# 3-D SPH Modelling of Sediment Scouring Induced by Rapid Flows

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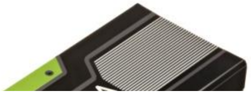
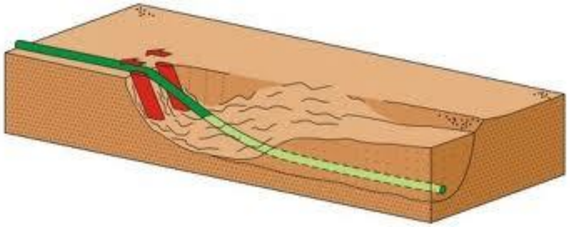
# Outline of the presentation

- Motivation
- Eulerian schemes and multi-phase flows
- Multi-phase model
  - Liquid - sediment model
    - Yield criteria
    - Constitutive equations
    - Sediment suspension
- GPU implementation
- Validation and applications
- Conclusions

# Motivation





- Real life engineering problems
  - Underwater sand bed trenching
  - Local scour around structures
  - Suspension of hazardous materials

- UK Nuclear industry application
  - Industrial tank
  - Hazardous material
  - Sediment agitation
  - Submerged jets

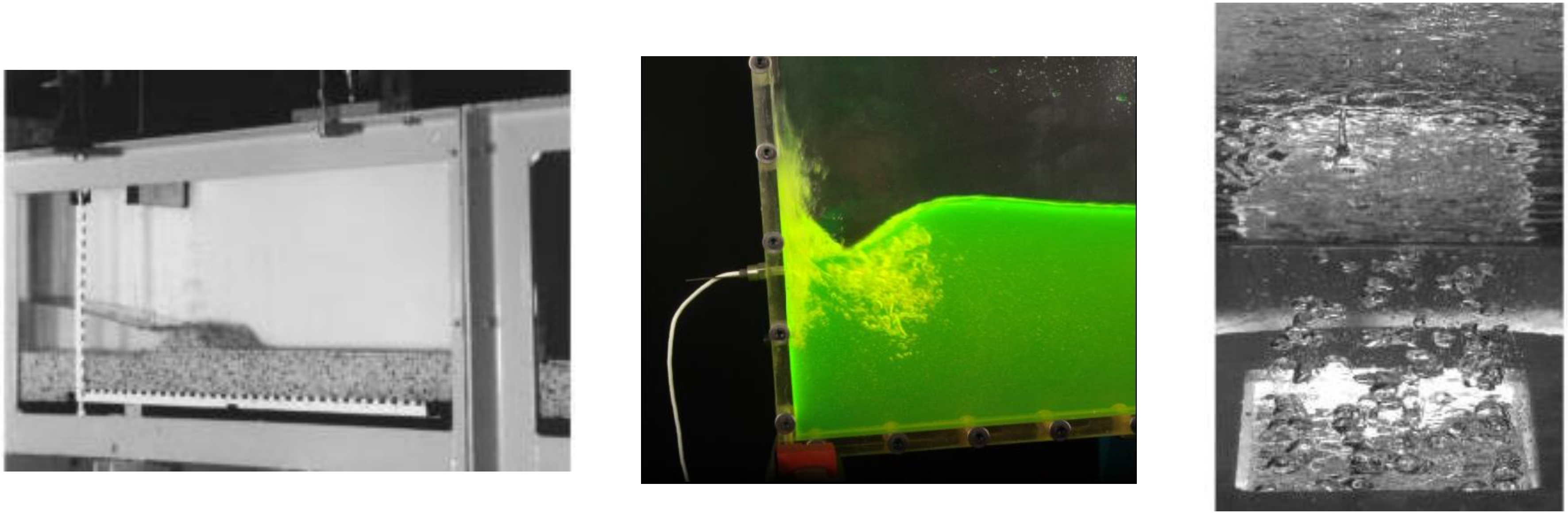
- GPUs Why?
  - Complicated geometry
  - Complex industrial flows
  - Computational cost

Courtesy of the National Nuclear Laboratory, UK

# Traditional CFD methods (Eulerian)

## Grid based methods

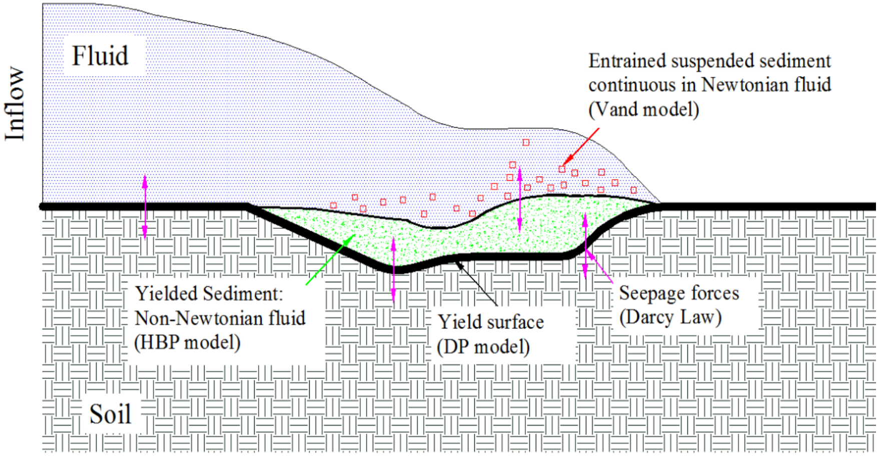
- Mesh generation can be expensive
- Mesh refinement in areas of interest (some knowledge a priori)
- Not applicable to highly non-linear deformations, (or very expensive)
- Multi-phase, free surfaces and phase-change flows



# Multi-phase model

## Liquid – sediment model

- Liquid phase
  - Newtonian flow
- Sediment phase
  - Yield criteria
    - Surface yielding
    - Sediment skeleton pressure
  - Non-Newtonian flow
    - Sediment shear layer at the interface
    - Seepage forces
  - Sediment resuspension
    - Entrainment of soil grains by the liquid



# Multi-phase model

## Liquid phase

- Weakly compressible SPH (WCSPH)
  - Tait's equation of state to relate pressure to density
  - δ-SPH – Density diffusion term
- Particle shifting – Particle re-ordering
- Turbulence is modelled through a SPS model
- GPU implementation to DualSPHysics

# Liquid phase

- Newtonian constitutive equation
  - Single phase DualSPHysics

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}} + \nu\nabla^2\mathbf{u} + \mathbf{g} + SPS \longrightarrow \frac{d\mathbf{u}_i}{dt} = -\sum_j^N m_j\left(\frac{P_j+P_i}{\rho_j\rho_i}\right)\nabla W_{ij} + \sum_j^N \frac{m_j}{\rho_j\rho_i}\left(\mu_j+\mu_i\right)\mathbf{u}_{ij}\frac{\mathbf{x}_{ij}\cdot\nabla W_{ij}}{x_{ij}^2+\eta^2} + \mathbf{g} + SPS$$

  - Multi-phase implementation

$$\frac{d\mathbf{u}}{dt} = \frac{1}{r}\frac{\P s}{\P \mathbf{x}} + \mathbf{g} \longrightarrow \frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial \mathbf{x}} + \frac{1}{\rho}\frac{\partial \tau}{\partial \mathbf{x}} + \mathbf{g}$$

$$\frac{1}{\rho}\frac{\partial \tau_i^{\alpha\beta}}{\partial x^{\beta}} = \sum_j^N m_j\left(\frac{\tau_i^{\alpha\beta}+\tau_j^{\alpha\beta}}{\rho_i\rho_j}\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

since

$$\tau_i^{\alpha\beta} = f\left(\varepsilon_i^{\alpha\beta}\right)$$

$$\varepsilon_i^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_i^{\alpha}}{\partial x_i^{\beta}} + \frac{\partial u_i^{\beta}}{\partial x_i^{\alpha}}\right) - \frac{1}{3}\left(\frac{\partial u_i^{\gamma}}{\partial x_i^{\gamma}}\right)\delta^{\alpha\beta}$$

from

$$\left.\frac{\partial u^{\alpha}}{\partial x^{\beta}}\right|_i = \sum_j^N \frac{m_j}{\rho_j}u_{ij}^{\alpha}\frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

# Multi-phase model

## Sediment phase

- Treated as a semi-solid non-Newtonian fluid
- Yield criterion Drucker-Prager
  - Below a critical level of sediment deformation sediment particles remain still
  - Above a critical level of sediment deformation follow the governing equations
- Non-Newtonian flow
  - Herschel-Bulkley-Papanastasiou Bingham constitutive model
- Approximation of seepage forces on the surface
  - Darcy law
- Entrained suspended sediment
  - Concentration based apparent viscosity based on a Newtonian formulation, Vand model

# Sediment phase

- Surface Yielding
  - Drucker-Prager (DP) yield criterion
    - For an isotropic material

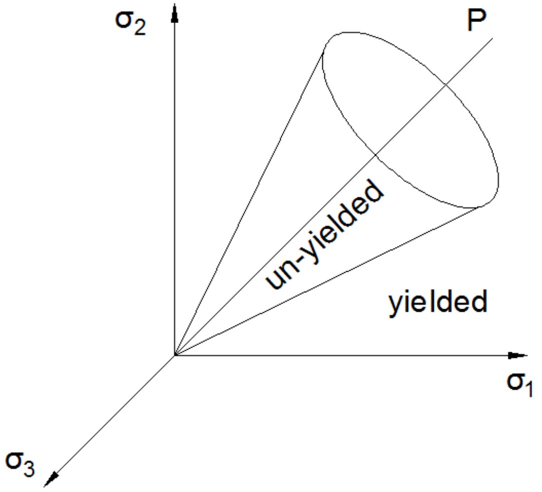
$$\sqrt{J_2} - \left|\tau_y\right| = 0$$

