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TÉCNICO LISBOA

cpu gpu DualSPHysics

Current Developments:

Multi-phase Sediment-liquid

Georgios Fourtakas

Multi-phase Liquid-gas

Athanasios Mokos

DEM-SPH coupling

Ricardo Canelas

3-D SPH Modelling of Sediment Scouring Induced by Rapid Flows

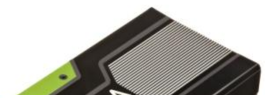
G. Fourtakas, B. D. Rogers, D. Laurence

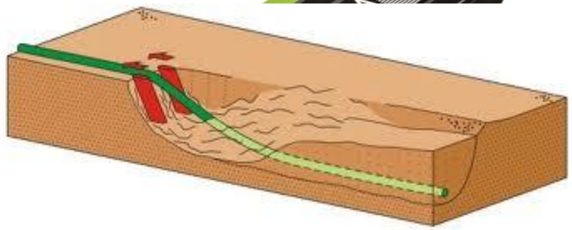
School of Mechanical, Aerospace and Civil Engineering,
University of Manchester,
UK

Outline of the presentation

- Motivation
- Eulerian schemes and multi-phase flows
- SPH methodology
- Multi-phase model
    - Liquid - sediment model
        - Yield criteria
        - Constitutive equations
        - Sediment suspension
- GPU implementation
- Validation and applications
- Conclusions

Motivation





- Real life engineering problems
  - Underwater sand bed trenching
  - Local scour around structures
  - Suspension of hazardous materials
- UK Nuclear industry application
  - Industrial tank
  - Hazardous material
  - Sediment agitation
  - Submerged jets

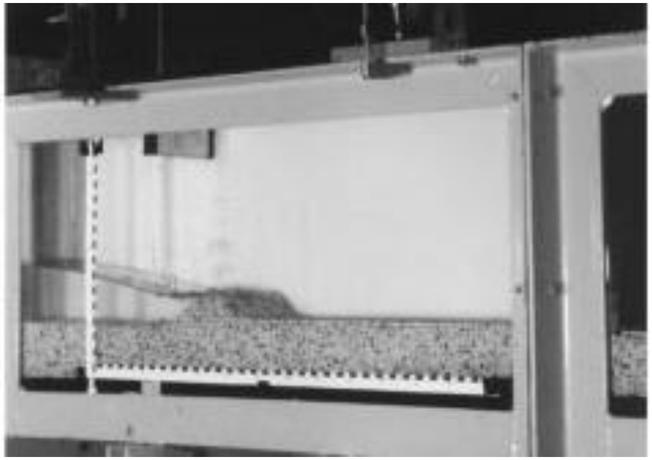
- GPUs Why?
  - Complicated geometry
  - Complex industrial flows
  - Computational cost

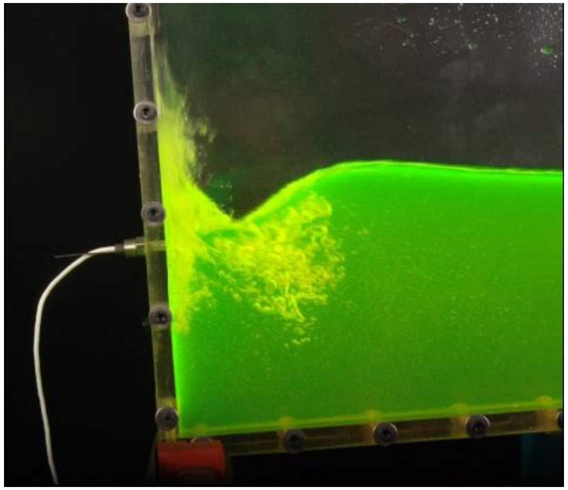
Courtesy of the National Nuclear Laboratory, UK

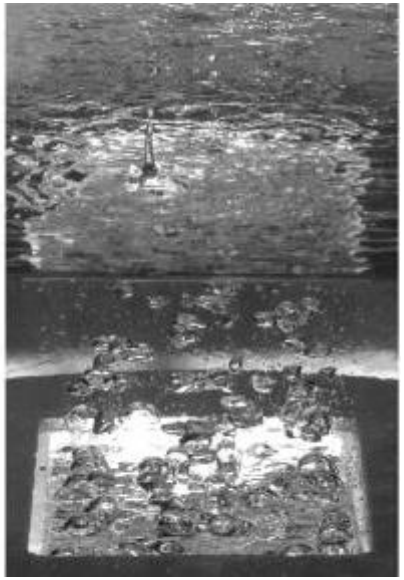
Traditional CFD methods (Eulerian)

Grid based methods

- Mesh generation can be expensive
- Mesh refinement in areas of interest (some knowledge a priori)
- Not applicable to highly non-linear deformations, (or very expensive)
- Multi-phase, free surfaces and phase-change flows







The SPH method

Navier Stokes equations in Lagrangian form and SPH formalism

- Using the total derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}_i \cdot \nabla$$

the Lagrangian form of the Navier-Stokes equations is:

- Continuity equation (conservation of mass)

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{u} = 0 \longrightarrow \frac{d\rho_i}{dt} = \sum_j^N m_j (\mathbf{u}_i - \mathbf{u}_j) \cdot \nabla W_{ij}$$

- Momentum equation (conservation of momentum)

$$\frac{d\mathbf{u}}{dt} = \frac{1}{\rho}\frac{\partial \sigma}{\partial \mathbf{x}} + \mathbf{g} \longrightarrow \frac{d\mathbf{u}_i}{dt} = \sum_j^N m_j \left(\frac{\sigma_j + \sigma_i}{\rho_j \rho_i} \right) \nabla W_{ij} + \mathbf{g}$$

- Tait's equation of State (weakly compressible SPH (**WCSPH**))

$$p = B\left(\left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right)$$

- Plus other closure models

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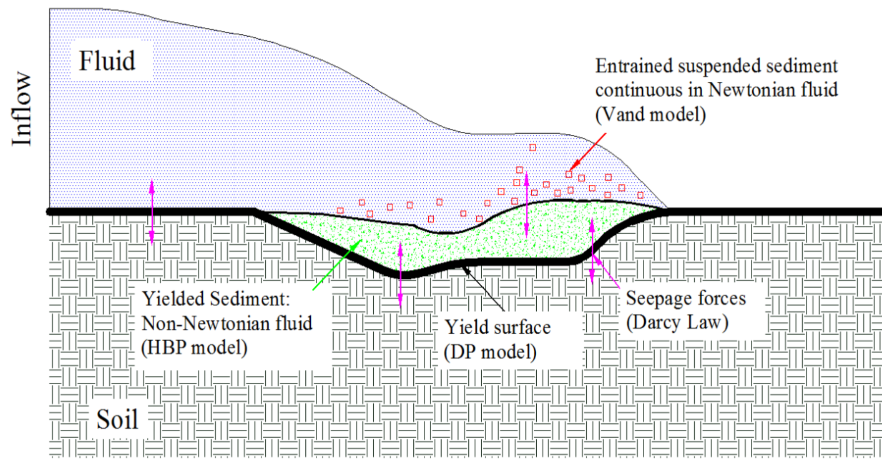
DualSPHysics

Multi-phase model

Liquid – sediment model

- Liquid phase
  - Newtonian flow
- Sediment phase
  - Yield criteria
    - Surface yielding
    - Sediment skeleton pressure
  - Non-Newtonian flow
    - Sediment shear layer at the interface
    - Seepage forces
  - Sediment resuspension
    - Entrainment of soil grains by the liquid



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DualSPHysics

Multi-phase model

Liquid phase

- Weakly compressible SPH (WCSPH)
  - Tait's equation of state to relate pressure to density
  - δ-SPH – Density diffusion term
- Particle shifting – Particle re-ordering
- Turbulence is modelled through a SPS model
- GPU implementation to DualSPHysics

