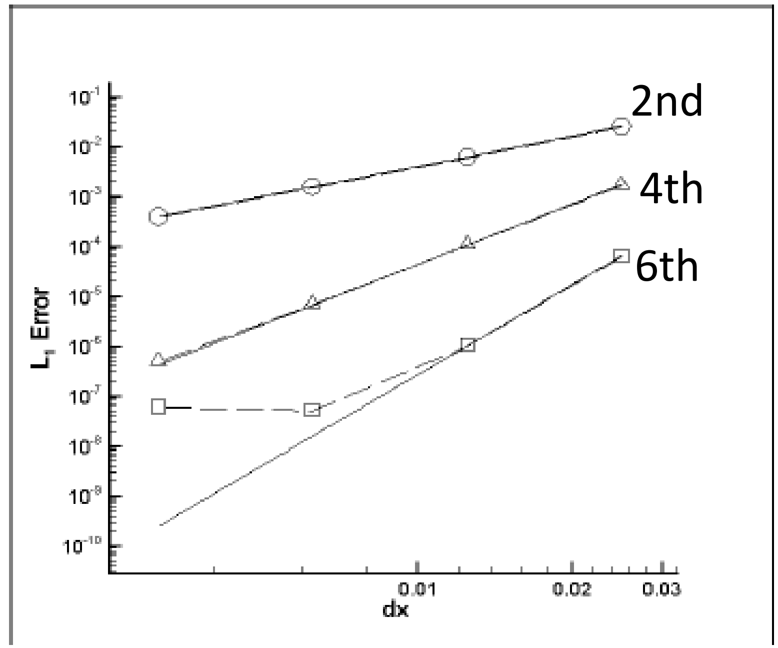
○: Second order
△: Fourth order
□: Sixth order

Errors at t=0.1

L1 error in u at $t = 0.1$, $Re = 1000$



L1 error in p at $t = 0.1$, $Re = 1000$

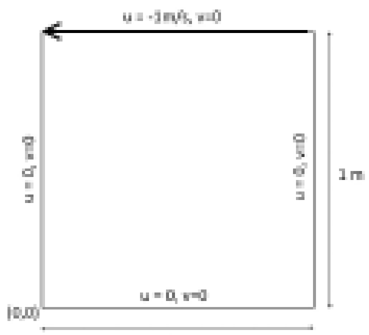


- 2nd and 4th order kernels converge as theoretical
- 6th order limited by time integration error and solver tolerance

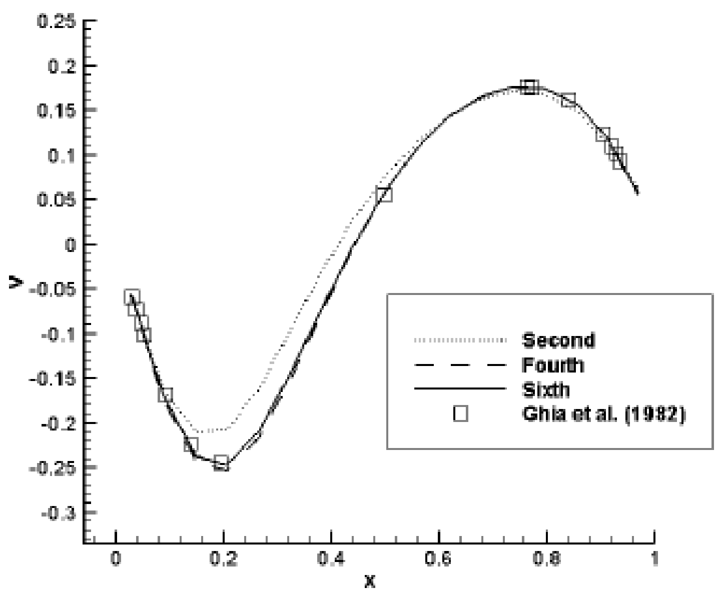
Ideal convergence recovered with small time steps and solver convergence