    - Apply the yield criterion

$$\left|\tau_y\right| = -\alpha \mathrm{I}_1 + \kappa$$

    - Yielding occurs when

$$\sqrt{J_2} \geq \boxed{\alpha P_{skeleton}} + \kappa$$



Drucker-Prager (DP) yield surface in principal stress space

<u>Constants</u>

$$\alpha = \frac{\tan\phi}{\sqrt{9 + 12\tan^2\phi}} \qquad \kappa = \frac{3c}{\sqrt{9 + 12\tan^2\phi}}$$

where *c* is the cohesion and *φ* angle of repose

# Sediment phase

- Sediment skeleton pressure
  - For a fully saturated soil
    - Terzaghi relationship

$$P_{total} = P_{skeleton} + P_{pw}$$

    - or

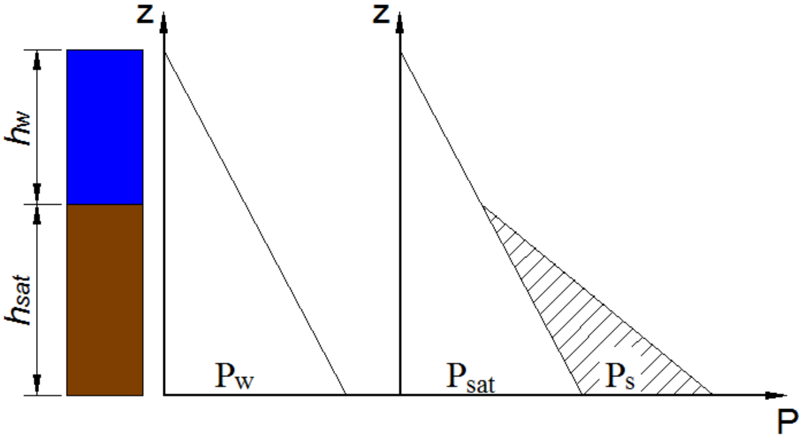
$$P_{total} = h_w \gamma_w + h_s \gamma'_{sat}$$

    - Pore water pressure

$$p_{pw,i} = B\left(\left(\frac{\rho_{sat,i}}{\rho_{sat,0}}\right)^{\gamma} - 1\right)$$

$$\downarrow$$

$$B = \frac{c^2{}_{w,s0}\rho_{w,0}}{\gamma_w}$$



Sediment skeleton pressure

$$P_{skeleton} = P_t - P_{pw}$$

# Sediment phase

- Sediment constitutive equation
  - Simple Bingham

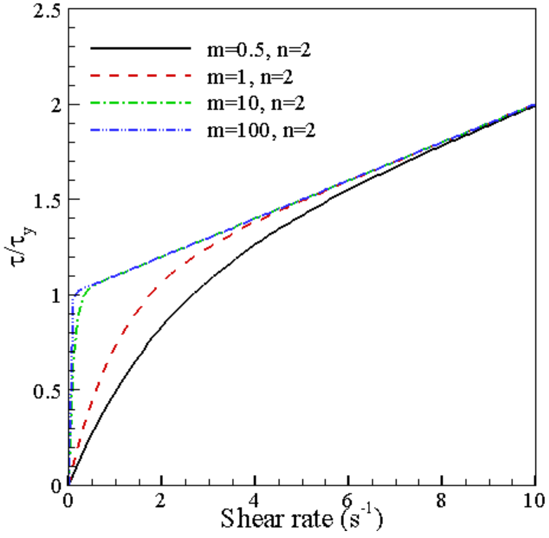
$$\mu_{Bingh} = \frac{|\tau_y|}{\sqrt{II_D}} + \mu_d$$

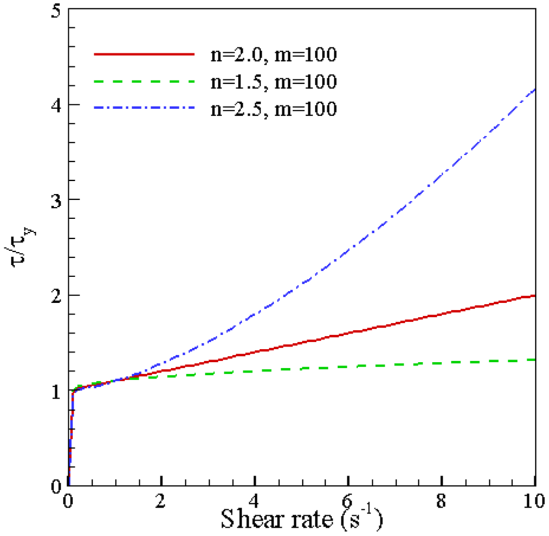
  - Herschel-Bulkley-Papanastasiou (HBP)
    - Viscous – Plastic (*m* exponential growth)
    - Shear thinning or thickening (*n* power law*)*

$$\mu_{pap} = \frac{|\tau_y|}{\sqrt{II_D}}\boxed{\left[1 - e^{-m\sqrt{II_D}}\right]} + \boxed{K D^{(n-1)/2}}$$

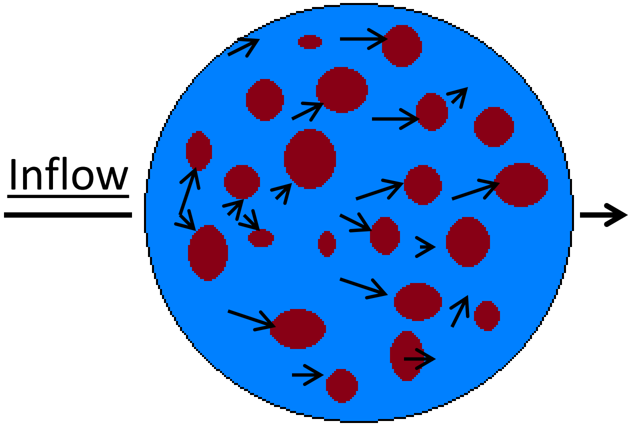
$$\big\downarrow$$

$$\tau_i^{\alpha\beta} = 2\mu_{pap} D^{\alpha\beta} \longrightarrow D^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u^{\alpha}}{\partial x^{\beta}} + \frac{\partial u^{\beta}}{\partial x^{\alpha}}\right)$$





# Sediment phase



- Seepage force
  - Generalised Darcy law

$$S = K\left(u_w - u_s\right) \longrightarrow \quad K = \frac{n_r \gamma_w}{k}$$

(Soil properties)

SPH formalism

$$S^{\alpha}_{s,i} = \sum_{j \in W,Sat}^{N} \frac{m_j}{\rho_i \rho_j} S^{\alpha}_{ij} W_{ij}$$

- Suspension

Vand equation

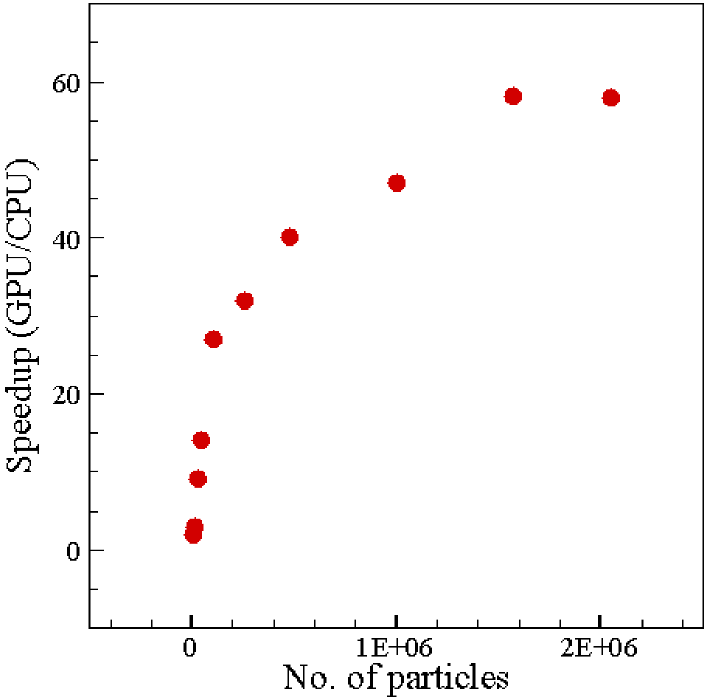
$$\mu_{susp} = \mu_{fluid} e^{\frac{2.5c_v}{1-\frac{39}{64}c_v}} \qquad c_v \le 0.3 \qquad \longrightarrow$$

Concentration volume fraction of sediment

$$c_{v,i} = \frac{\sum\limits_{j_{sat} \in 2h}^{N} \frac{m_j}{\rho_j}}{\sum\limits_{j \in 2h}^{N} \frac{m_j}{\rho_j}}$$

# GPU implementation in DualSPHysics

- Multi-phase issues
  - Branching
  - Registers
  - Arithmetic operations
  - Larger data size
- Resolve
  - Memory operations
  - Smaller kernels
  - Combine similar operations