Liquid phase

- Newtonian constitutive equation
  - Single phase DualSPHysics

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}} + \nu\nabla^2\mathbf{u} + \mathbf{g} + SPS \longrightarrow \frac{d\mathbf{u}_i}{dt} = -\sum_j^N m_j\left(\frac{P_j + P_i}{\rho_j\rho_i}\right)\nabla W_{ij} + \sum_j^N \frac{m_j}{\rho_j\rho_i}\left(\mu_j + \mu_i\right)\mathbf{u}_{ij}\frac{\mathbf{x}_{ij}\cdot\nabla W_{ij}}{x_{ij}^2 + \eta^2} + \mathbf{g} + SPS$$

  - Multi-phase implementation

$$\frac{d\mathbf{u}}{dt} = \frac{1}{r}\frac{\P s}{\P \mathbf{x}} + \mathbf{g} \longrightarrow \frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}} + \frac{1}{\rho}\frac{\partial \tau}{\partial \mathbf{x}} + \mathbf{g}$$

$$\frac{1}{\rho}\frac{\partial \tau_i^{\alpha\beta}}{\partial x^\beta} = \sum_j^N m_j\left(\frac{\tau_i^{\alpha\beta} + \tau_j^{\alpha\beta}}{\rho_i\rho_j}\right)\frac{\partial W_{ij}}{\partial x_i^\beta}$$

since

$$\tau_i^{\alpha\beta} = f(\varepsilon_i^{\alpha\beta})$$

$$\varepsilon_i^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_i^\alpha}{\partial x_i^\beta} + \frac{\partial u_i^\beta}{\partial x_i^\alpha}\right) - \frac{1}{3}\left(\frac{\partial u_i^\gamma}{\partial x_i^\gamma}\right)\delta^{\alpha\beta}$$

from

$$\left.\frac{\partial u^\alpha}{\partial x^\beta}\right|_i = \sum_j^N \frac{m_j}{\rho_j}u_{ij}^\alpha\frac{\partial W_{ij}}{\partial x_i^\beta}$$

Liquid phase

- δ-SPH
  - Diffusion term

$$D_{d\text{-}SPH} = d_d h C_{s0} \sum_j^N \frac{m_j}{r_j} T_{ij} \cdot \nabla W_{ij}$$

where *T*

$$T_{ij} = 2(r_j - r_i) \frac{\mathbf{x}_{ij}}{\left|\mathbf{x}_{ij}\right|^2 + 0.1h^2}$$

  - The continuity equation

$$\frac{d\rho_i}{dt} = \rho_i \sum_j^N \frac{m_j}{\rho_j} (u_i^\alpha - u_j^\alpha) \frac{\partial W_{ij}}{\partial x_i^\alpha} + D_{\delta-SPH,i}$$

Liquid phase

- Shifting scheme [Lind *et al.* 2012, Skillen *et al.* 2013]
  - Interior fluid domain

$$\delta r_i^{\alpha} = -Ah|u_i|\Delta t \frac{\partial C_i}{\partial x_i^{\beta}}$$

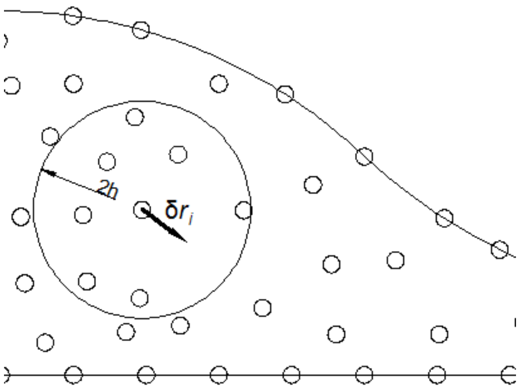
  - Surface of the fluid

$$\delta r_i^{\alpha} = -Ah|u_i|\Delta t \left(\frac{\partial C_i}{\partial s_i^{\beta}} s_i^{a} \right)$$

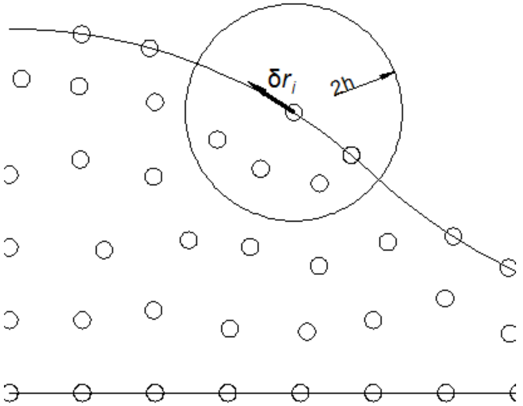
where

$$\frac{\partial C_i}{\partial x^{\alpha}} = \sum_j^N \frac{m_j}{\rho_j} \frac{\partial W_{ij}}{\partial x_i^{\alpha}}$$

Interior domain shifting



Surface shifting



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Multi-phase model

Sediment phase

- Treated as a semi-solid non-Newtonian fluid
- Yield criterion Drucker-Prager
  - Below a critical level of sediment deformation sediment particles remain still
  - Above a critical level of sediment deformation follow the governing equations
- Non-Newtonian flow
  - Herschel-Bulkley-Papanastasiou Bingham constitutive model
- Approximation of seepage forces on the surface
  - Darcy law
- Entrained suspended sediment
  - Concentration based apparent viscosity based on a Newtonian formulation, Vand model

Sediment phase

- Surface Yielding
  - Drucker-Prager (DP) yield criterion
    - For an isotropic material

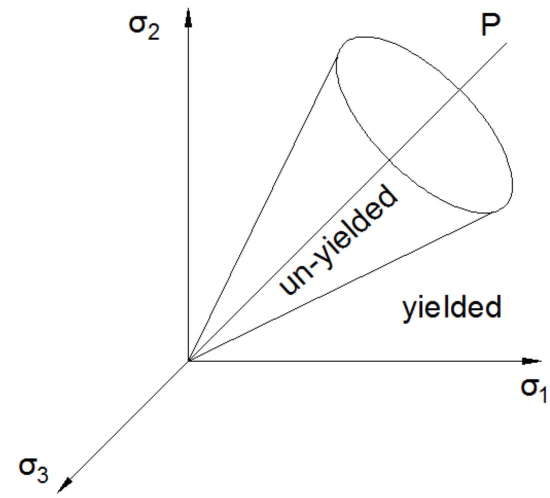
$$\sqrt{J_2} - |\tau_y| = 0$$

    - Apply the yield criterion

$$|\tau_y| = -\alpha I_1 + \kappa$$

    - Yielding occurs when

$$\sqrt{J_2} \geq \alpha P_{skeleton} + \kappa$$



Drucker-Prager (DP) yield surface in principal stress space

<u>Constants</u>

$$\alpha = \frac{\tan\phi}{\sqrt{9 + 12\tan^2\phi}} \qquad \kappa = \frac{3c}{\sqrt{9 + 12\tan^2\phi}}$$

where *c* is the cohesion and *φ* angle of repose

Sediment phase

- Sediment skeleton pressure
  - For a fully saturated soil
    - Terzaghi relationship

$$P_{total} = P_{skeleton} + P_{pw}$$

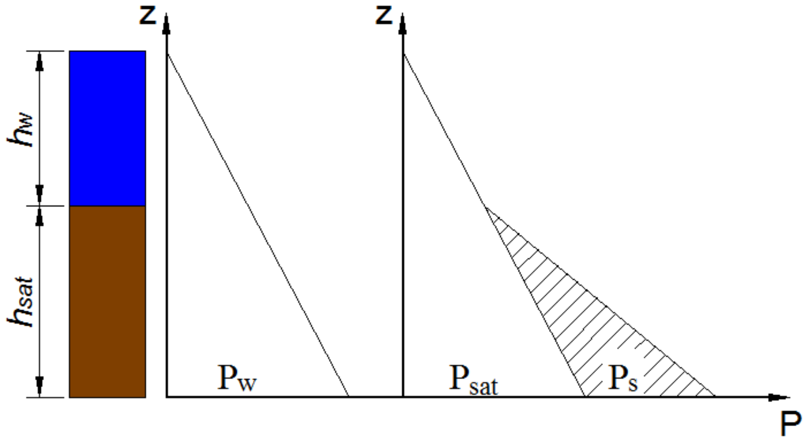
    - or

$$P_{total} = h_w \gamma_w + h_s \gamma'_{sat}$$

    - Pore water pressure

$$p_{pw,i} = B\left(\left(\frac{\rho_{sat,i}}{\rho_{sat,0}}\right)^{\gamma} - 1\right)$$