Compares well with high order finite difference

Accuracy with small number particles Lid driven cavity example



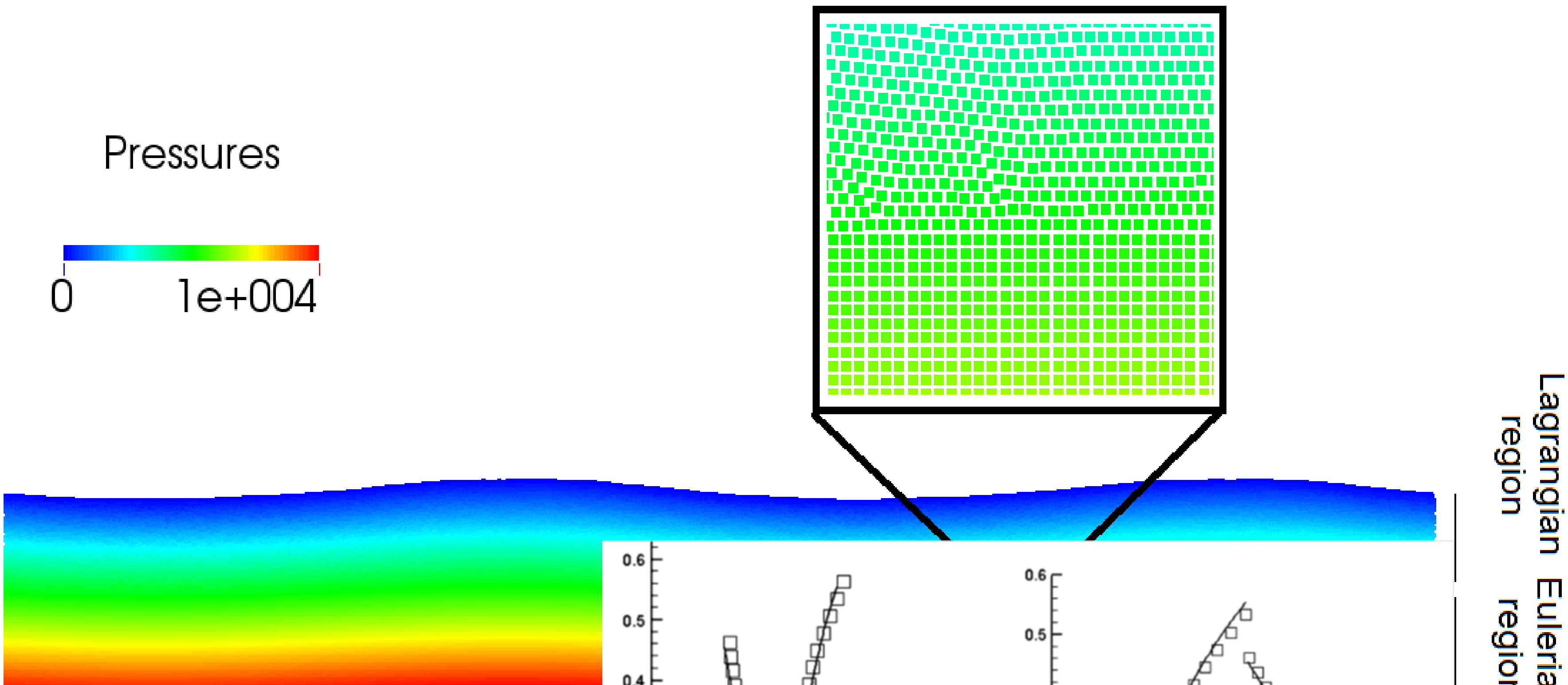
- Gains to be had low resolution
- Consider 17×17 particles
- $Re = 100$