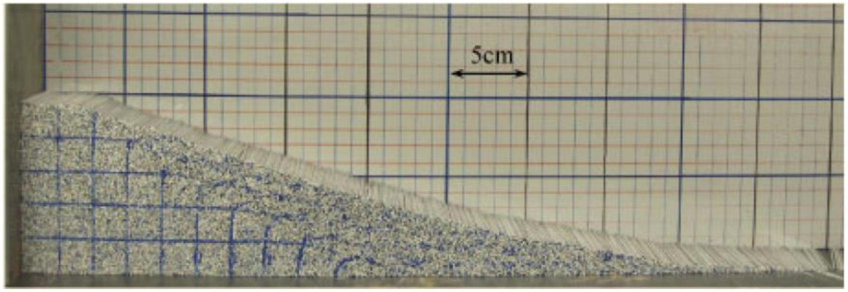


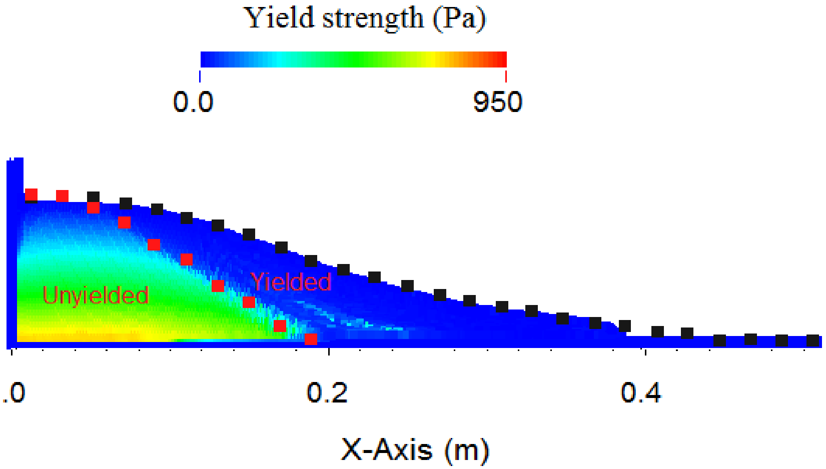
GPU algorithm speed up curve
(x58 compared to a single thread CPU)

# Numerical results

## Soil Dam break

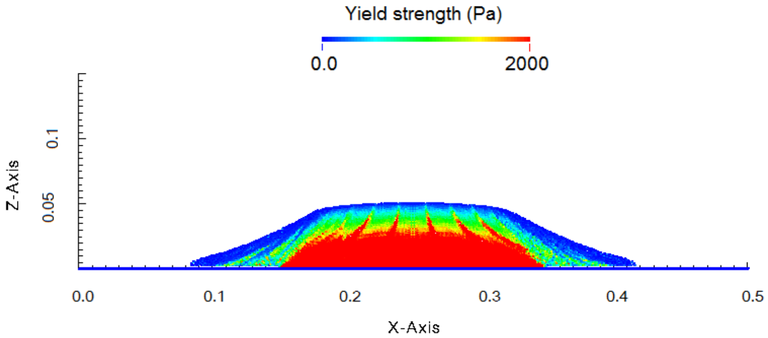
Bui *et al.*, Langrangian method for large deformation and failure flows of geo-material, 2008