$$B = \frac{c^2{}_{w,s0}\,\rho_{w,0}}{\gamma_w}$$



Sediment skeleton pressure

$$P_{skeleton} = P_t - P_{pw}$$

Sediment phase

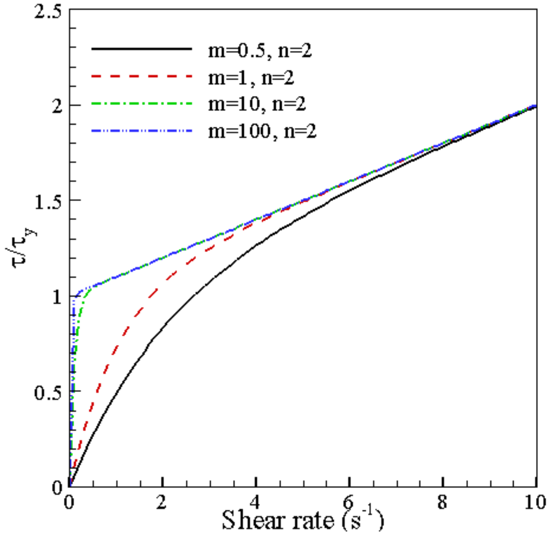
- Sediment constitutive equation
  - Simple Bingham

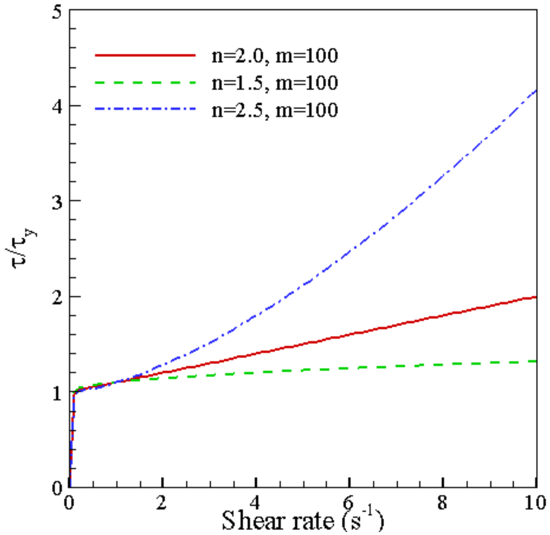
$$\mu_{Bingh} = \frac{|\tau_y|}{\sqrt{\mathrm{II}_D}} + \mu_d$$

  - Herschel-Bulkley-Papanastasiou (HBP)
    - Viscous – Plastic (*m* exponential growth)
    - Shear thinning or thickening (*n* power law)

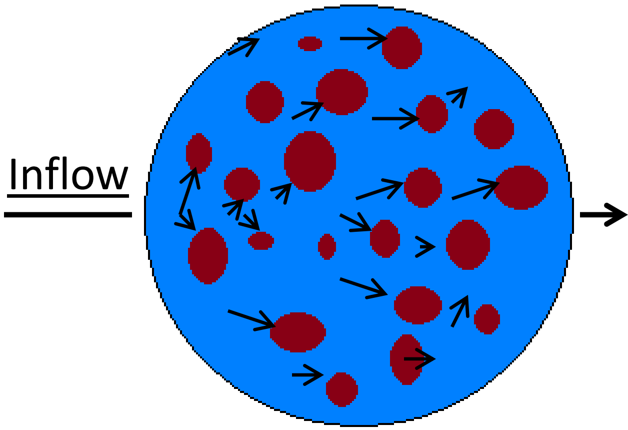
$$\mu_{pap} = \frac{|\tau_y|}{\sqrt{\mathrm{II}_D}}\left[1 - e^{-m\sqrt{\mathrm{II}_D}}\right] + K D^{(n-1)/2}$$

$$\tau_i^{\alpha\beta} = 2\mu_{pap} D^{\alpha\beta} \longrightarrow D^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u^\alpha}{\partial x^\beta} + \frac{\partial u^\beta}{\partial x^\alpha}\right)$$





Sediment phase



- Seepage force
  - Generalised Darcy law

$$S = K(u_w - u_s) \longrightarrow \quad K = \frac{n_r \gamma_w}{k}$$

(Soil properties)

SPH formalism

$$S^{\alpha}_{s,i} = \sum_{j \in W,Sat}^{N} \frac{m_j}{\rho_i \rho_j} S^{\alpha}_{ij} W_{ij}$$

- Suspension

Vand equation

$$\mu_{susp} = \mu_{fluid} e^{\frac{2.5 c_v}{1 - \frac{39}{64} c_v}} \qquad c_v \leq 0.3 \qquad \longrightarrow$$

Concentration volume fraction of sediment

$$c_{v,i} = \frac{\sum_{j_{sat} \in 2h}^{N} \frac{m_j}{\rho_j}}{\sum_{j \in 2h}^{N} \frac{m_j}{\rho_j}}$$

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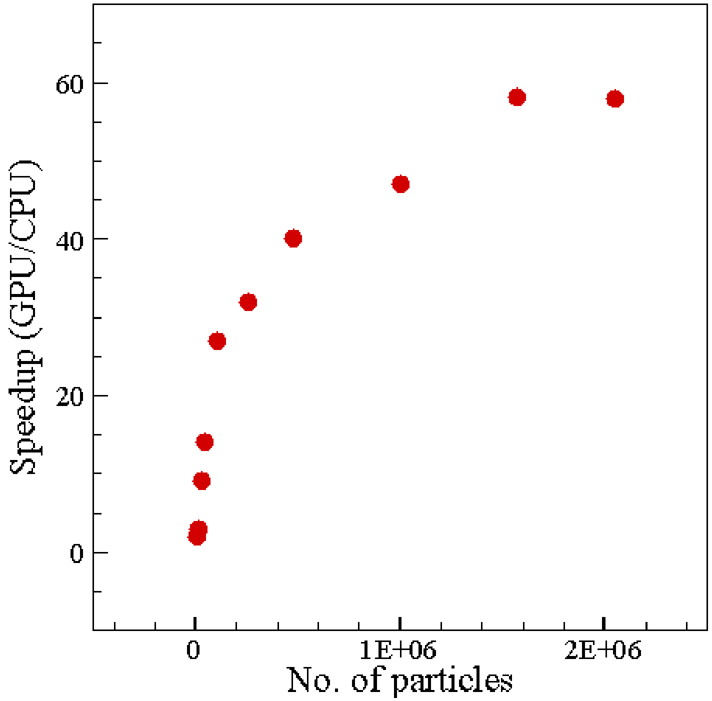
GPU implementation in DualSPHysics

- Multi-phase issues
  - Branching
  - Registers
  - Arithmetic operations
  - Larger data size

- Resolve
  - Memory operations
  - Smaller kernels
  - Combine similar operations



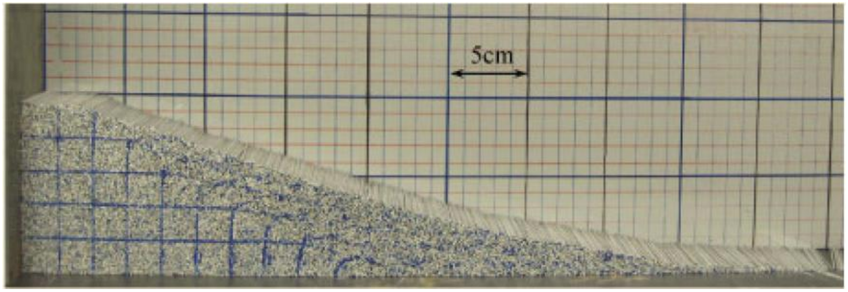
GPU algorithm speed up curve
(x58 compared to a single thread CPU)