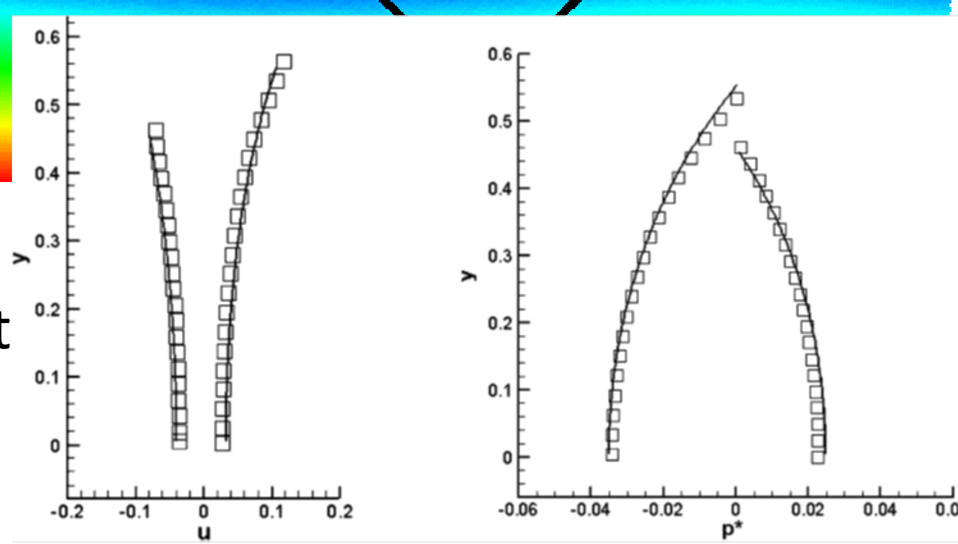


ESPH

- Opens opportunities
- Straight ESPH for internal flows with no free surface – ease of initial particle distribution generation at expense of larger number of particle connections
- Accurate boundary representation with regular fixed particles
- High order opens up accurate turbulence simulation
- Couple Eulerian with Lagrangian e.g. where free surface occurs – always need Lagrangian boundary particles

Eulerian – Lagrangian (ELI-SPH)

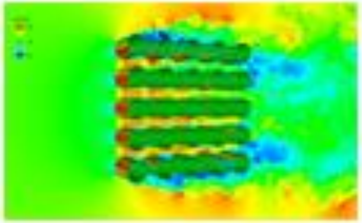




Eulerian region (fixed part ... ccuracy
suitable below surface

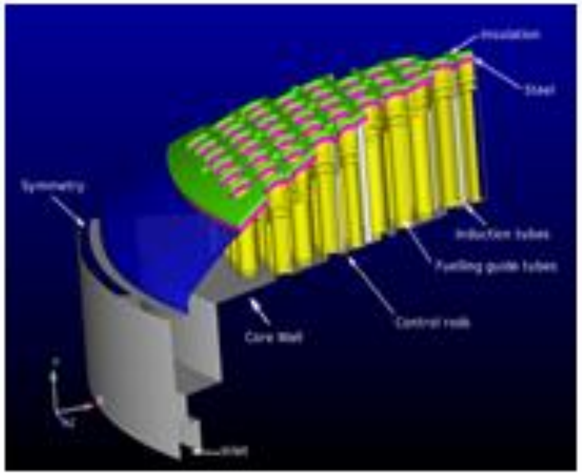
Fourtakas et al 2017 CMAME

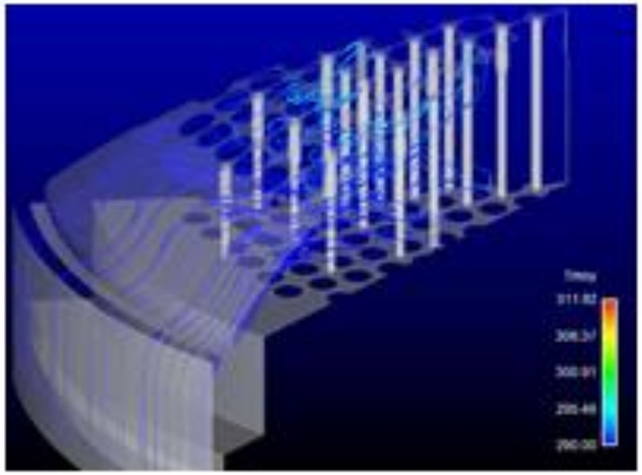
Challenge for ESPH in thermal hydraulics