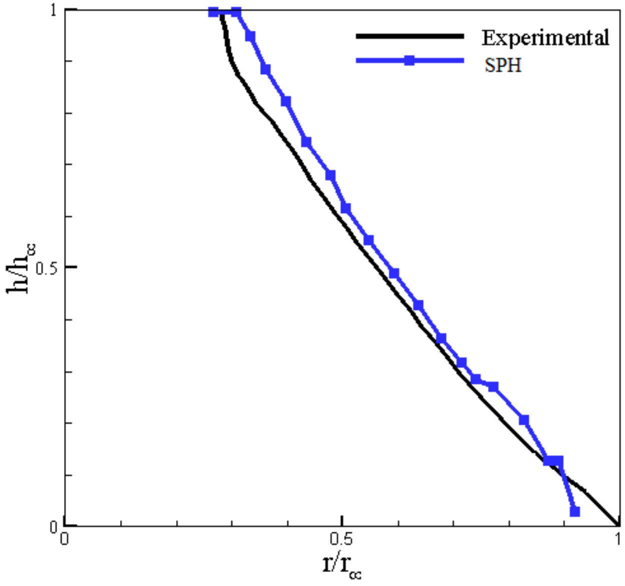




## Sediment block collapse

Lude *et al.*, Axisymmetric collapses of granular columns, 2014



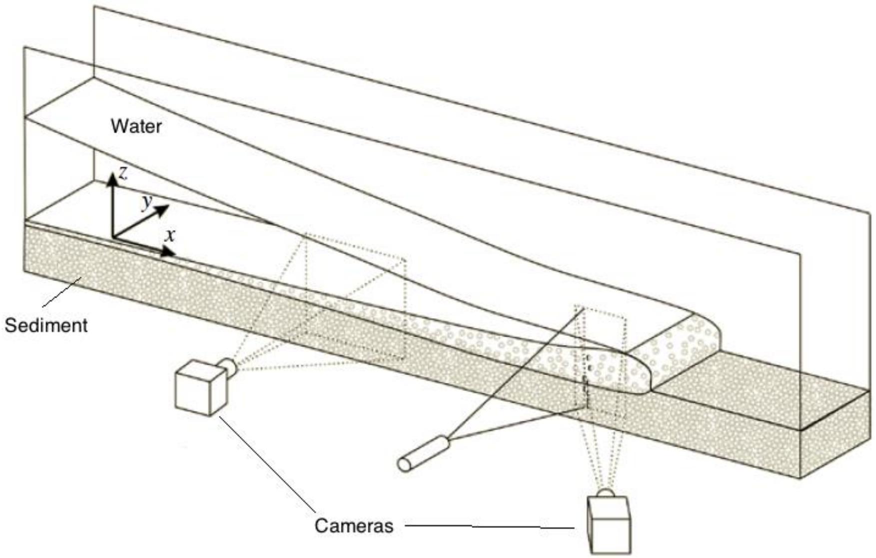


# Numerical results

## Case definition

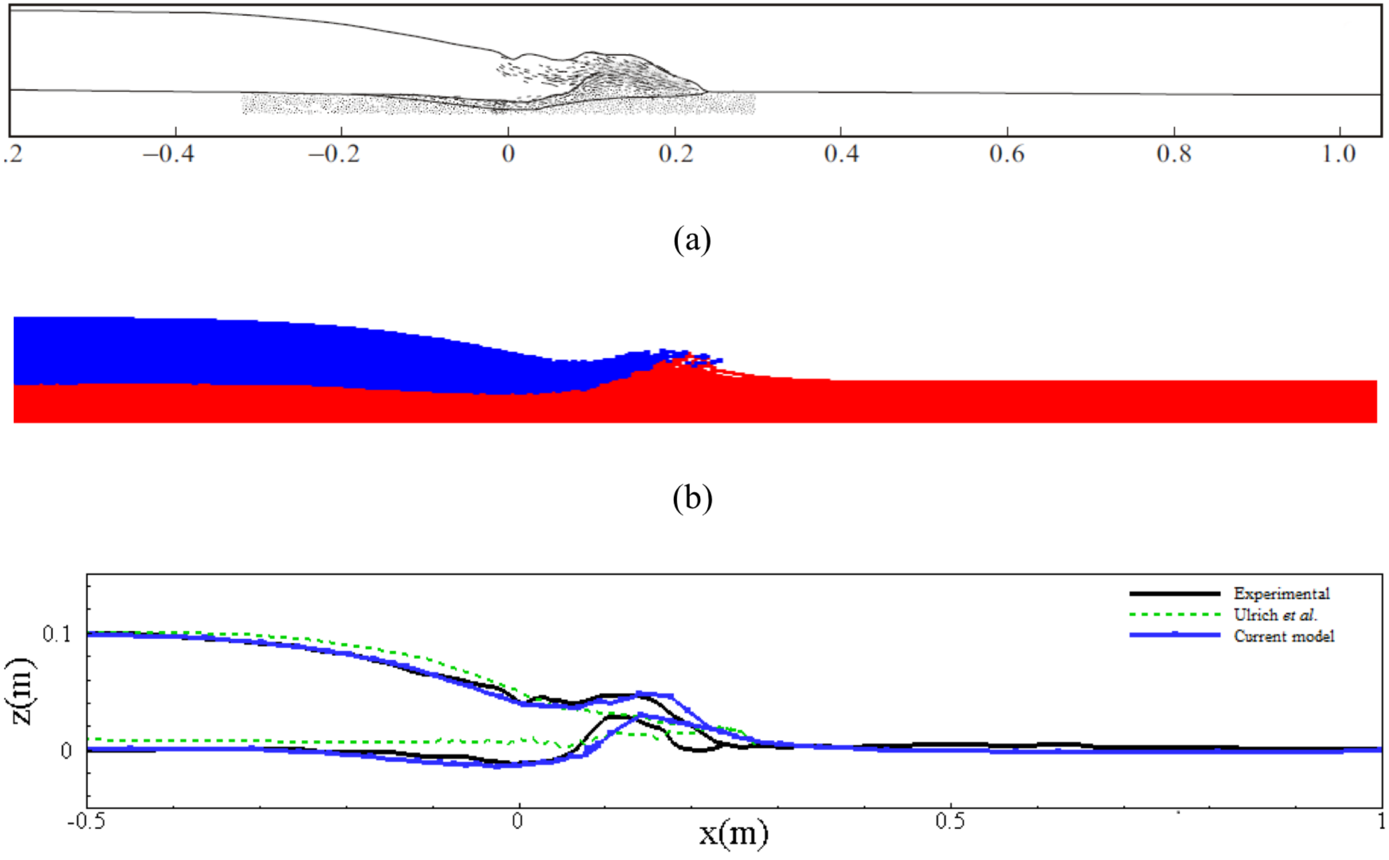
### Erodible Dam break

Spinewine *et al.*, Intense bed-load due to sudden dam break, 2013



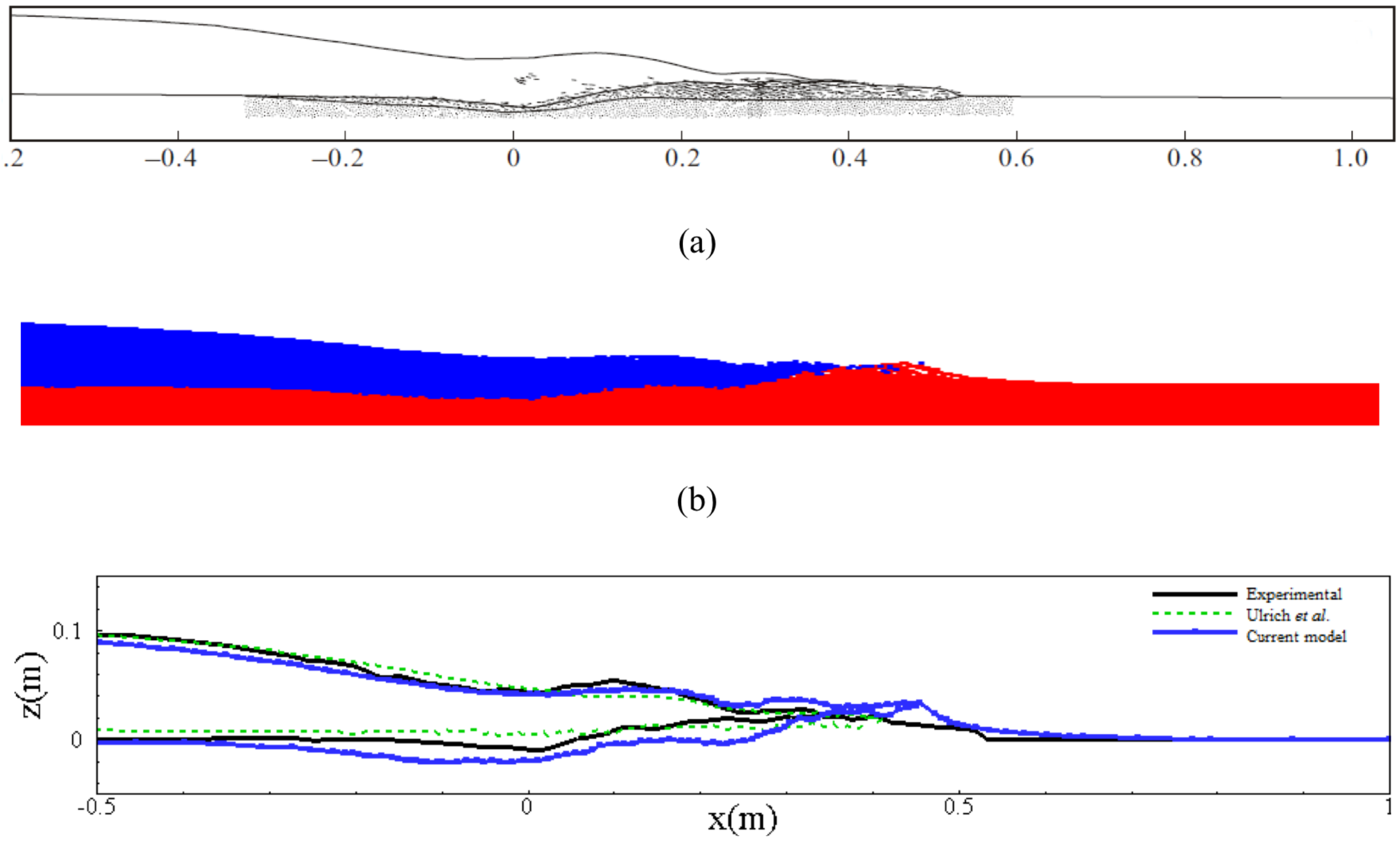
| Parameter | Value | Units |
|---|---|---|
| Liquid height | 0.1 | m |
| Sediment height | 0.6 | m |
| Density ratio | 1.54 | |