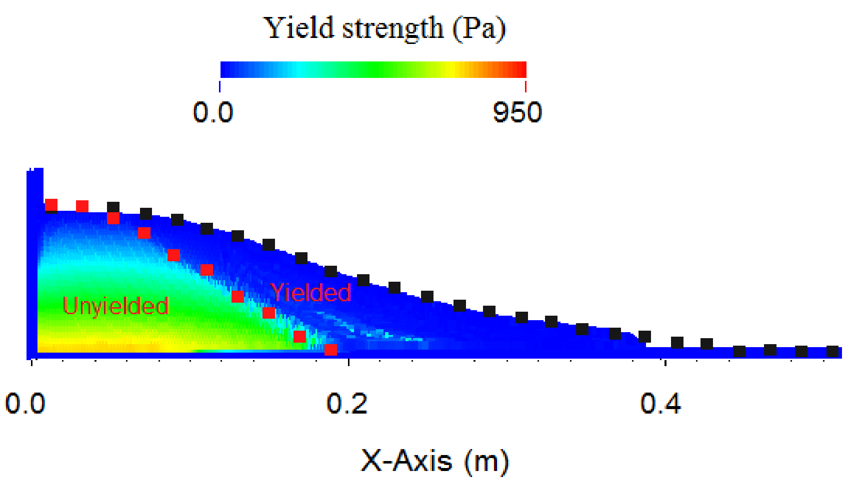
(See Mokos *et al*. 2015)

Numerical results

Soil Dam break

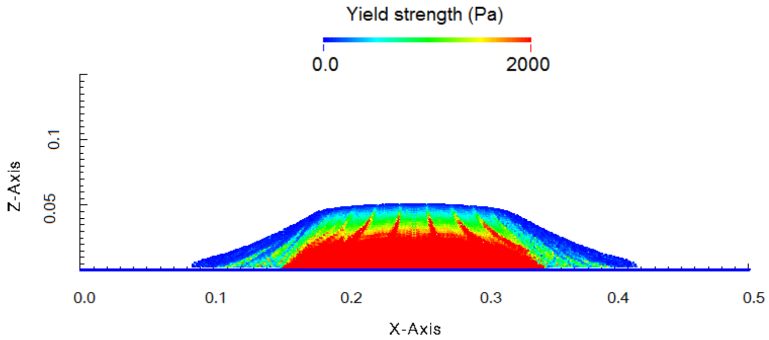
Bui *et al.*, Langrangian method for large deformation and failure flows of geo-material, 2008

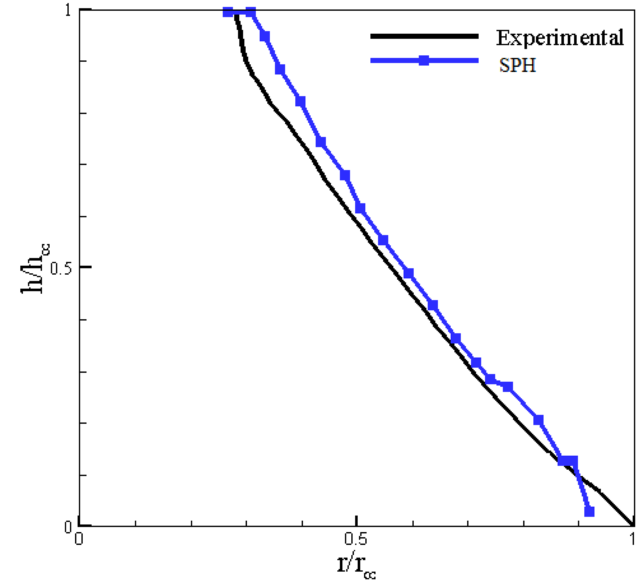




Sediment block collapse

Lude *et al.*, Axisymmetric collapses of granular columns, 2014



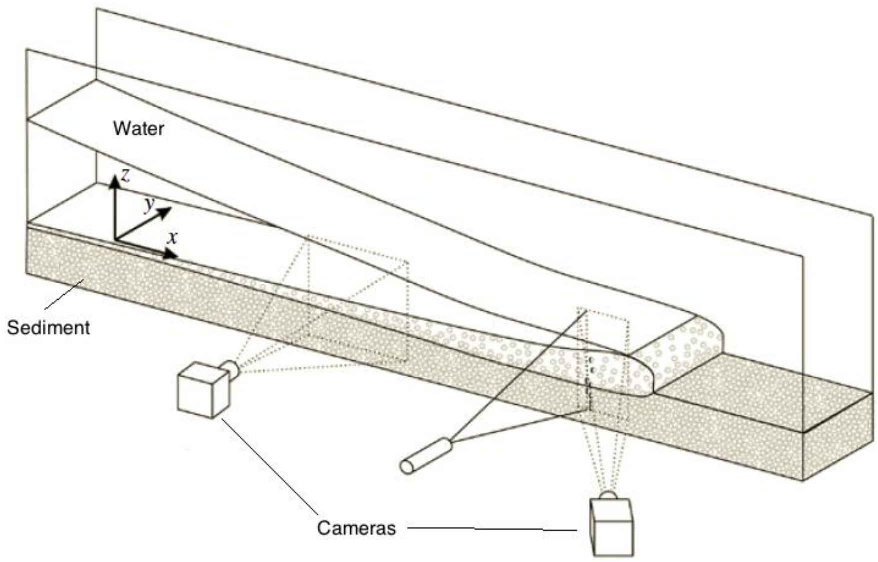


Numerical results

Case definition

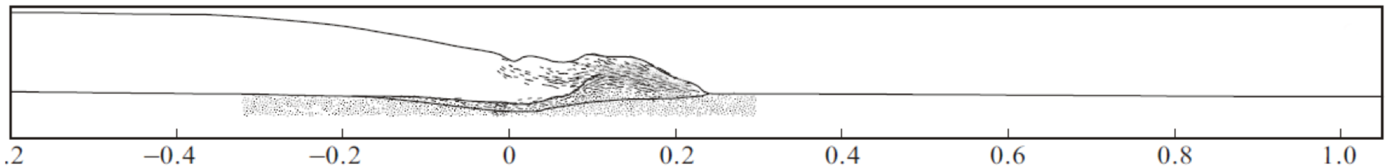
Erodible Dam break

Spinewine *et al.*, Intense bed-load due to sudden dam break, 2013



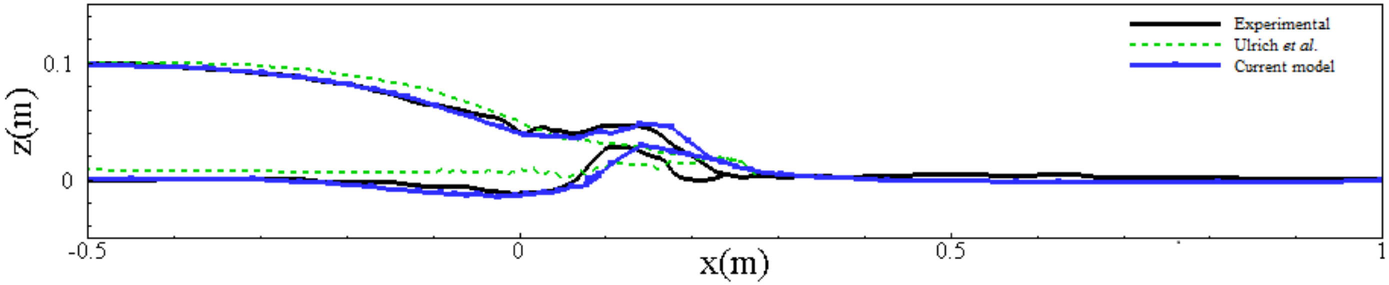
Parameter	Value	Units
Liquid height	0.1	m
Sediment height	0.6	m
Density ratio	1.54	
Porosity		
Numerical cohesion	100	Pa
Sediment viscosity	500	Pa.sec
m (HBP)	100	
n (HBP)	1.6	
Runtime	1.5	sec
No. Particle	328 000	

2-D Erodible dam break



(a)