Real problems invariably complex

Hot box dome AGR





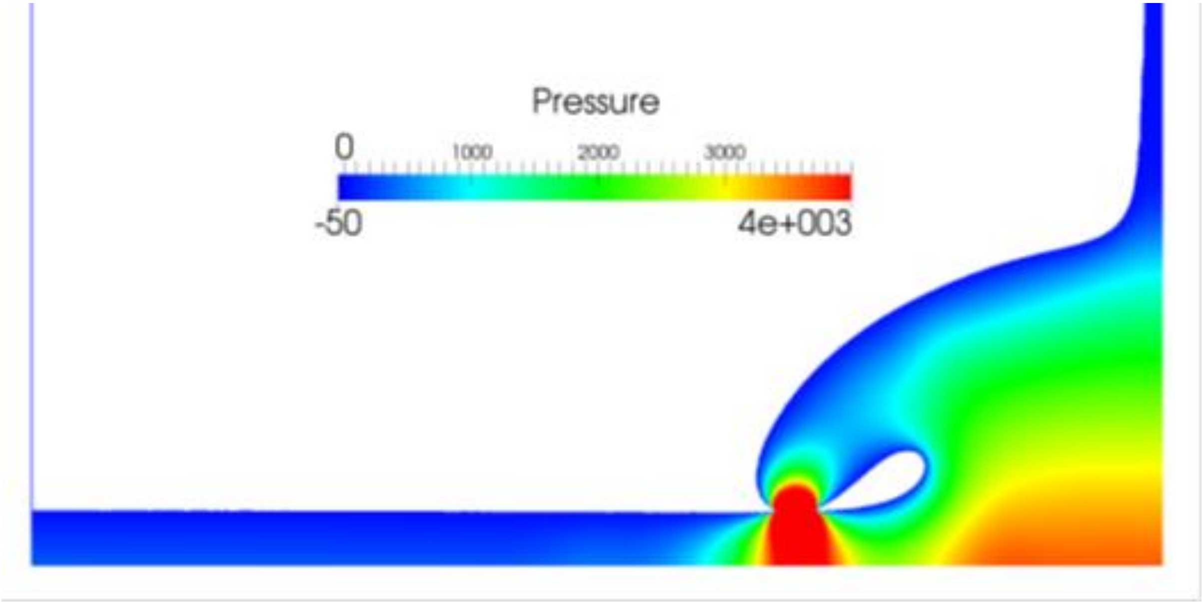
Computations expensive - 500 hours on 2048 cores typically
BUT physical testing many times more expensive or impossible
OK if Computational Fluid Dynamics reliable and accurate – surrogate for reality

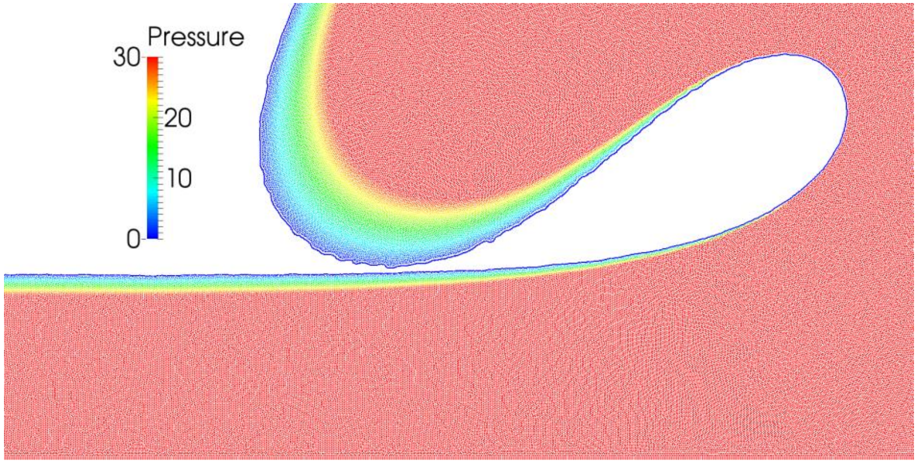
VALIDATION VITAL

ISPH speedup

- Particle adaptivity – coalescing/splitting (Renato Vacondio Parma, UoM)
- MPC – 10^8 particles , PetSC Poisson solver, Zoltan library, Hilbert space filling curve, 12000 partitions with MPI, typically 40% efficient, petascale computing (Xiaohu Guo STFC, UoM)
- GPU – DualSPHysics+ViennaCL for PPE (Alex Chow UoM)

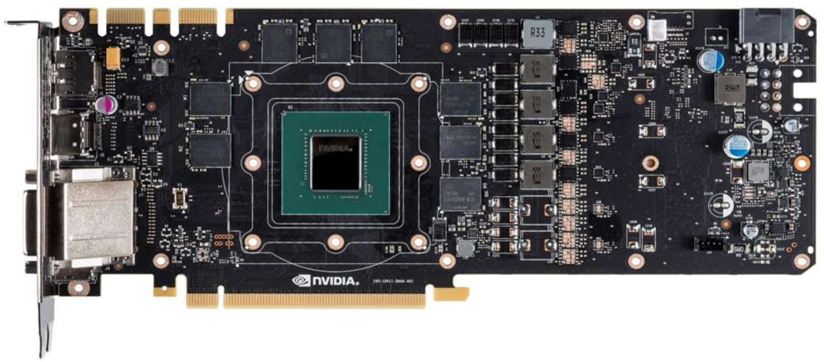
GPU Alex Chow PhD





The Graphics Processing Unit (GPU)