| Porosity | | |
| Numerical cohesion | 100 | Pa |
| Sediment viscosity | 500 | Pa.sec |
| *m* (HBP) | 100 | |
| *n* (HBP) | 1.6 | |
| Runtime | 1.5 | sec |
| No. Particle | 328 000 | |

# 2-D Erodible dam break



Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at *t* = 0.25 s.

# 2-D Erodible dam break



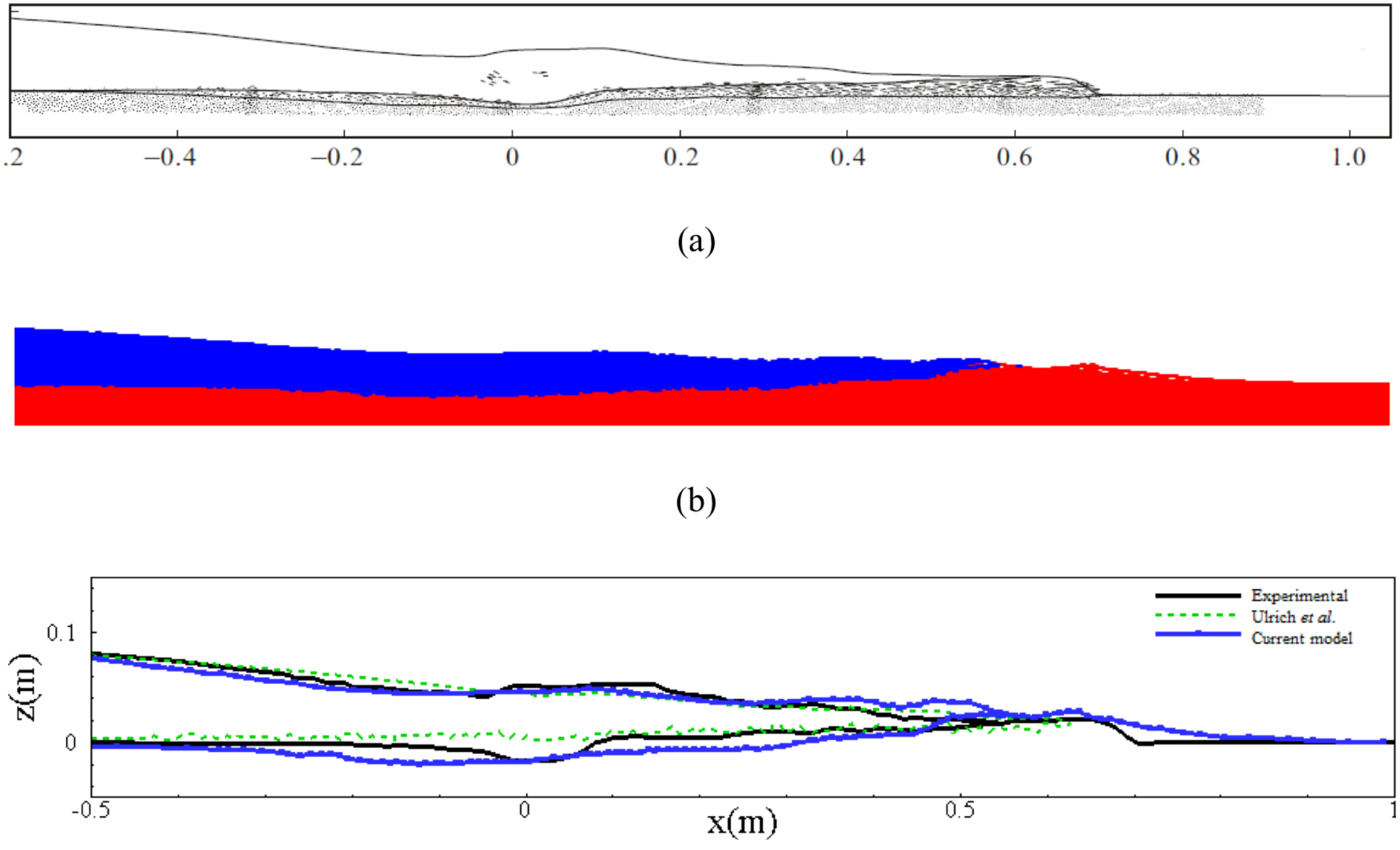
(a)

(b)

(c)

Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at *t* = 0.50 s.

# 2-D Erodible dam break



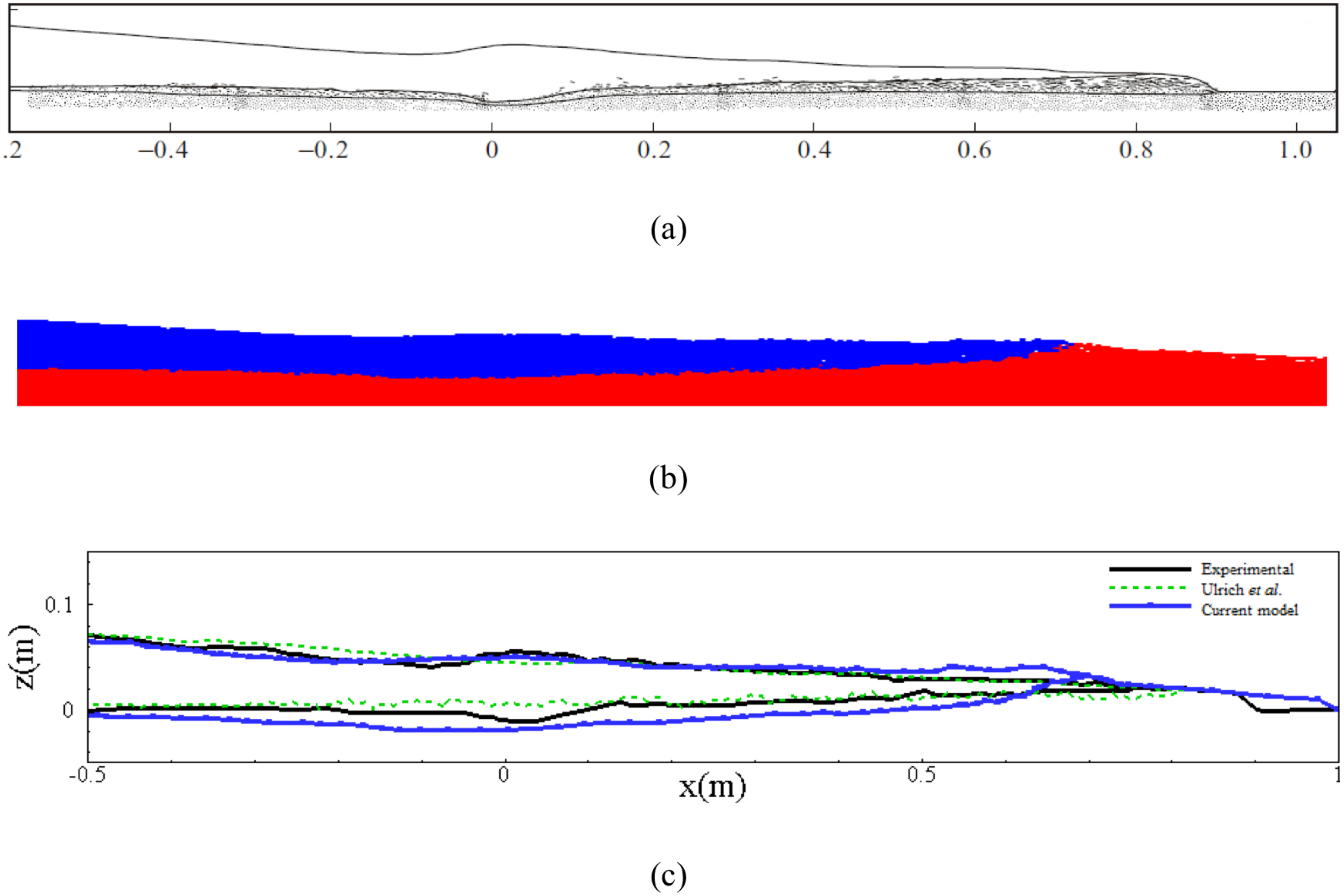
(a)