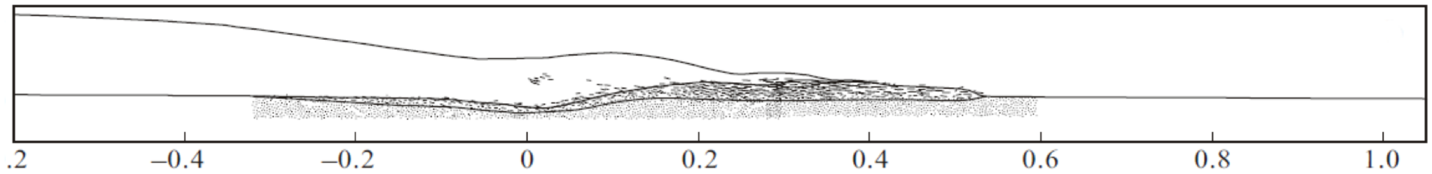
(b)



(c)

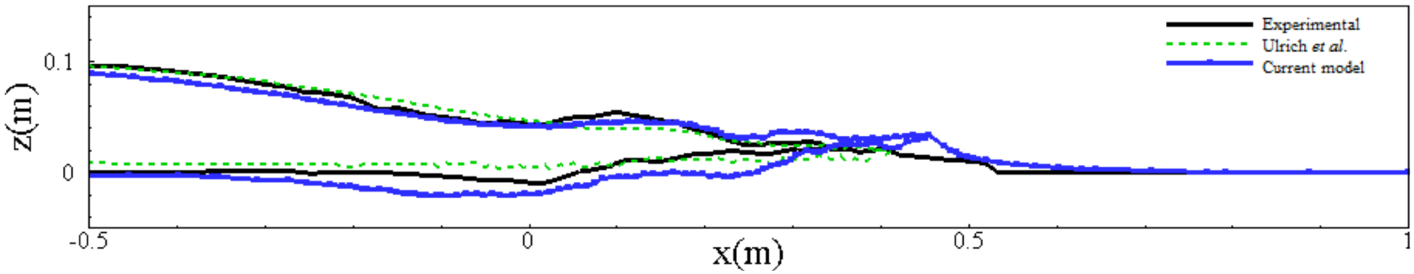
Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at t = 0.25 s.

2-D Erodible dam break



(a)

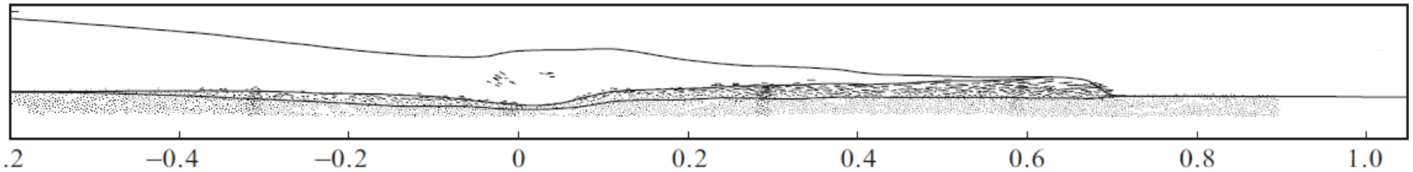
(b)



(c)

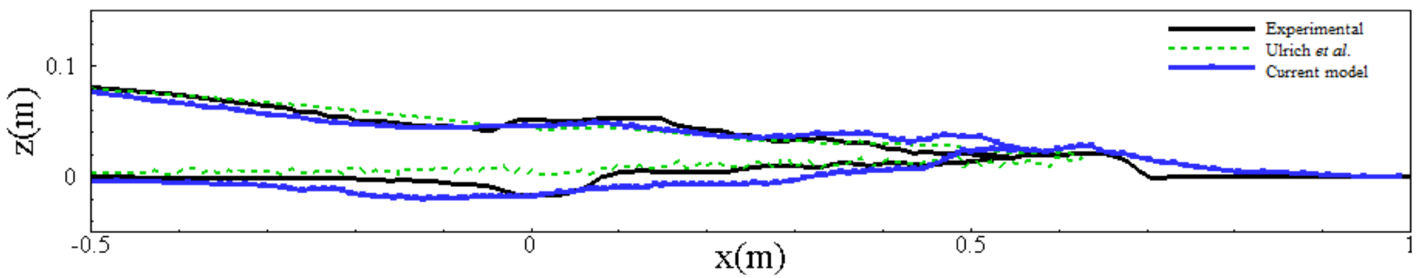
Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at *t* = 0.50 s.

2-D Erodible dam break



(a)

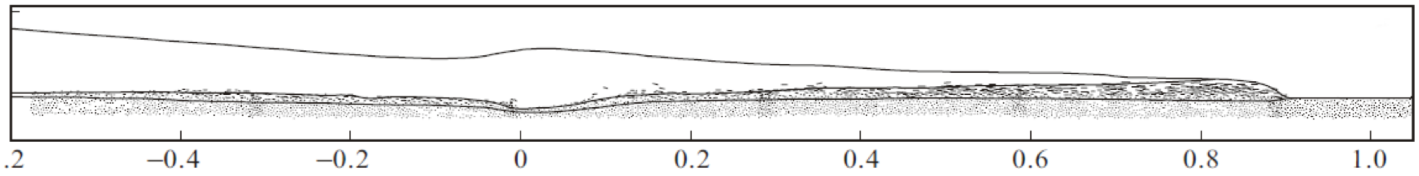
(b)



(c)

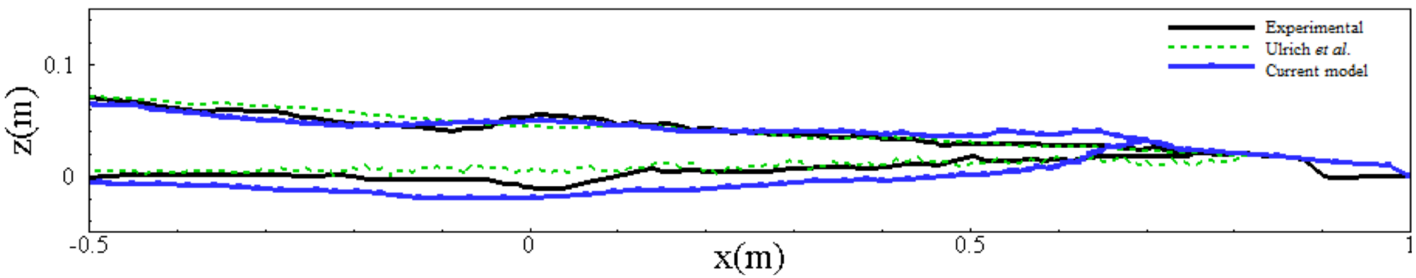
Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at t = 0.75 s.

2-D Erodible dam break



(a)

(b)



(c)

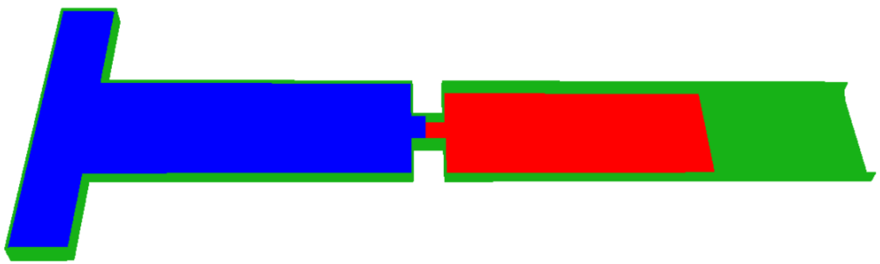
Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at t = 1.00 s.