- Thousands of computing cores => very powerful and fast
- Relatively cheap option for hardware acceleration
- Energy efficient high performance computing
- Acceleration of Poisson solver
- Highly portable

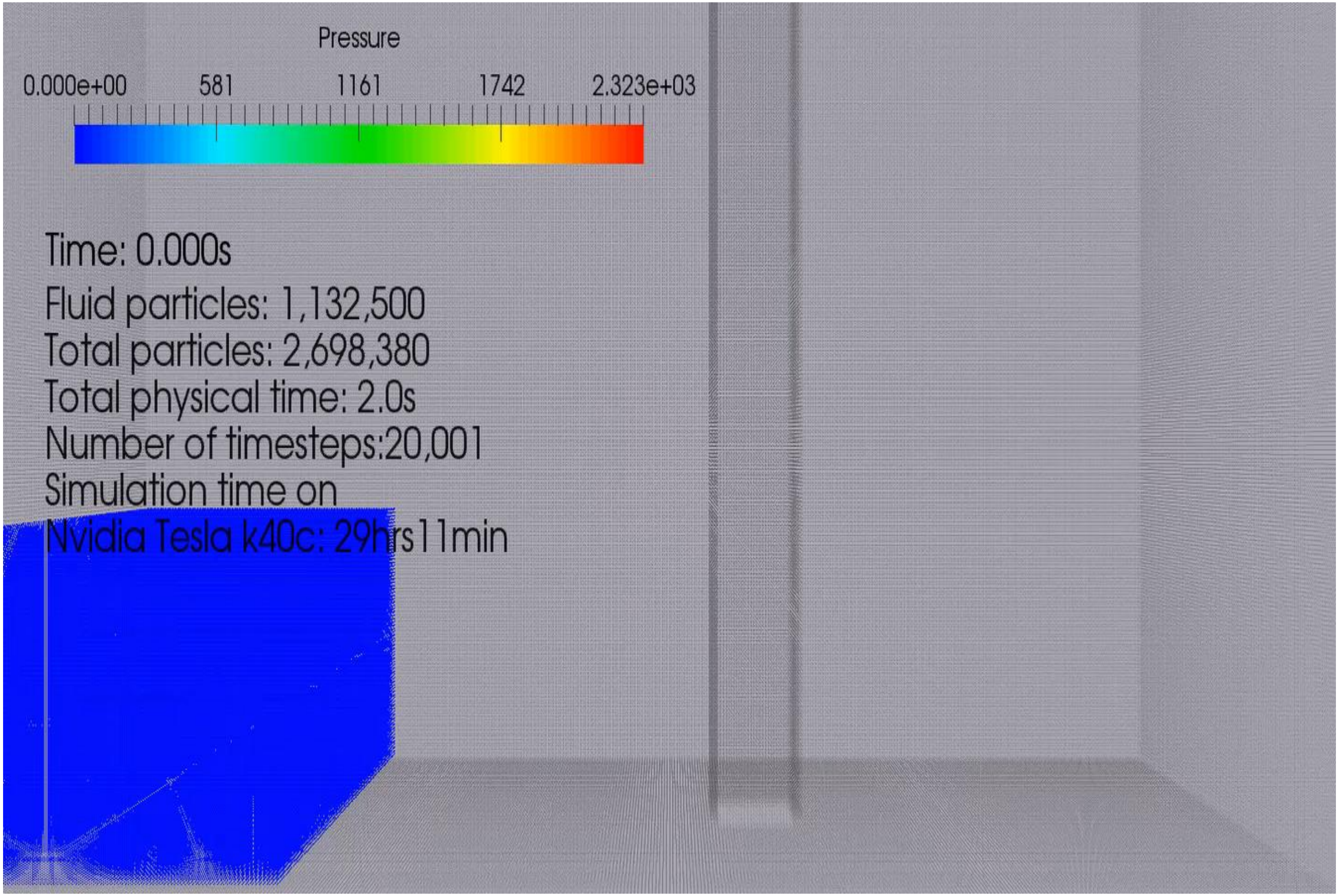


Nvidia
Tesla k40c
GTX 1070

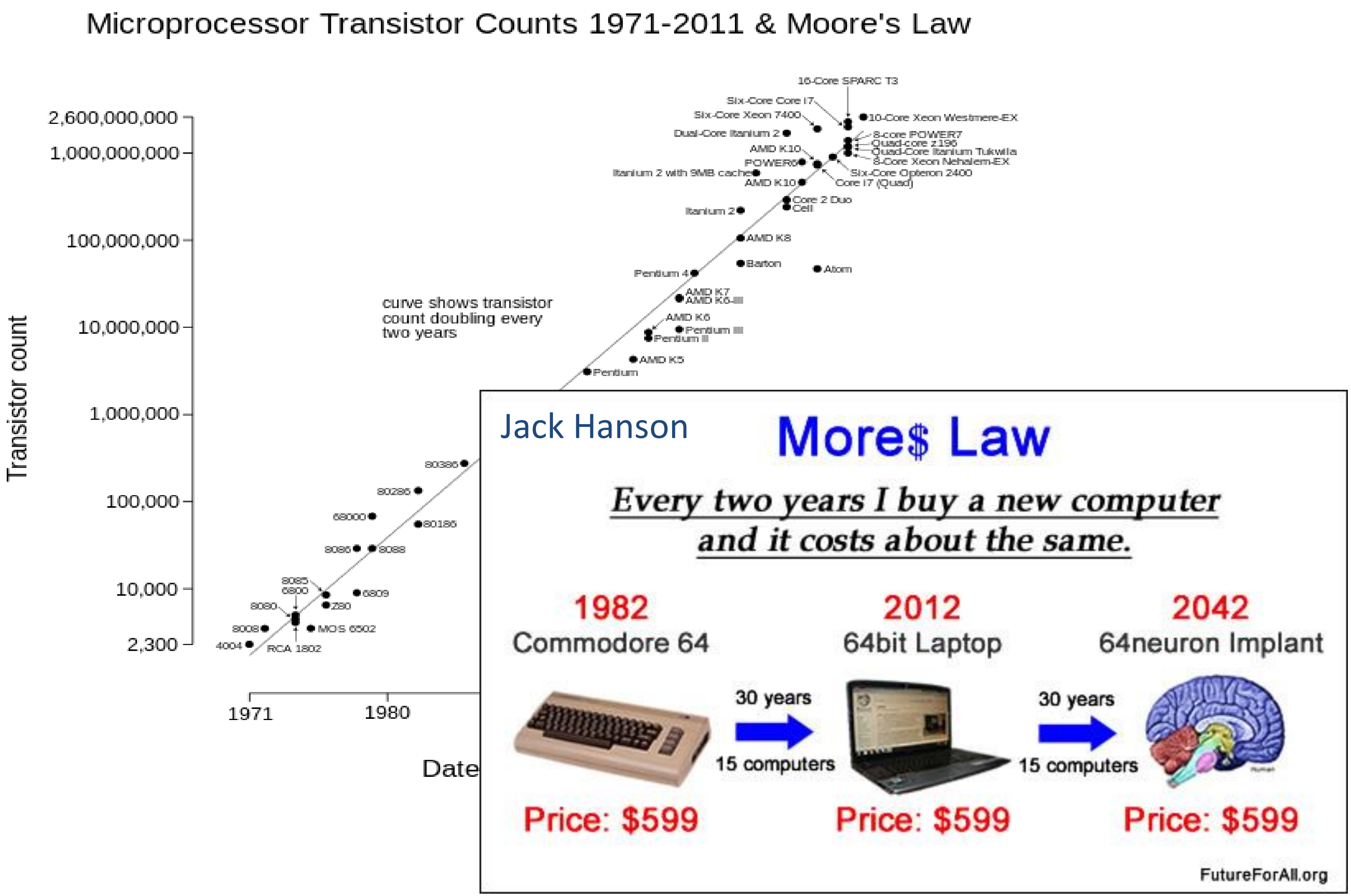
Performance Comparison:
GPU speed ups

GPU vs CPU single thread:
=> 12-17 times
GPU vs CPU 16-threads:
=> 2.5-4.0 times

3D dambreak with column



Hardware always advances



How is hardware accelerating?

- Today GPUs – multi-GPUs, MPC - petascale
- Tomorrow - exascale , Tensor Processing Units
- FPGAs – ‘field programmable gate array’
- With quantum computing, graphene, nanotubes etc speed massive exoscale +++
- But occasional faults possible – need fault tolerant algorithms – SPH with multiple particle connections potentially suited
- Need to plan algorithms for hardware

What are we doing tomorrow ?

- 3D hybrid QALE – ISPH
- Include two phase formulation
- ESPH to high order for thermal hydraulics

Thanks for your attention

and questions