(b)

(c)

Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at *t* = 0.75 s.

# 2-D Erodible dam break



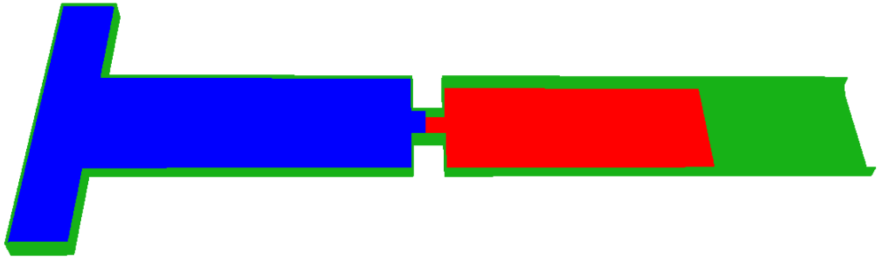
Qualitative comparison of (a) experimental and (b) current numerical results and (c) comparison of liquid-sediment profiles of the experiments, numerical results of Ulrich *et al*. and current model at $t$ = 1.00 s.

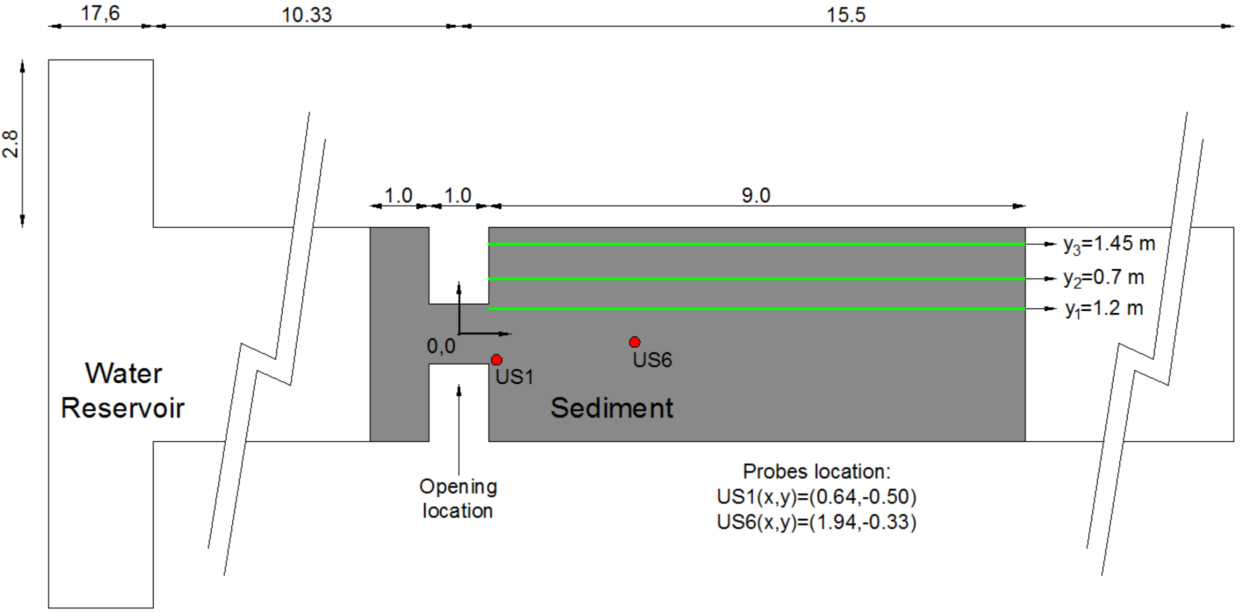
# Numerical results

## Case definition

### 3-D Erodible dam break

Soares-Frazão, S., et al., Dam-break flows over mobile beds., 2013



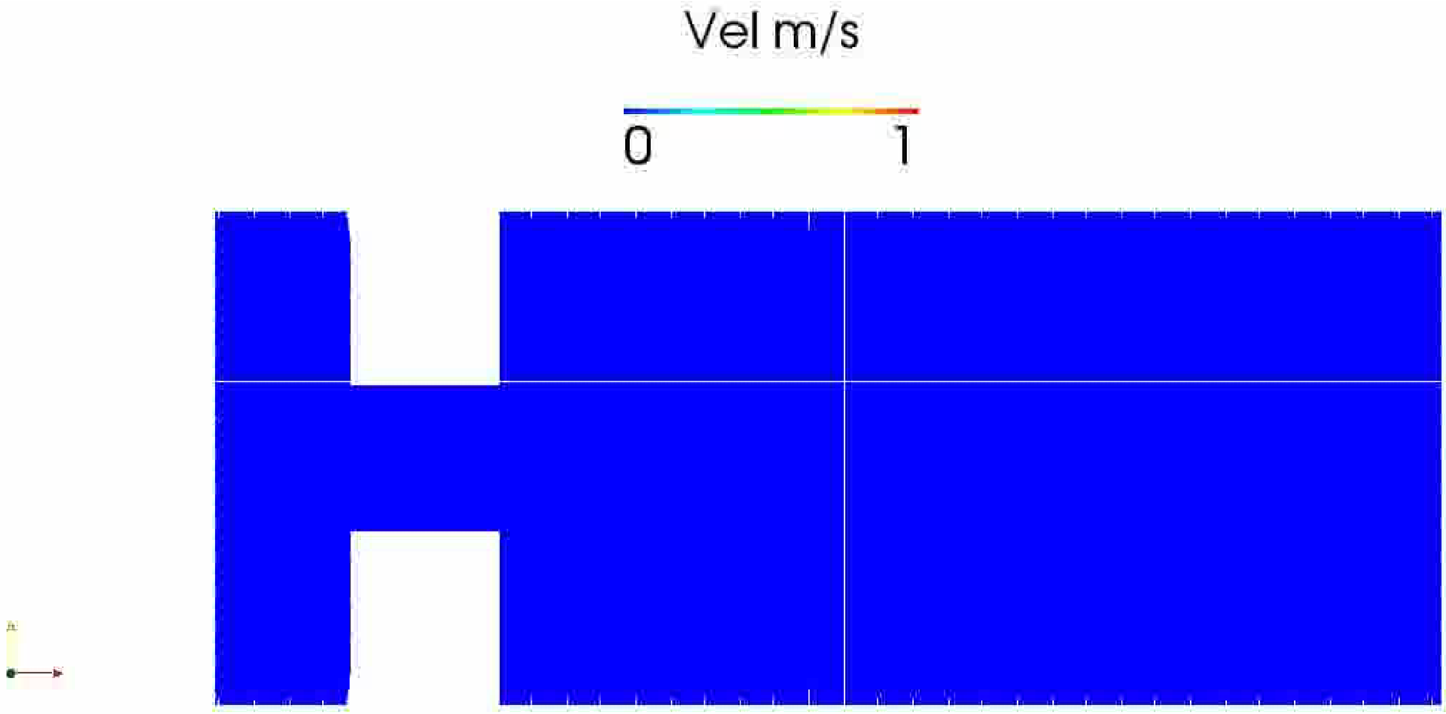


Probes location:
US1(x,y)=(0.64,-0.50)
US6(x,y)=(1.94,-0.33)

| Parameter | Value | Units |
|---|---|---|
| Liquid height | 0.47 | m |
| Sediment height | 0.085 | m |
| Density ratio | 2.63 | |
| Porosity | 0.42 | |
| Numerical cohesion | 100 | Pa |
| Sediment viscosity | 150 | Pa.sec |
| *m* (HBP) | 100 | |
| *n* (HBP) | 1.8 | |
| Runtime | 20 | sec |
| No. Particle | 4 million | |