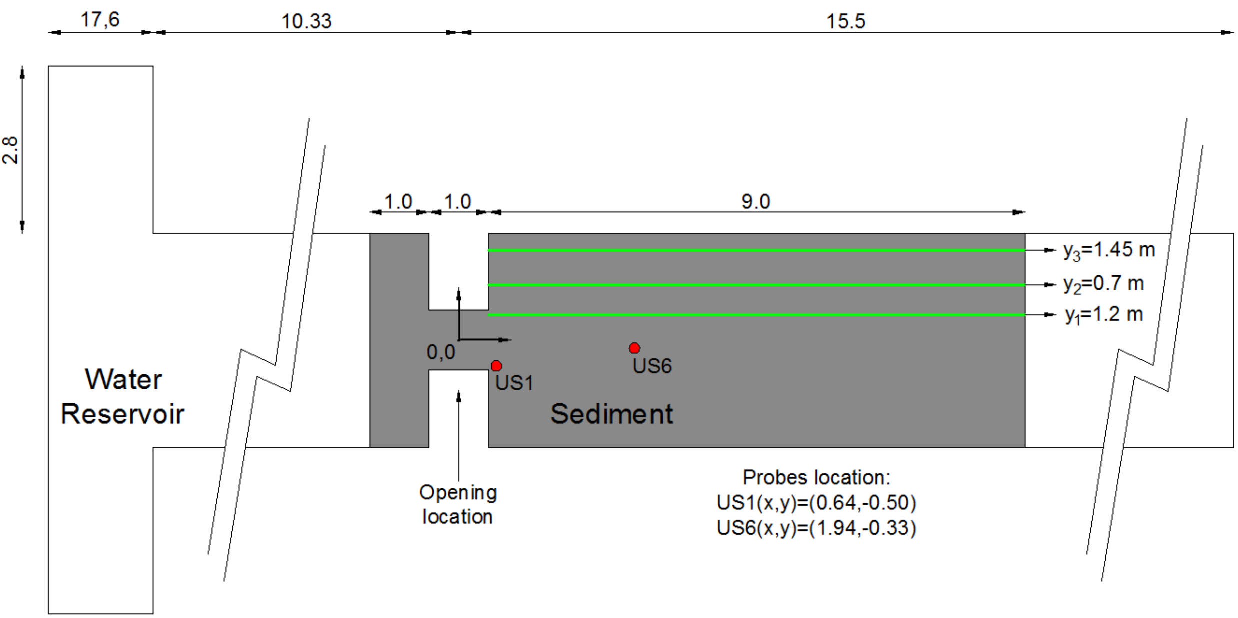
Numerical results

Case definition

3-D Erodible dam break

Soares-Frazão, S., et al., Dam-break flows over mobile beds., 2013





Parameter	Value	Units
Liquid height	0.47	m
Sediment height	0.085	m
Density ratio	2.63	
Porosity	0.42	
Numerical cohesion	100	Pa
Sediment viscosity	150	Pa.sec
m (HBP)	100	
n (HBP)	1.8	
Runtime	20	sec
No. Particle	4 million	

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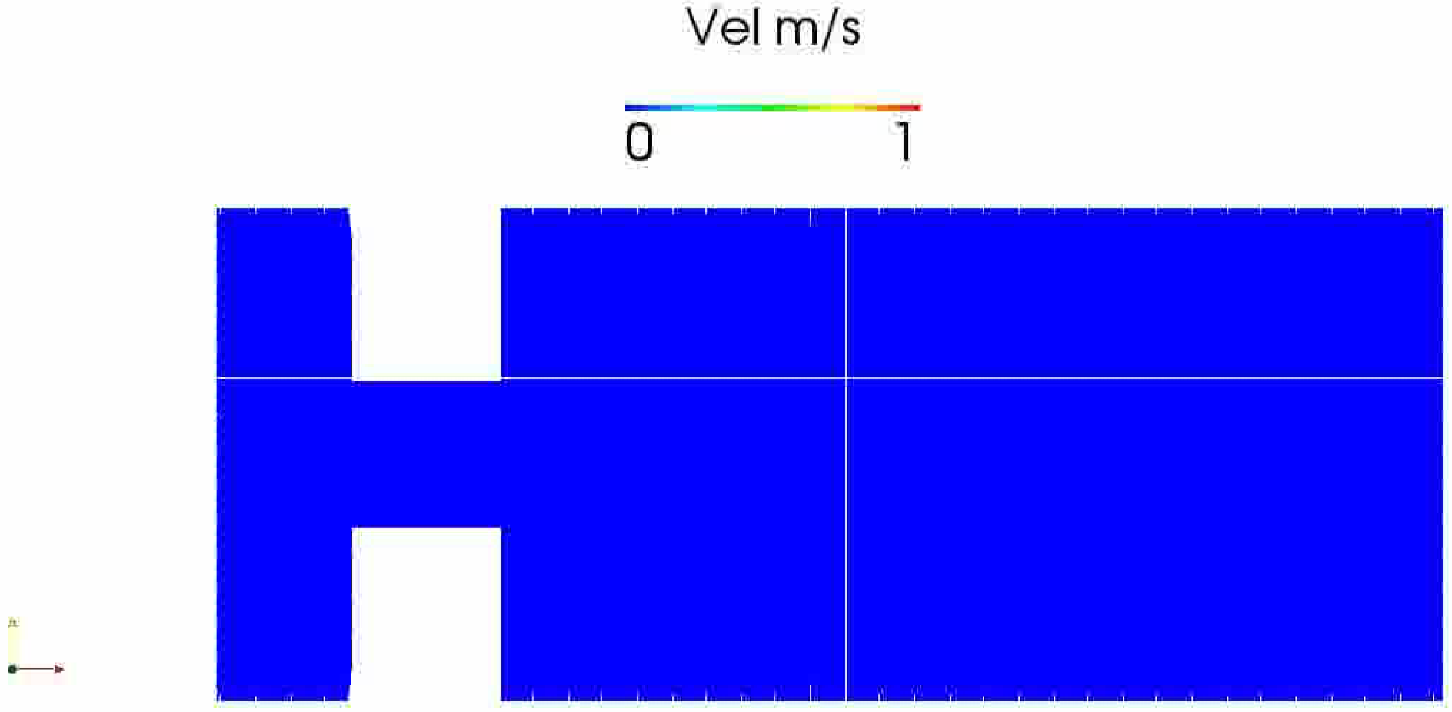
3-D Erodible dam break