# 3-D Erodible dam break

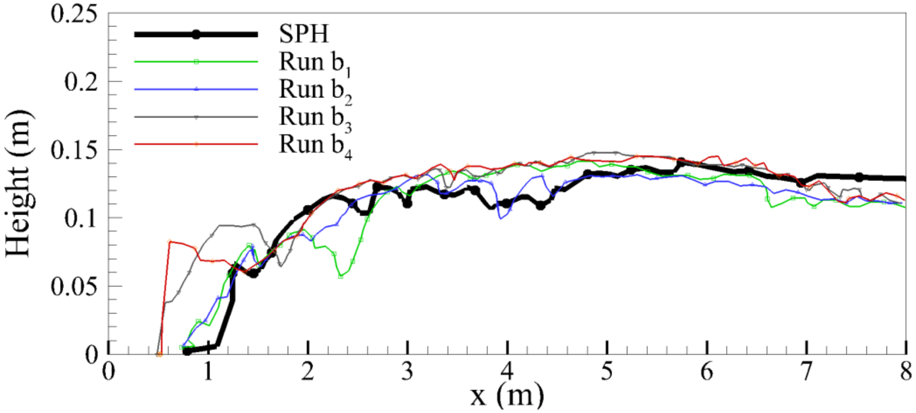
- Sediment bed profile evolution video



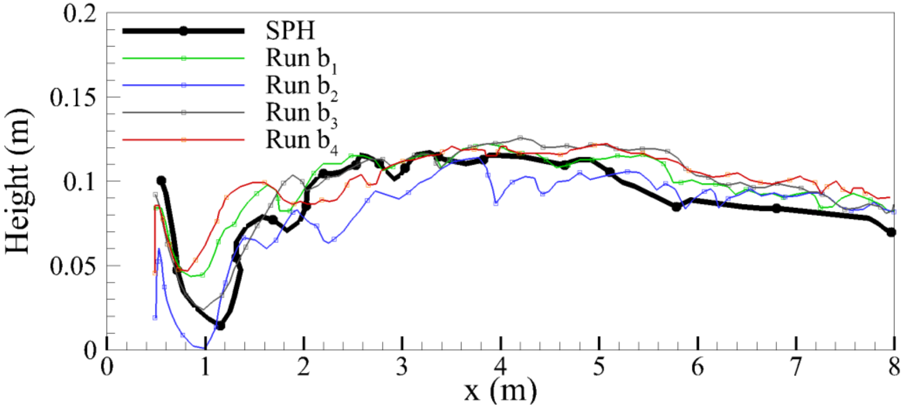
# 3-D Erodible dam break

- Sediment bed profile at *t* = 20 s

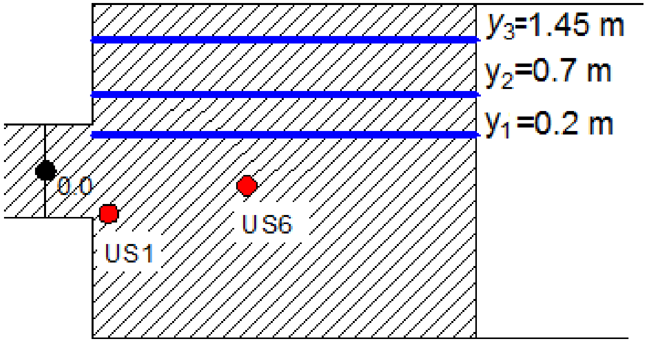
Bed profile at locations $y_1$



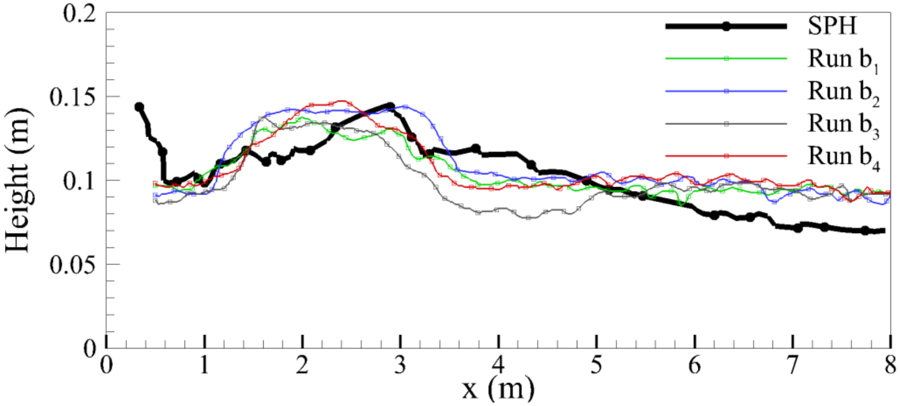
Bed profile at locations $y_2$



Schematic of probes and Profile locations

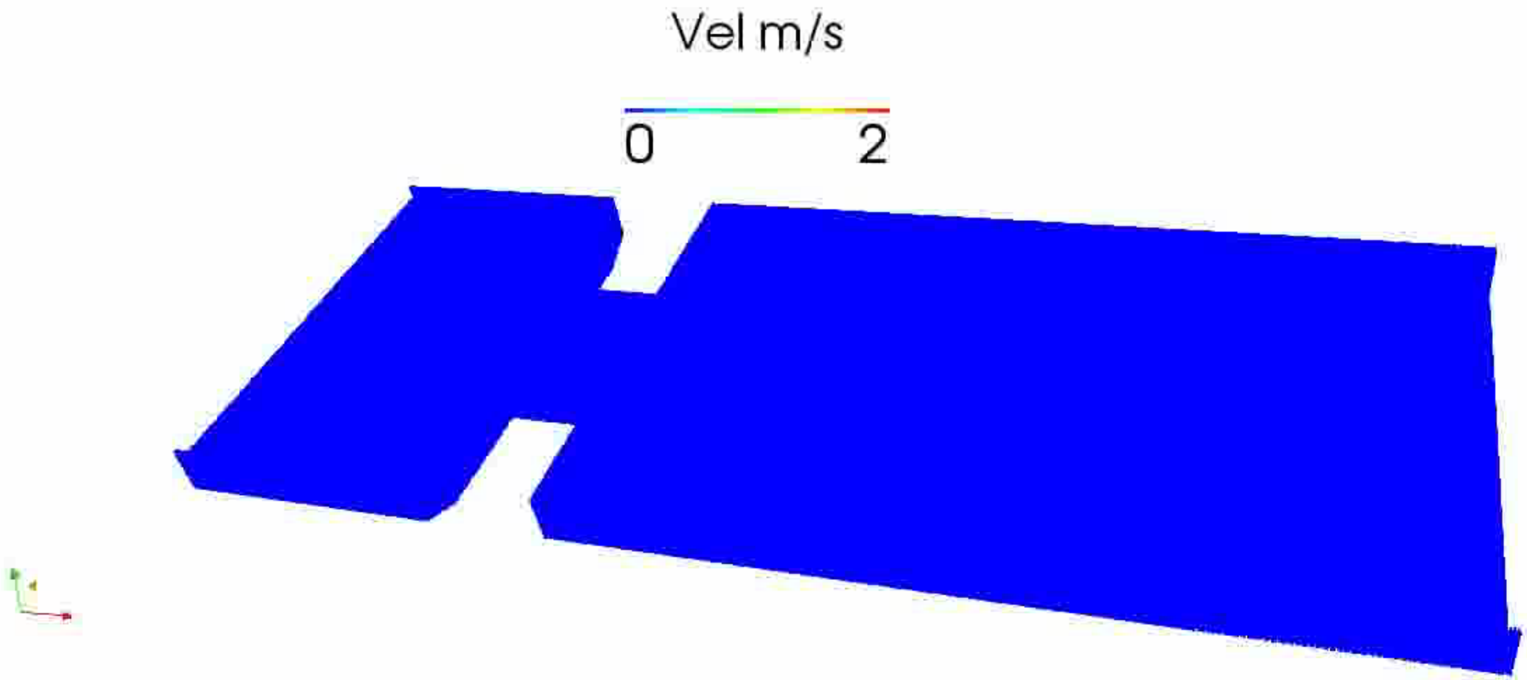


Bed profile at locations $y_3$



# 3-D Erodible dam break

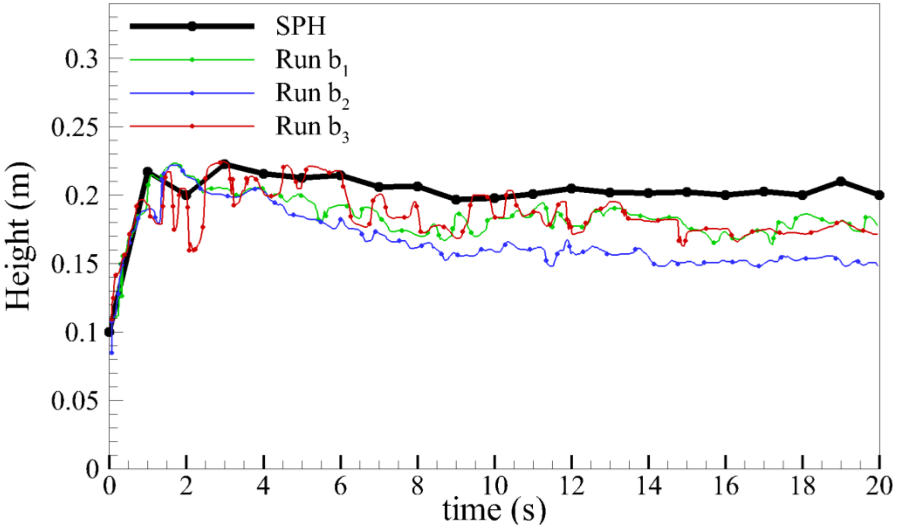
- Water level elevation video



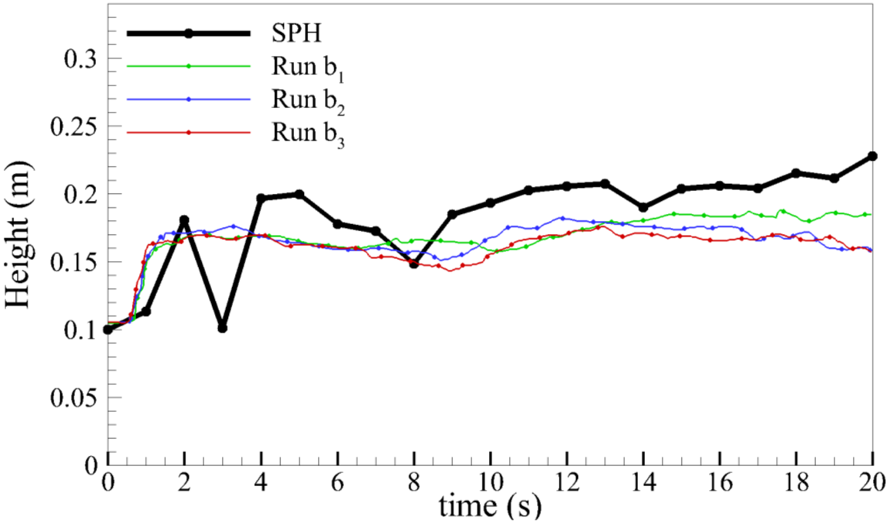
# 3-D Erodible dam break

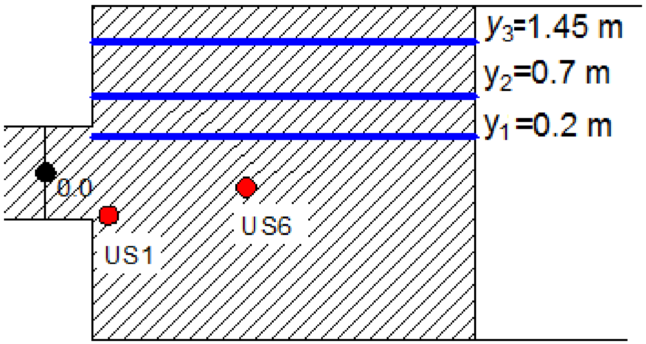
- Water level elevation from 0 to 20 s

Water level at probe US1



Water level at probe US6





Schematic of probes and Profile locations

# Current developments

**But..**

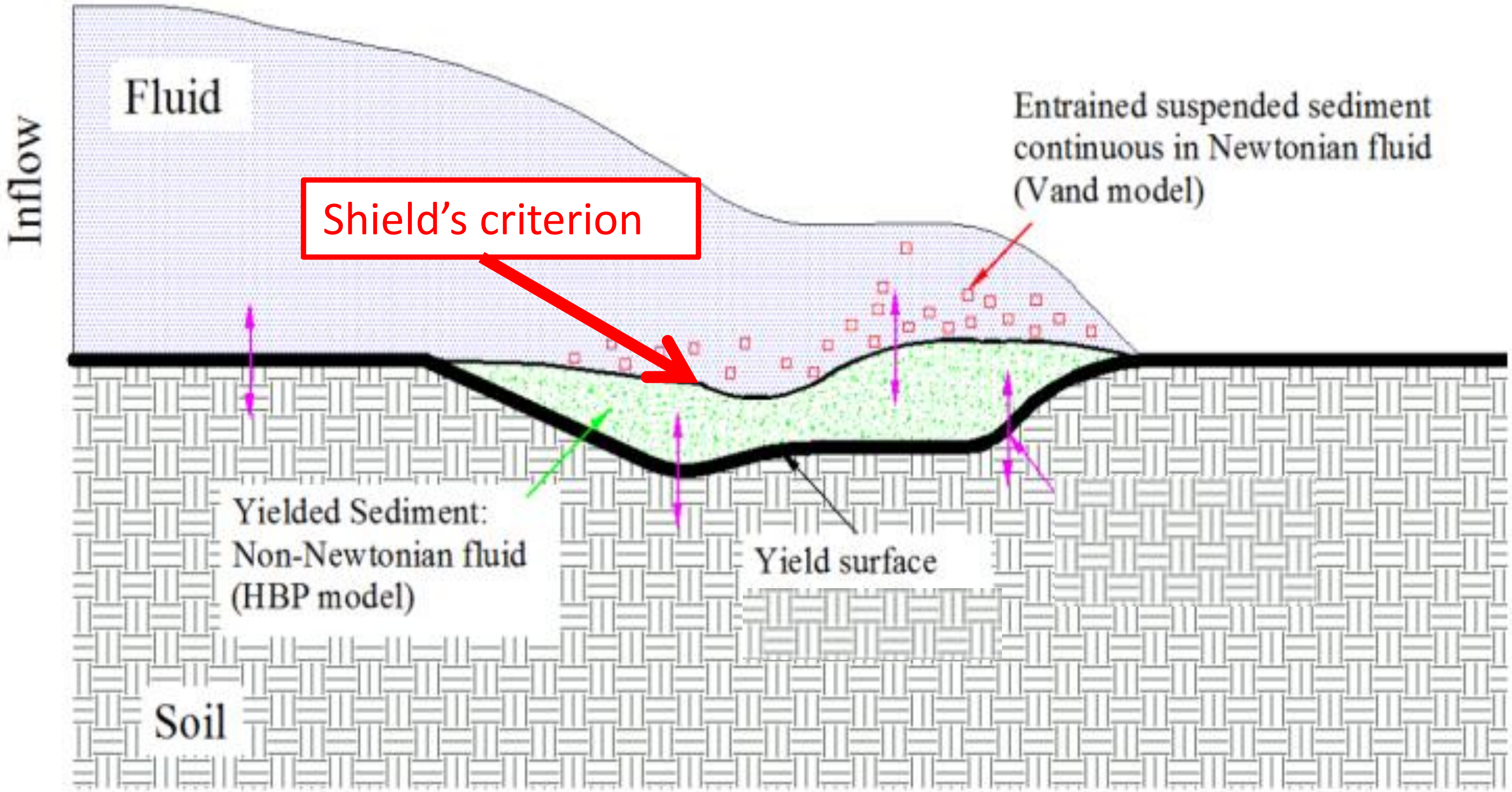
**We can do better!!!**

- **Sediment is eroded due to:**
    - **Shear forces at the interface**
    - **Turbulence at the interface**

- **Shield's yield criterion (**Zubeldia et al, 2016, SPHERIC workshop**)**

# Current developments

(Zubeldia et al, 2016)



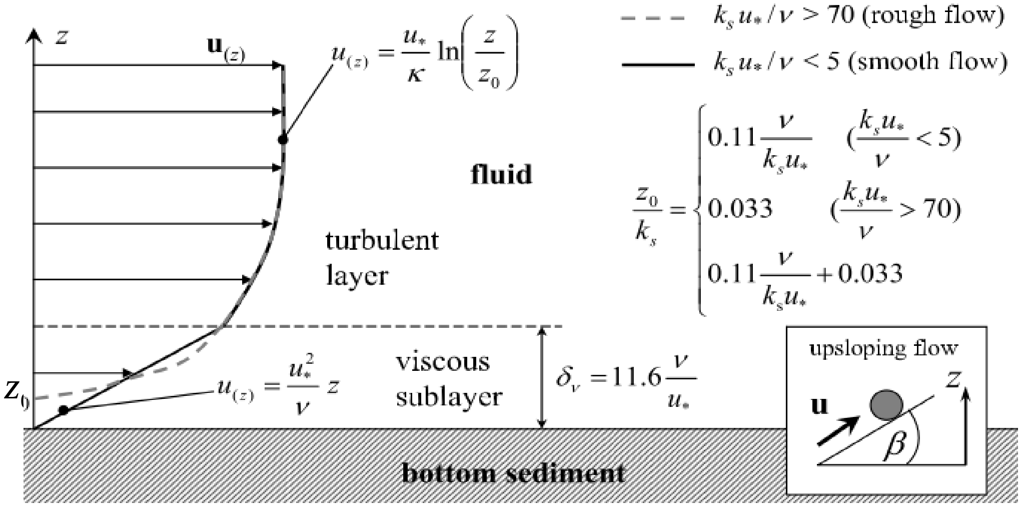
# Current developments

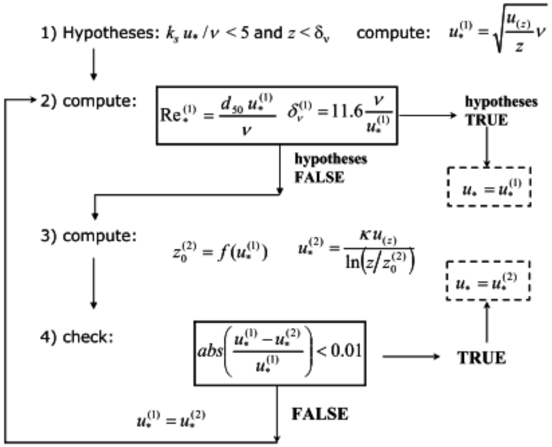
(Zubeldia et al, 2016)

## Shields erosion criterion

$$\boxed{\theta_{cr} = \frac{\tau_{b,cr,0}}{(\rho_s - \rho)gd} = f(\mathrm{Re}_*)}$$

$$\mathrm{Re}_* = du_*/\nu^w$$



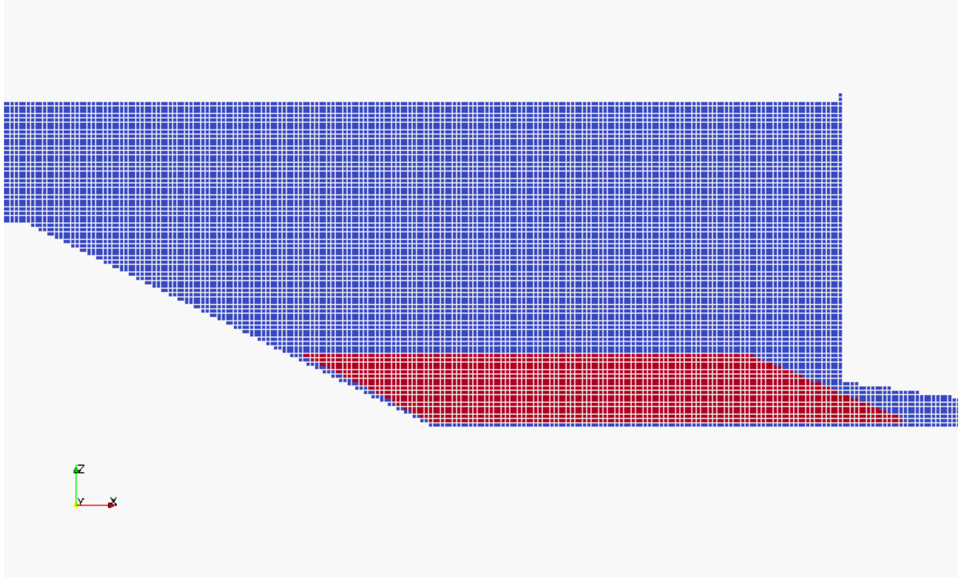


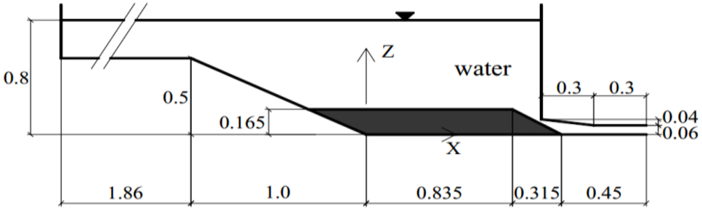
Calulate $(u_*)$

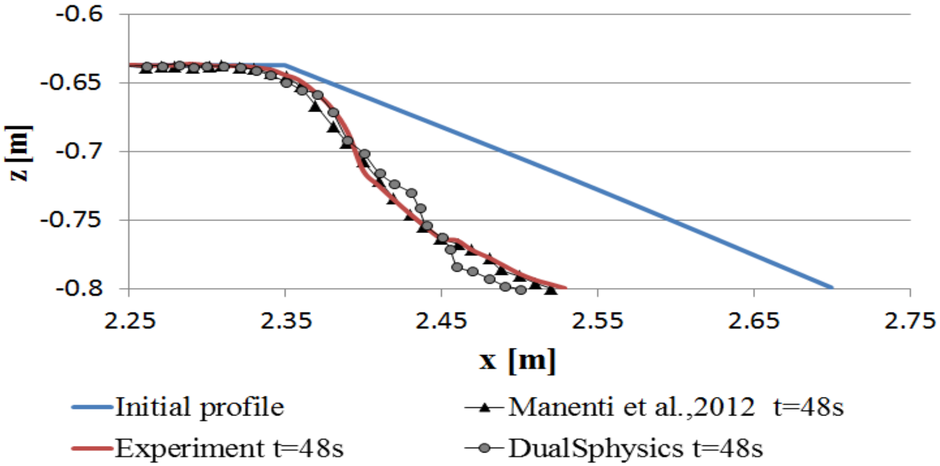
$$u_* \rightarrow \mathrm{Re}_* \rightarrow \theta_{cr} \rightarrow \tau_{b,cr,0}$$

# Current developments

(Zubeldia et al, 2016)







# Conclusions

- A novel sediment model has been presented and implemented in DualSPHysics with improvements to the yielding, shear layer constitutive modelling and sediment resuspension
- Good speed up characteristics achieved by the multi-phase GPU implementation (x58)
- The 2-D and 3-D results where in good agreement with the experimental data especially for the 3-D case:
  - The sediment profile at different locations
  - The water level elevation at the probe locations
- Future work
  - Inclusion of more physics, Shield's criterion (in progress)
  - Turbulence modelling (cheaper mixing length / RANS model) (Next development)

Executable available in DualSPHysics package!!!

# Thank you

## Acknowledgments

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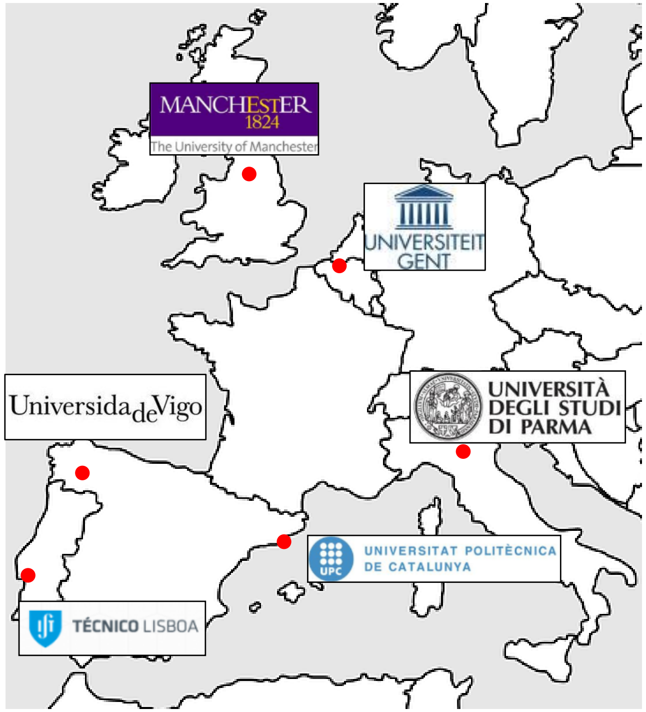
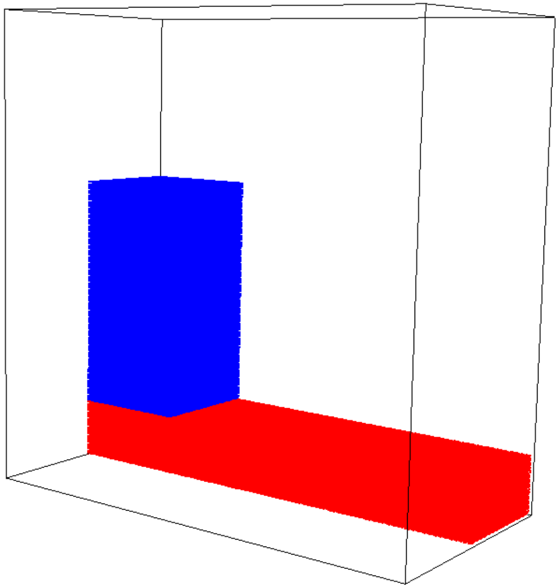
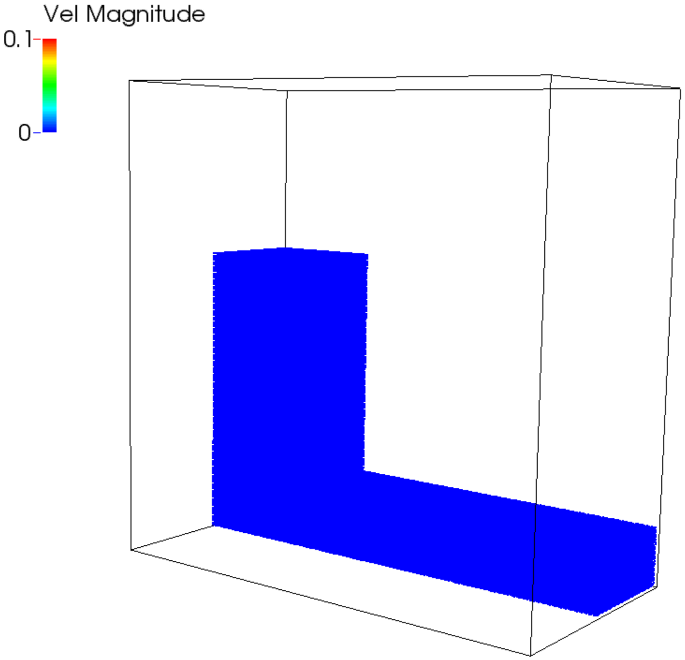
- U-Vigo: Jose Dominguez, Alex Crespo, Anxo Barreiro, Moncho Gomez-Gesteira
- U-Parma: Renato Vacondio

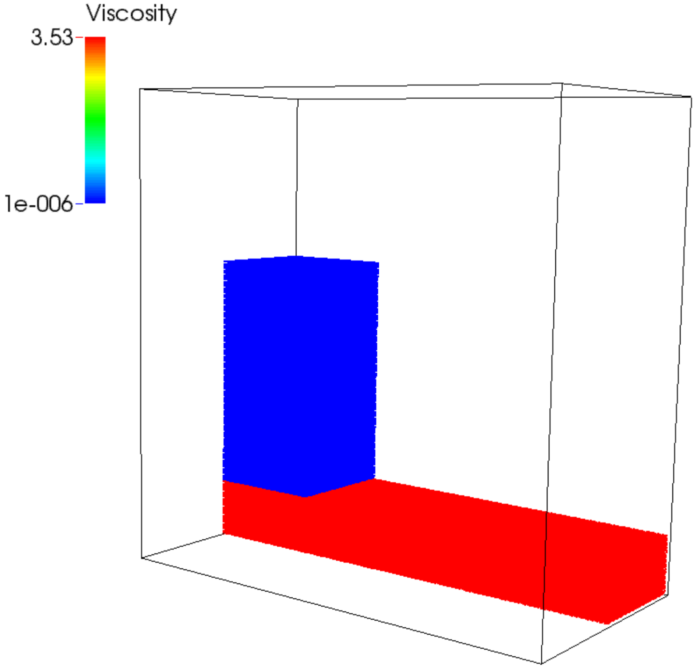
## Websites

- http://www.dual.sphysics.org/
- https://wiki.manchester.ac.uk/sphysics
- http://www.mace.manchester.ac.uk/...sph






Vel Magnitude
0.1
0


Viscosity
3.53
1e-006