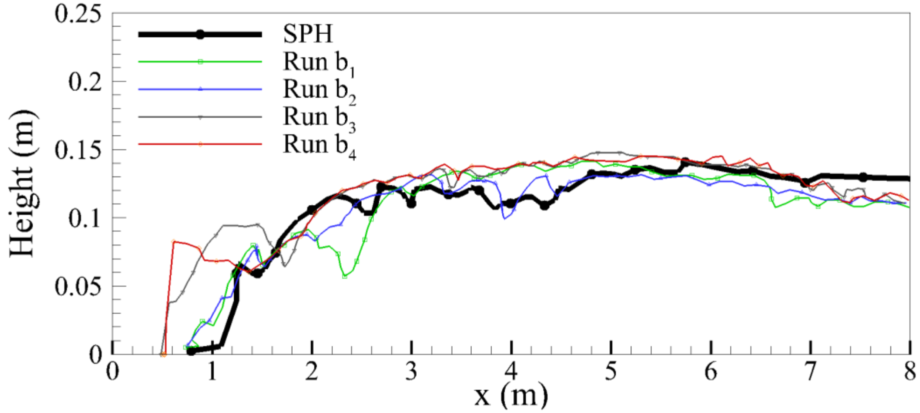
- Sediment bed profile evolution video



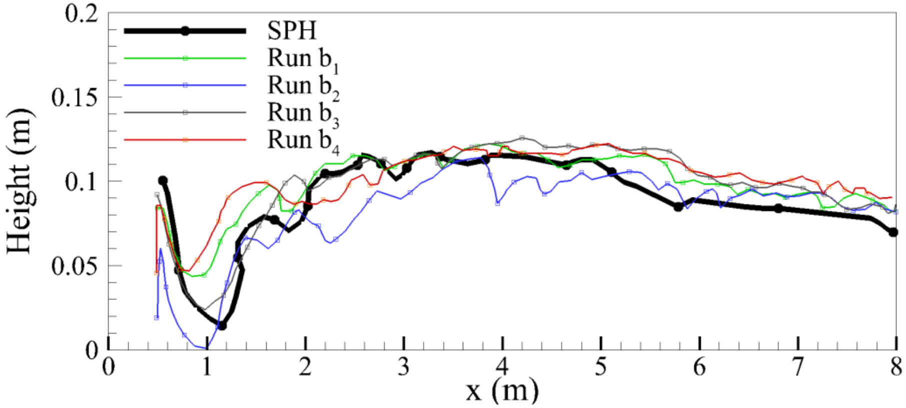
3-D Erodible dam break

- Sediment bed profile at *t* = 20 s

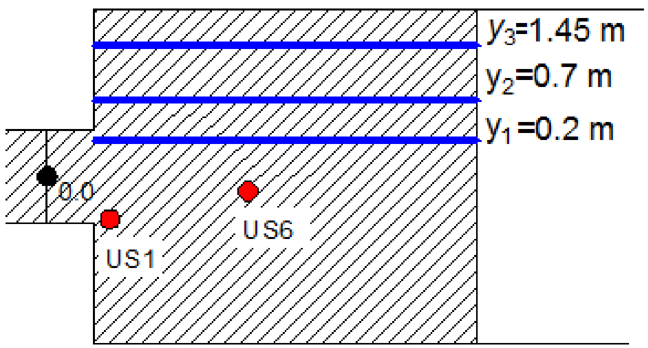
Bed profile at locations y_1



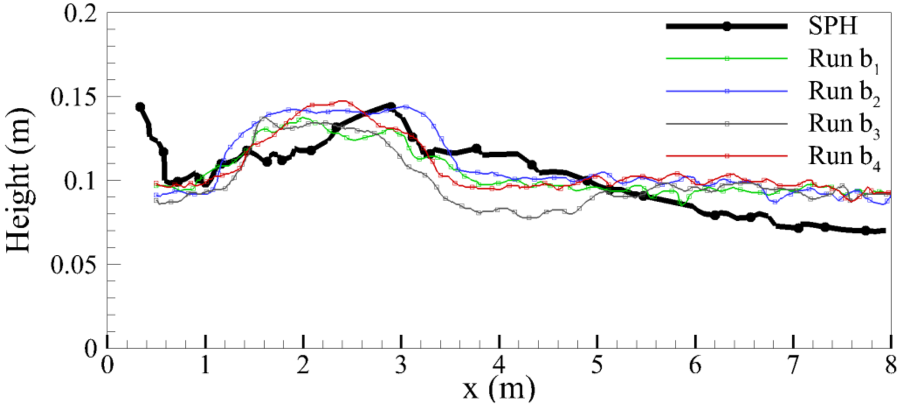
Bed profile at locations y_2



Schematic of probes and Profile locations

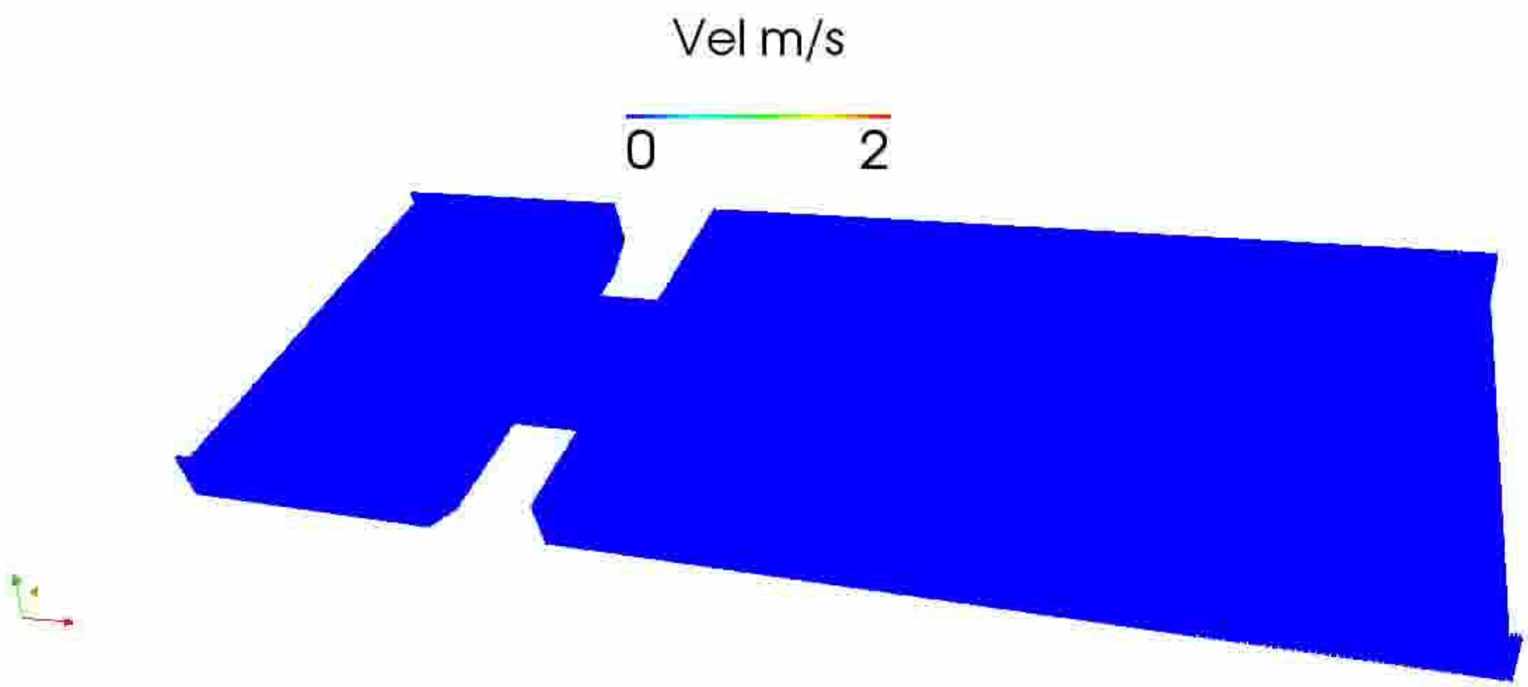


Bed profile at locations y_3



3-D Erodible dam break

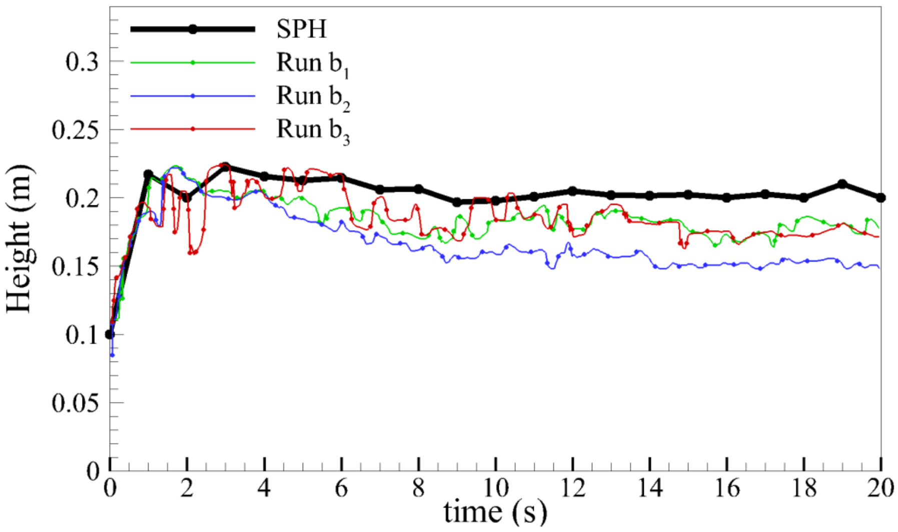
- Water level elevation video



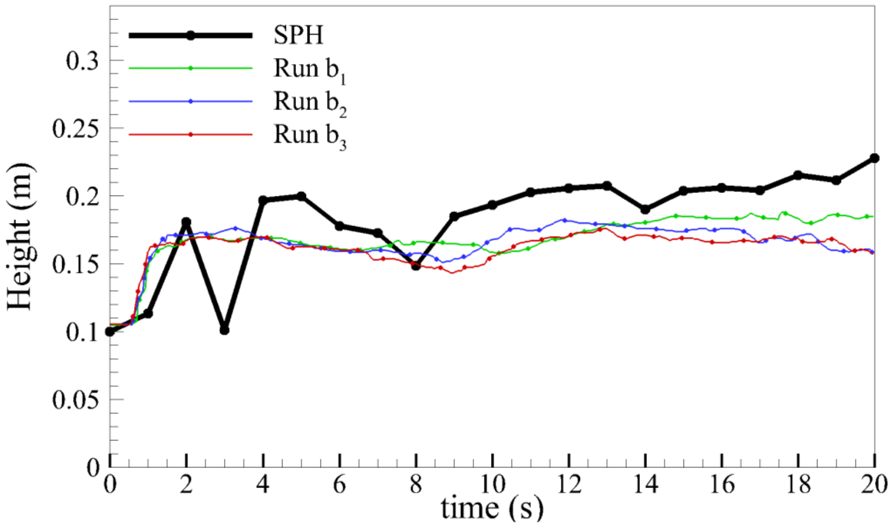
3-D Erodible dam break

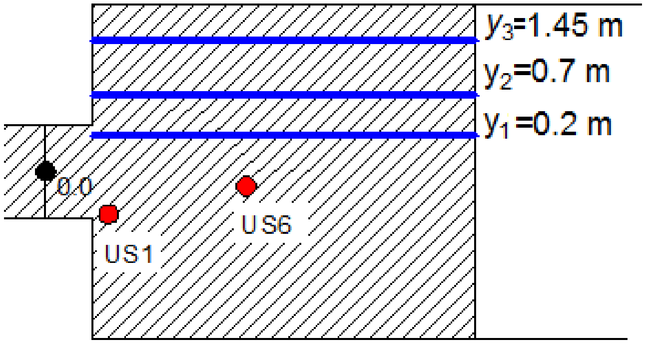
- Water level elevation from 0 to 20 s

Water level at probe US1



Water level at probe US6





Schematic of probes and Profile locations

Conclusions

- A novel sediment model has been presented with improvements to the yielding, shear layer constitutive modelling and sediment resuspension
- Good speed up characteristics achieved by the multi-phase GPU implementation (x58)
- The 2-D and 3-D results where in good agreement with the experimental data especially for the 3-D case:
  - The sediment profile at different locations
  - The water level elevation at the probe locations
- Future work
  - Inclusion of more physics, Shield's criterion
  - Turbulence modelling (cheaper mixing length / RANS model)
  - Subaqueous sediment flows e.g. sea bed slope failure

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Thank you

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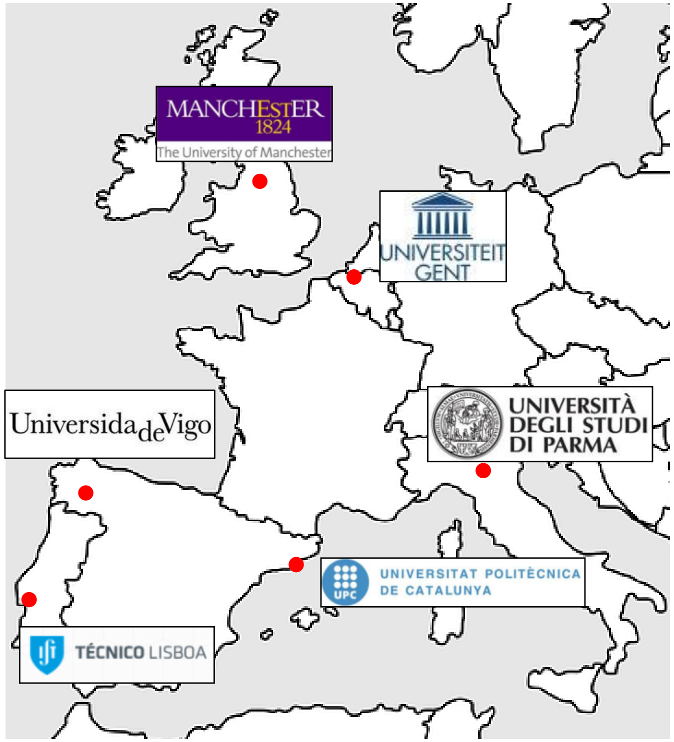
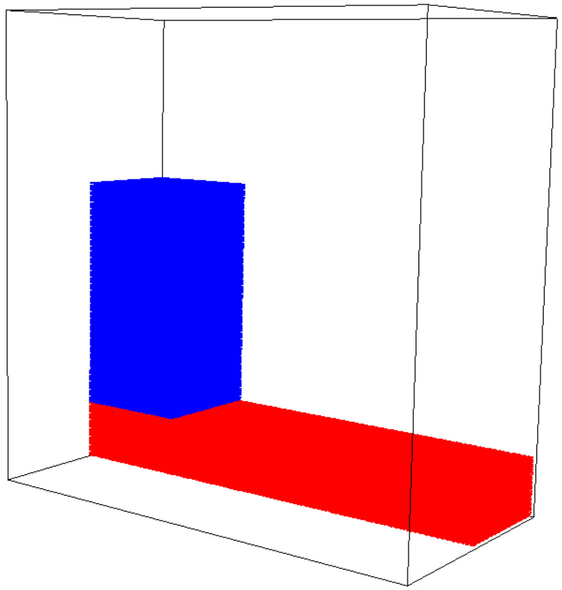
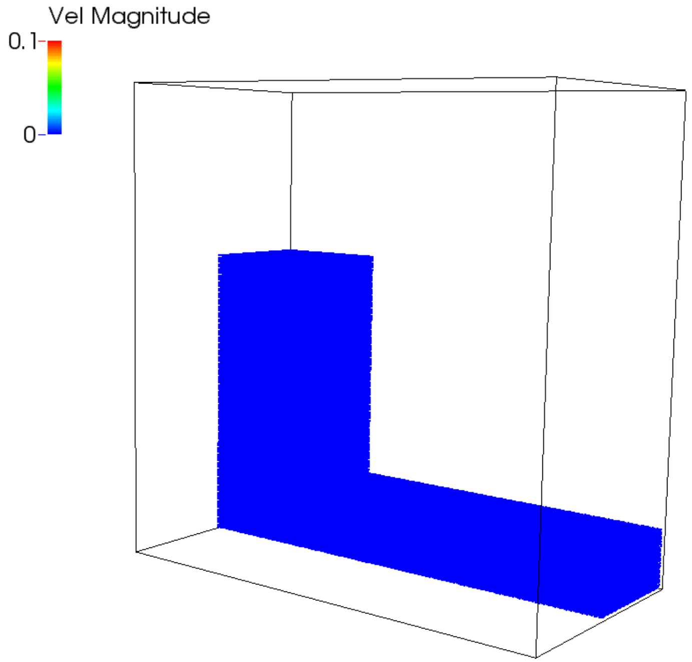
- U-Vigo: Jose Dominguez, Alex Crespo, Anxo Barreiro, Moncho Gomez-Gesteira
- U-Parma: Renato Vacondio

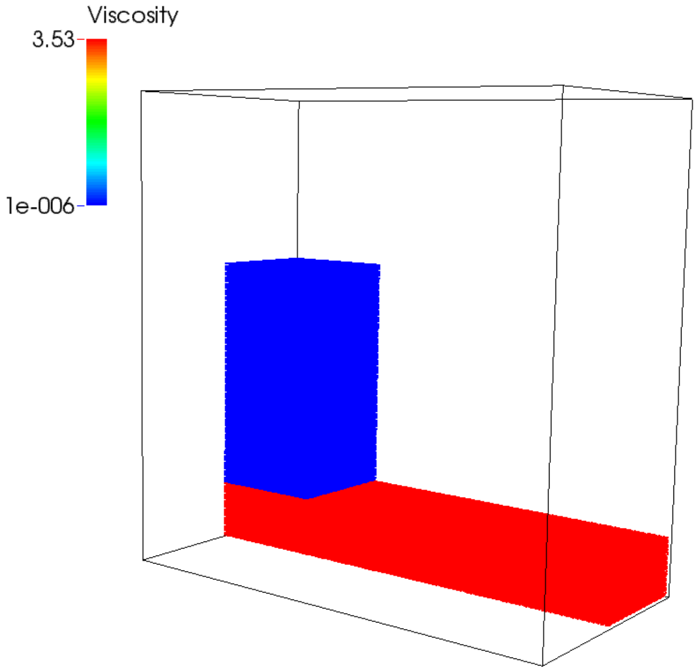
Websites

- http://www.dual.sphysics.org/
- https://wiki.manchester.ac.uk/sphysics
- http://www.mace.manchester.ac.uk/...sph






Vel Magnitude
0.1
0


Viscosity
3.53
1e-006

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